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CAMBRIDGE
International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

3205142238

FURTHER MATHEMATICS **9231/24**

Paper 2 Further Pure Mathematics 2 **May/June 2026**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen. Do **not** use correction fluid or tape.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

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1 **(a)** Find the set of values of k for which the system of equations

$$\begin{aligned} 3x + ky + 7z &= 1, \\ 8x + \ y + 2z &= 2, \\ 8x + 2y + 4z &= 3, \end{aligned}$$

has a unique solution and interpret this situation geometrically. [4]

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(b) Given that $k = \frac{7}{2}$, show that the system of equations in part **1(a)** is inconsistent and interpret this situation geometrically. [2]

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2 Let $\mathbf{A} = \begin{pmatrix} a & -1 \\ 5 & 1 \end{pmatrix}$, where a is a constant. It is given that 2 is an eigenvalue of $\mathbf{A}$.

(a) Find a and the other eigenvalue of $\mathbf{A}$. [4]

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(b) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^5 = \mathbf{PDP}^{-1}$. [5]

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3 Find the particular solution of the differential equation

$$\frac{dy}{dx} + y = e^{\frac{1}{2}x},$$

given that $y = 3$ when $x = 0$. Give your answer in the form $y = f(x)$. [7]

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4 The curve C has parametric equations

$$x = 2\cosh 2t + 3t, \quad y = \frac{3}{2}\cosh 2t - 4t, \quad \text{for } 0 \leqslant t \leqslant 1.$$

(a) Find, in terms of e, the length of C. [5]

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(b) Show that

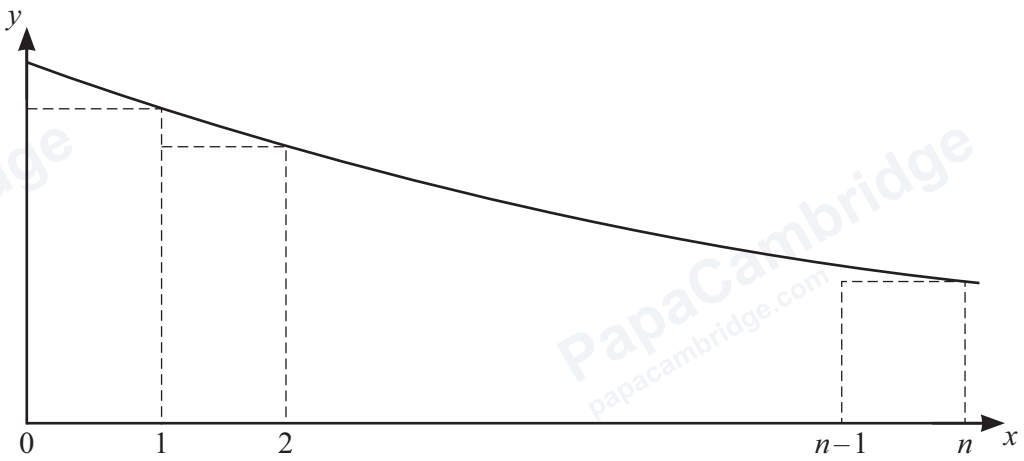
$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{a\cosh 2t}{\left(b\sinh 2t + c\right)^n},$$

where a, b, c and n are integers to be determined. [5]

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The diagram shows the curve $y = \dfrac{1}{\sqrt{4x^2+4x+3}}$ for $x \geqslant 0$, together with a set of n rectangles of unit width. By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^{n} \frac{1}{\sqrt{4r^2+4r+3}} < \tfrac{1}{2}\ln\left(\frac{1+2n+\sqrt{4n^2+4n+3}}{1+\sqrt{3}}\right). \qquad [9]$$

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6 Let $I_n = \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} e^x \cos^n x \, dx$, where n is a non-negative integer.

(a) Show that, for $n \geqslant 2$,

$$I_n = n(n-1)\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} e^x \cos^{n-2} x \sin^2 x \, dx - nI_n,$$

and deduce the reduction formula

$$(n^2+1)I_n = n(n-1)I_{n-2}. \qquad [6]$$

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(b) Find the exact value of I_4. [4]

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7 **(a)** Use de Moivre's theorem to show that

$$\sec 7\theta = \frac{\sec^7\theta}{64 - 112\sec^2\theta + 56\sec^4\theta - 7\sec^6\theta}.$$ [6]

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(b) Hence obtain the roots of the equation

$$x^7 - 14x^6 + 112x^4 - 224x^2 + 128 = 0$$

in the form $\sec q\pi$, where q is rational. [5]

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8 It is given that $v = xy^3$ and

$$xy^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2xy\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 + (3x+2)y^2\frac{\mathrm{d}y}{\mathrm{d}x} + (x+1)y^3 = x+5.$$

(a) Show that

$$\frac{1}{3}\frac{\mathrm{d}^2v}{\mathrm{d}x^2} + \frac{\mathrm{d}v}{\mathrm{d}x} + v = x+5. \qquad [4]$$

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(b) Find the general solution for y in terms of x. [7]

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(c) Deduce that, for large positive values of x and for any initial conditions,

$$y \approx \sqrt[3]{a+\frac{b}{x}},$$

where the constants a and b are to be determined. [2]

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Additional page

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