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Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

CANDIDATE NUMBER | | | | |

6271108655

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **18** printed pages and **2** blank pages.

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1 Find the coefficient of x^3 in the expansion of $(3 - x)(1 + 3x)^{\frac{1}{3}}$ in ascending powers of x. [4]

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2 Showing all necessary working, solve the equation $9^x = 3^x + 12$. Give your answer correct to 2 decimal places. [4]

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3 Showing all necessary working, solve the equation $\cot 2\theta = 2\tan\theta$ for $0° < \theta < 180°$. [5]

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4 Find the exact coordinates of the point on the curve $y = \dfrac{x}{1 + \ln x}$ at which the gradient of the tangent is equal to $\frac{1}{4}$. [7]

5 Throughout this question the use of a calculator is not permitted.

It is given that the complex number $-1 + (\sqrt{3})\mathrm{i}$ is a root of the equation

$$kx^3 + 5x^2 + 10x + 4 = 0,$$

where k is a real constant.

(i) Write down another root of the equation. [1]

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(ii) Find the value of k and the third root of the equation. [6]

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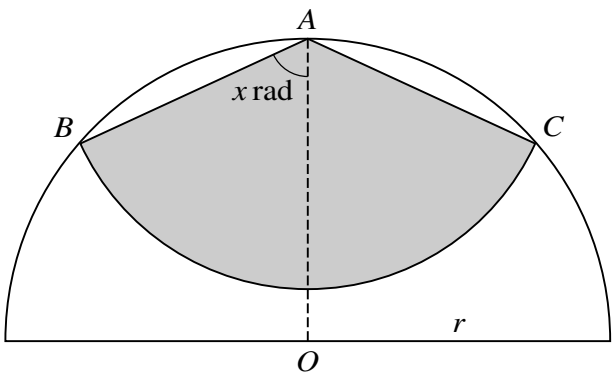
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In the diagram, A is the mid-point of the semicircle with centre O and radius r. A circular arc with centre A meets the semicircle at B and C. The angle OAB is equal to x radians. The area of the shaded region bounded by AB, AC and the arc with centre A is equal to half the area of the semicircle.

(i) Use triangle OAB to show that $AB = 2r\cos x$. [1]

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(ii) Hence show that $x = \cos^{-1}\sqrt{\left(\frac{\pi}{16x}\right)}$. [2]

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(iii) Verify by calculation that x lies between 1 and 1.5. [2]

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(iv) Use an iterative formula based on the equation in part **(ii)** to determine x correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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7 The variables x and y satisfy the differential equation $\frac{dy}{dx} = xe^{x+y}$. It is given that $y = 0$ when $x = 0$.

(i) Solve the differential equation, obtaining y in terms of x. [7]

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(ii) Explain why x can only take values that are less than 1. [1]

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8 Let $\mathrm{f}(x) = \dfrac{10x + 9}{(2x + 1)(2x + 3)^2}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

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(ii) Hence show that $\int_0^1 \mathrm{f}(x)\,\mathrm{d}x = \frac{1}{2}\ln\frac{9}{5} + \frac{1}{5}$. [5]

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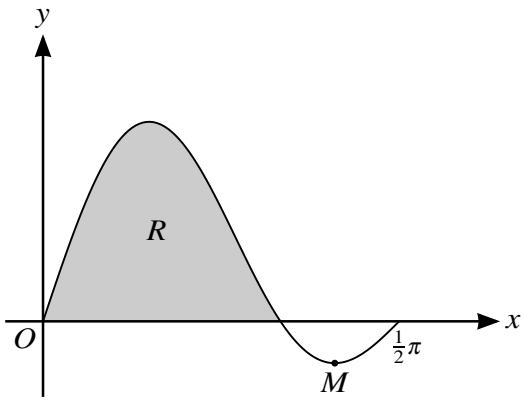
9 The points A and B have position vectors $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} + \mathbf{j} + \mathbf{k}$ respectively. The line l has equation $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + \mathbf{j} + 2\mathbf{k})$.

(i) Show that l does not intersect the line passing through A and B. [5]

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(ii) The plane m is perpendicular to AB and passes through the mid-point of AB. The plane m intersects the line l at the point P. Find the equation of m and the position vector of P. [5]

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The diagram shows the curve $y = \sin 3x \cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$ and its minimum point M. The shaded region R is bounded by the curve and the x-axis.

(i) By expanding $\sin(3x + x)$ and $\sin(3x - x)$ show that

$$\sin 3x \cos x = \tfrac{1}{2}(\sin 4x + \sin 2x).$$ [3]

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(ii) Using the result of part **(i)** and showing all necessary working, find the exact area of the region R. [4]

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(iii) Using the result of part **(i)**, express $\frac{dy}{dx}$ in terms of $\cos 2x$ and hence find the x-coordinate of M, giving your answer correct to 2 decimal places. [5]

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Additional Page

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