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Cambridge International AS & A Level

CANDIDATE NAME

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8034282211

FURTHER MATHEMATICS **9231/44**

Paper 4 Further Probability & Statistics **May/June 2026**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen. Do **not** use correction fluid or tape.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

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1 A random sample of 40 observations of a random variable X and a random sample of 60 observations of a random variable Y are taken. The results are summarised as follows.

$$\sum x = 67.1 \qquad \sum (x-\overline{x})^2 = 934.3 \qquad \sum y = 84.1 \qquad \sum (y-\overline{y})^2 = 756.2$$

Find a 95% confidence interval for the difference between the population means of X and Y. [6]

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2 The probability generating function $G_X(t)$ of a random variable X is given by

$$G_X(t) = \tfrac{1}{6}(t+1)(t^2+2).$$

(a) Use $G_X(t)$ to find $E(X)$ and $\mathrm{Var}(X)$. [4]

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(b) The random variable Y is the sum of three independent observations of X.

Find $P(Y \leqslant 2)$. [3]

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3 A biologist believes that the masses of a particular species of frog are normally distributed. He randomly selects 120 frogs of this species and records the mass, x, of each frog in grams. The results are summarised as follows.

mass, x	$x < 210$	$210 \leqslant x < 220$	$220 \leqslant x < 230$	$230 \leqslant x < 240$	$240 \leqslant x < 250$	$250 \leqslant x < 260$	$x \geqslant 260$
observed frequency	2	9	20	39	40	8	2

The biologist uses the masses of these 120 frogs to calculate unbiased estimates of the mean and variance of the underlying population. He calculates these as 236.5 g and 134.56 g^2 respectively.

Most of the relevant expected frequencies, correct to 2 decimal places, are given in the following table. The expected frequencies have been calculated using the biologist's model.

mass, x	$x < 210$	$210 \leqslant x < 220$	$220 \leqslant x < 230$	$230 \leqslant x < 240$	$240 \leqslant x < 250$	$250 \leqslant x < 260$	$x \geqslant 260$
expected frequency	1.34	7.94	25.22	39.73	31.11	c	d

(a) Find the value of c and the value of d. Give your answers correct to two decimal places. [2]

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(b) Carry out a goodness of fit test at the 5% significance level to investigate whether a normal distribution is a suitable model for the data. [7]

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4 Members of a cycling club use the same circular route every week. Throughout the year, the members change between completing the route in a clockwise direction one week and completing the route in an anticlockwise direction the following week.

The club captain believes that the anticlockwise route takes longer to complete on average than the clockwise route. To test this belief, the club captain selects a random sample of 10 members. Each of the 10 club members gives their personal best times, in minutes, for completing the route in both directions over the previous year. These times are as follows.

club member	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
clockwise	127.1	129.1	139.3	151.4	142.5	123.6	153.4	129.7	140.9	137.4
anticlockwise	131.0	132.1	147.3	150.0	144.3	127.3	153.5	129.9	140.6	140.3

(a) Explain the advantage of using paired samples to investigate the club captain's belief. [1]

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(b) Carry out a Wilcoxon matched-pairs signed-rank test at the 5% significance level to test the club captain's belief. [6]

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(c) State an assumption that is necessary for the test in **4(b)** to be valid. [1]

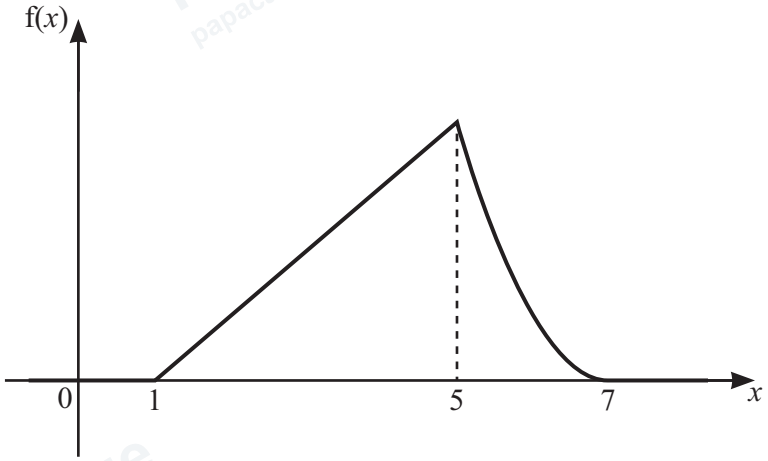
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The diagram shows the continuous random variable X with probability density function f given by

$$f(x) = \begin{cases} a(x-1) & 1 \leqslant x \leqslant 5, \\ b(x-7)^2 & 5 \leqslant x \leqslant 7, \\ 0 & \text{otherwise.} \end{cases}$$

The upper quartile of X is equal to 5.

(a) Show that $a = \frac{3}{32}$ and $b = \frac{3}{32}$. [3]

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(b) Find the cumulative distribution function of X. [3]

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(c) The random variable Y is defined by $Y = 10 - X$.

Find the cumulative distribution function of Y. [4]

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6 A researcher is studying household waste recycling in two different cities, city *A* and city *B*. She randomly selects 8 households in city *A* and 9 households in city *B*. The researcher records the percentage of waste that each household recycles. Her results are as follows.

City *A*	25	49	41	38	31	21	25	40	
City *B*	18	35	30	22	13	23	24	36	24

The population variances of the percentages of waste recycled in each city are assumed to be equal.

(a) Carry out a *t*-test at the 10% significance level to investigate whether there is a difference in the average percentage of waste that is recycled in city *A* and city *B*. [9]

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(b) State another assumption that is necessary for the test in **6(a)** to be valid. [1]

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Additional page

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