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CAMBRIDGE
International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

2270653792

FURTHER MATHEMATICS **9231/23**

Paper 2 Further Pure Mathematics 2 **May/June 2026**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen. Do **not** use correction fluid or tape.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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[Turn over

1 Find the roots of the equation $w^5 + 32 = 0$, giving your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi \leqslant \theta < \pi$. [5]

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2 A curve has equation $y = \sqrt{x}$, for $0 \leqslant x \leqslant 2$. Find, in terms of π, the area of the surface generated when the curve is rotated through 2π radians about the x-axis. [5]

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3 Let $I_n = \int_0^a \left(\ln(x+1)\right)^n \mathrm{d}x$, where n is a non-negative integer and $a > 0$.

(a) By considering the derivative of $(x+1)\left(\ln(x+1)\right)^n$, or otherwise, show that for $n \geqslant 1$

$$I_n = (a+1)\left(\ln(a+1)\right)^n - nI_{n-1}.$$

[3]

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(b) Find I_2 in terms of a. [3]

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4 Find the particular solution of the differential equation

$$49\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 14\frac{\mathrm{d}x}{\mathrm{d}t} + x = 196\mathrm{e}^{\frac{1}{7}t},$$

given that, when $t = 0$, $x = 7$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 0$. [10]

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5 **(a)** Starting from the definitions of sech and tanh in terms of exponentials, prove that

$$1 - \operatorname{sech}^2 t = \tanh^2 t.$$

[3]

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It is given that

$$x = t\sinh t \qquad \text{and} \qquad y = t\cosh t.$$

(b) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{t\tanh t + 1}{t + \tanh t}$. [3]

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(c) Show that $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = \dfrac{\left(t^2-2\right)\mathrm{sech}^n t}{(t+\tanh t)^n}$, where n is an integer to be determined. [6]

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6 **(a)** Show that an appropriate integrating factor for

$$\left(1-x^2\right)\frac{\mathrm{d}y}{\mathrm{d}x}+y=\left(1-x^2\right)\sqrt{1-x}$$

is $\sqrt{\frac{1+x}{1-x}}$. [4]

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(b) Hence find the solution of the differential equation

$$\left(1-x^2\right)\frac{\mathrm{d}y}{\mathrm{d}x}+y=\left(1-x^2\right)\sqrt{1-x}$$

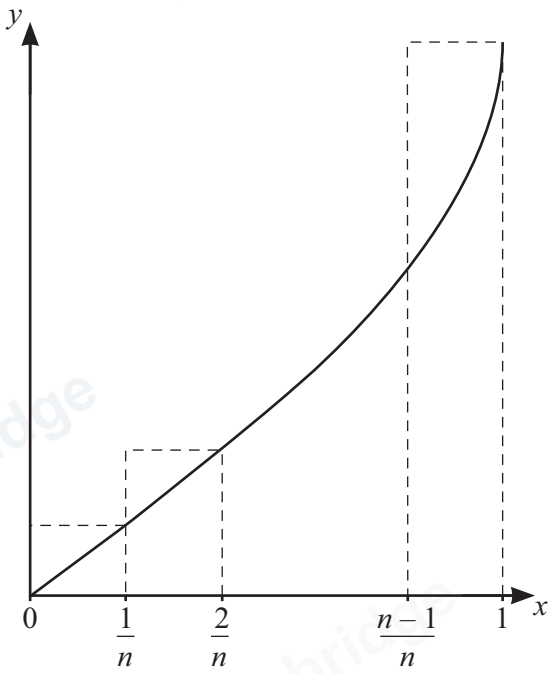
for which $y = 1$ when $x = 0$. [6]

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7 **(a)** Use the substitution $u = 1 - x^2$ to find $\int \frac{x}{\sqrt{1-x^2}} \mathrm{d}x$. [3]



The diagram shows the curve with equation for $y = \sin^{-1} x$ for $0 \leqslant x \leqslant 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(b) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^{n} \frac{1}{n} \sin^{-1}\left(\frac{r}{n}\right) \geqslant \tfrac{1}{2}\pi - 1.$$

[5]

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(c) Use a similar method to find, in terms of n, an upper bound for $\sum_{r=1}^{n} \frac{1}{n} \sin^{-1}\left(\frac{r}{n}\right)$. [3]

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(d) Deduce the exact value of $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n} \sin^{-1}\left(\frac{r}{n}\right)$. [1]

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8 **(a)** The constants a and b are such that the system of equations

$$\begin{aligned} 6x - 12y - 12z &= 1, \\ ax + 4y + z &= 2, \\ 2x + by - 8z &= 3, \end{aligned}$$

does not have a unique solution. Find a in terms of b. [3]

(b) Given that $a = -\frac{1}{4}$ and $b = -32$, show that the system of equations in part **(a)** is inconsistent. Interpret this situation geometrically. [3]

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The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 6 & -12 & -12 \\ -1 & 4 & 1 \\ 2 & -8 & -8 \end{pmatrix},$$

(c) Show that $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ is an eigenvector of **A** and state the corresponding eigenvalue. [2]

(d) Find a matrix **P** and a diagonal matrix **D** such that $\mathbf{A} = \mathbf{PDP}^{-1}$. [7]

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