GradeMax

Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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7232044465

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

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1 An arithmetic progression has first term -12 and common difference 6. The sum of the first n terms exceeds 3000. Calculate the least possible value of n. [4]

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2 Find the set of values of a for which the curve $y = -\frac{2}{x}$ and the straight line $y = ax + 3a$ meet at two distinct points. [4]

3 **(i)** Find the term independent of x in the expansion of $\left(\frac{2}{x} - 3x\right)^6$. [2]

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(ii) Find the value of a for which there is no term independent of x in the expansion of

$$(1 + ax^2)\left(\frac{2}{x} - 3x\right)^6.$$ [3]

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4 The function f is such that $f(x) = (2x - 1)^{\frac{3}{2}} - 6x$ for $\frac{1}{2} < x < k$, where k is a constant. Find the largest value of k for which f is a decreasing function. [5]

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5 **(i)** Show that the equation $\frac{\cos\theta + 4}{\sin\theta + 1} + 5\sin\theta - 5 = 0$ may be expressed as $5\cos^2\theta - \cos\theta - 4 = 0$. [3]

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(ii) Hence solve the equation $\frac{\cos\theta + 4}{\sin\theta + 1} + 5\sin\theta - 5 = 0$ for $0° \leqslant \theta \leqslant 360°$. [4]

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6 The functions f and g are defined by

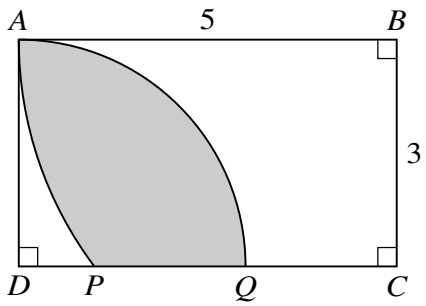
$$\mathrm{f}(x) = \frac{2}{x^2 - 1} \text{ for } x < -1,$$
$$\mathrm{g}(x) = x^2 + 1 \text{ for } x > 0.$$

(i) Find an expression for $\mathrm{f}^{-1}(x)$. [3]

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(ii) Solve the equation $\mathrm{gf}(x) = 5$. [4]

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The diagram shows a rectangle $ABCD$ in which $AB = 5$ units and $BC = 3$ units. Point P lies on DC and AP is an arc of a circle with centre B. Point Q lies on DC and AQ is an arc of a circle with centre D.

(i) Show that angle $ABP = 0.6435$ radians, correct to 4 decimal places. [1]

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(ii) Calculate the areas of the sectors BAP and DAQ. [3]

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(iii) Calculate the area of the shaded region. [3]

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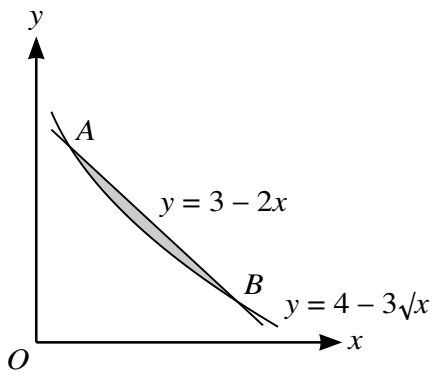
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The diagram shows parts of the graphs of $y = 3 - 2x$ and $y = 4 - 3\sqrt{x}$ intersecting at points A and B.

(i) Find by calculation the x-coordinates of A and B. [3]

(ii) Find, showing all necessary working, the area of the shaded region. [5]

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9 Relative to an origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 8 \\ -6 \\ 5 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} -10 \\ 3 \\ -13 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}.$$

A fourth point, D, is such that the magnitudes $|\overrightarrow{AB}|$, $|\overrightarrow{BC}|$ and $|\overrightarrow{CD}|$ are the first, second and third terms respectively of a geometric progression.

(i) Find the magnitudes $|\overrightarrow{AB}|$, $|\overrightarrow{BC}|$ and $|\overrightarrow{CD}|$. [5]

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(ii) Given that D is a point lying on the line through B and C, find the two possible position vectors of the point D. [4]

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10 A curve has equation $y = f(x)$ and it is given that $f'(x) = ax^2 + bx$, where a and b are positive constants.

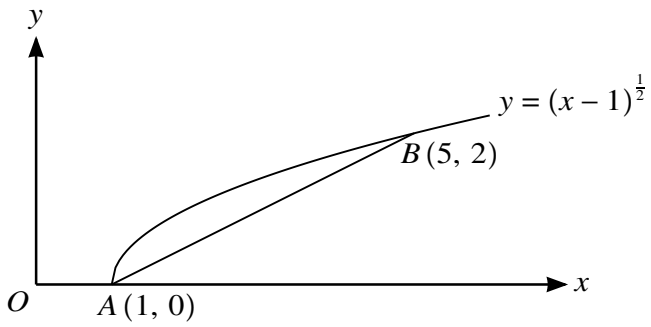
(i) Find, in terms of a and b, the non-zero value of x for which the curve has a stationary point and determine, showing all necessary working, the nature of the stationary point. [3]

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(ii) It is now given that the curve has a stationary point at $(-2, -3)$ and that the gradient of the curve at $x = 1$ is 9. Find $f(x)$. [6]

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The diagram shows the curve $y = (x - 1)^{\frac{1}{2}}$ and points $A\,(1, 0)$ and $B\,(5, 2)$ lying on the curve.

(i) Find the equation of the line AB, giving your answer in the form $y = mx + c$. [2]

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(ii) Find, showing all necessary working, the equation of the tangent to the curve which is parallel to AB. [5]

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(iii) Find the perpendicular distance between the line AB and the tangent parallel to AB. Give your answer correct to 2 decimal places. [3]

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