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CAMBRIDGE
International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 1 6 7 9 8 3 4 3 5 0 *

FURTHER MATHEMATICS **9231/41**

Paper 4 Further Probability & Statistics **May/June 2026**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen. Do **not** use correction fluid or tape.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 The manager of a village store is interested in the shopping habits of his customers. From previous records, the median number of items bought per customer was found to be 8. The manager believes the median number of items bought per customer has increased. He selects a random sample of 15 customers and records the number of items each customer buys at his store. The results are shown in the stem-and-leaf diagram.

key

2	3	3 represents 23 items

0	6	6	7	9	9
1	0	5	7	7	
2	3	6			
3	4				
4	0				
5	6				
6					
7	4				

(a) Suggest a reason why using a Wilcoxon signed-rank test to examine the manager's belief might **not** be appropriate in this situation. [1]

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(b) Use a sign test at the 5% significance level to investigate the manager's belief. [6]

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2 Three different bus companies, *A*, *B* and *C*, record the reliability of their service. Each bus journey is recorded as being on time, late or cancelled. A reliable bus service is considered to be one that is on time. The records from a random sample of buses for each company are taken. These records are summarised in the table.

	company *A*	**company *B***	**company *C***	**total**
on time	7	22	20	49
late	14	11	12	37
cancelled	4	6	4	14
total	25	39	36	100

(a) Test at the 5% significance level whether the reliability of the service is independent of the bus company. [8]

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(b) Further evidence indicates that one company may be particularly unreliable compared with the other two companies. Use your calculations in **2(a)** to suggest which company this is likely to be. [2]

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3 Students studying geography at a college choose either human geography or physical geography. A teacher is investigating the times taken by students to complete assignments of equal difficulty in human geography and physical geography. He selects a random sample of 7 human geography students and records the times taken, x minutes, to complete an assignment. He also selects a random sample of 9 physical geography students and records the times taken, y minutes, to complete an assignment. The results are summarised as follows.

$$\Sigma x = 445 \qquad \Sigma x^2 = 29144 \qquad \Sigma y = 310 \qquad \Sigma y^2 = 11622$$

You should assume that the underlying distributions are normal and have equal population variances.

Find a 95% confidence interval for the difference in population mean times taken to complete assignments in human geography and physical geography. [6]

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4 The continuous random variable X has cumulative distribution function F given by

$$\mathrm{F}(x) = \begin{cases} 0 & x < 0, \\ -\frac{1}{108}x^3 + \frac{1}{12}x^2 & 0 \leqslant x \leqslant 3, \\ -\frac{1}{32}x^2 + \frac{7}{16}x + a & 3 < x \leqslant b, \\ 1 & x > b. \end{cases}$$

(a) Verify that the median of X is 3. [1]

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(b) Show that $a = -\frac{17}{32}$ and find the value of b. [3]

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(c) Find the probability density function of X. [2]

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(d) Find $\mathrm{E}\left(\frac{1}{X}\right)$. [3]

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5 Margaret has developed a new revision course to help students improve their ability in mathematics. The students each sit a mathematics test before attending the course and a test of similar difficulty after attending the course. Margaret claims that her course will increase a student's score by more than 5 marks on average. For a random sample of 8 students, the test scores before and after the revision course are given in the table.

student	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
test score before	43	29	67	54	34	29	52	60
test score after	49	53	79	62	39	55	65	62

(a) Use a *t*-test at the 10% significance level to investigate Margaret's claim. [7]

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(b) State an assumption that is necessary for the test in **5(a)** to be valid. [1]

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6 The discrete random variable X has probability generating function $G_X(t)$ defined by

$$G_X(t) = \frac{k}{t}(at+1)^2.$$

It is given that $E(X) = \frac{1}{3}$.

(a) Show that $a = 2$ and find the value of k. [5]

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(b) Find $\mathrm{Var}(X)$. [2]

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(c) Three independent observations of X are taken. The random variable Y is the sum of these observations.

Find $\mathrm{P}(Y = 0)$. [3]

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