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Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2016**

1 hour 45 minutes

Additional Materials: List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **4** printed pages and **1** insert.

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1 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 10 - 3x, \quad x \in \mathbb{R},$$
$$\mathrm{g} : x \mapsto \frac{10}{3 - 2x}, \quad x \in \mathbb{R},\ x \neq \tfrac{3}{2}.$$

Solve the equation $\mathrm{ff}(x) = \mathrm{gf}(2)$. [3]

2 A curve is such that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{8}{(5 - 2x)^2}$. Given that the curve passes through $(2, 7)$, find the equation of the curve. [4]

3 Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = 2\mathbf{i} - 5\mathbf{j} - 2\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 4\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}.$$

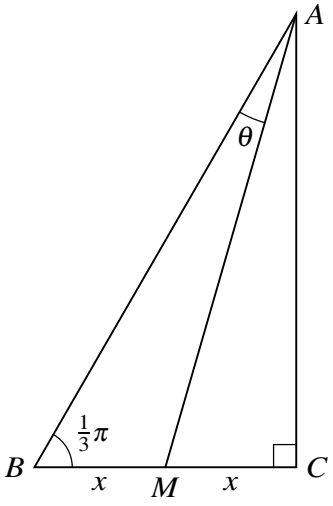
The point C is such that $\overrightarrow{AB} = \overrightarrow{BC}$. Find the unit vector in the direction of $\overrightarrow{OC}$. [4]

4 Find the term that is independent of x in the expansion of

(i) $\left(x - \frac{2}{x}\right)^6$, [2]

(ii) $\left(2 + \frac{3}{x^2}\right)\left(x - \frac{2}{x}\right)^6$. [4]

5



In the diagram, triangle ABC is right-angled at C and M is the mid-point of BC. It is given that angle $ABC = \frac{1}{3}\pi$ radians and angle $BAM = \theta$ radians. Denoting the lengths of BM and MC by x,

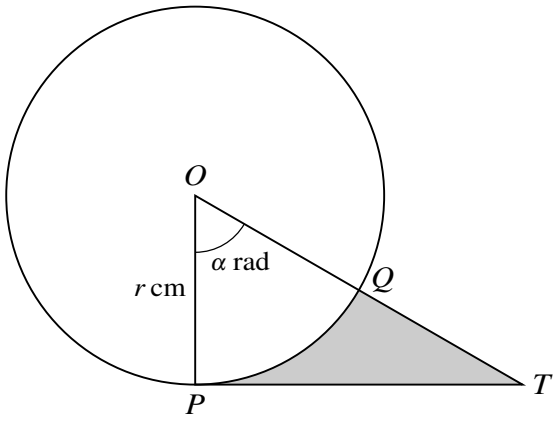
(i) find AM in terms of x, [3]

(ii) show that $\theta = \frac{1}{6}\pi - \tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$. [2]

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The diagram shows a circle with radius r cm and centre O. The line PT is the tangent to the circle at P and angle $POT = \alpha$ radians. The line OT meets the circle at Q.

(i) Express the perimeter of the shaded region PQT in terms of r and α. [3]

(ii) In the case where $\alpha = \frac{1}{3}\pi$ and $r = 10$, find the area of the shaded region correct to 2 significant figures. [3]

7 **(i)** Prove the identity $\dfrac{1+\cos\theta}{1-\cos\theta} - \dfrac{1-\cos\theta}{1+\cos\theta} \equiv \dfrac{4}{\sin\theta\tan\theta}$. [4]

(ii) Hence solve, for $0° < \theta < 360°$, the equation

$$\sin\theta\left(\frac{1+\cos\theta}{1-\cos\theta} - \frac{1-\cos\theta}{1+\cos\theta}\right) = 3.$$ [3]

8 Three points have coordinates $A\,(0,\ 7)$, $B\,(8,\ 3)$ and $C\,(3k,\ k)$. Find the value of the constant k for which

(i) C lies on the line that passes through A and B, [4]

(ii) C lies on the perpendicular bisector of AB. [4]

9 A water tank holds 2000 litres when full. A small hole in the base is gradually getting bigger so that each day a greater amount of water is lost.

(i) On the first day after filling, 10 litres of water are lost and this increases by 2 litres each day.

(a) How many litres will be lost on the 30th day after filling? [2]

(b) The tank becomes empty during the nth day after filling. Find the value of n. [3]

(ii) Assume instead that 10 litres of water are lost on the first day and that the amount of water lost increases by 10% on each succeeding day. Find what percentage of the original 2000 litres is left in the tank at the end of the 30th day after filling. [4]

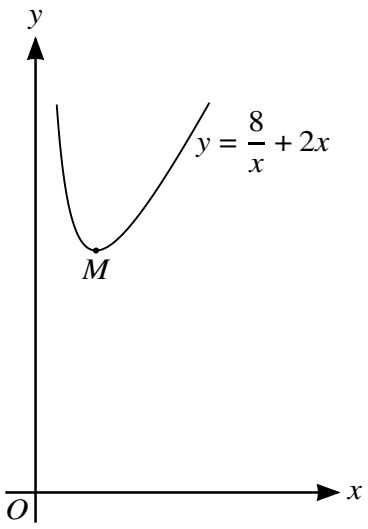
[Questions 10 and 11 are printed on the next page.]

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[Turn over

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The diagram shows the part of the curve $y = \frac{8}{x} + 2x$ for $x > 0$, and the minimum point M.

(i) Find expressions for $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\int y^2 \, dx$. [5]

(ii) Find the coordinates of M and determine the coordinates and nature of the stationary point on the part of the curve for which $x < 0$. [5]

(iii) Find the volume obtained when the region bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$ is rotated through 360° about the x-axis. [2]

11 The function f is defined by $f : x \mapsto 6x - x^2 - 5$ for $x \in \mathbb{R}$.

(i) Find the set of values of x for which $f(x) \leqslant 3$. [3]

(ii) Given that the line $y = mx + c$ is a tangent to the curve $y = f(x)$, show that $4c = m^2 - 12m + 16$. [3]

The function g is defined by $g : x \mapsto 6x - x^2 - 5$ for $x \geqslant k$, where k is a constant.

(iii) Express $6x - x^2 - 5$ in the form $a - (x - b)^2$, where a and b are constants. [2]

(iv) State the smallest value of k for which g has an inverse. [1]

(v) For this value of k, find an expression for $g^{-1}(x)$. [2]

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