GradeMax

Cambridge International AS & A Level

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

CANDIDATE NUMBER | | | | |

3117579859

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **20** printed pages.

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1 **(i)** Expand $(1 + y)^6$ in ascending powers of y as far as the term in y^2. [1]

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(ii) In the expansion of $\left(1 + (px - 2x^2)\right)^6$ the coefficient of x^2 is 48. Find the value of the positive constant p. [3]

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2 The function g is defined by $g(x) = x^2 - 6x + 7$ for $x > 4$. By first completing the square, find an expression for $g^{-1}(x)$ and state the domain of g^{-1}. [5]

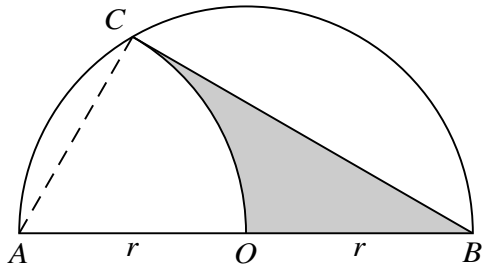
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3 The equation of a curve is $y = x^3 + x^2 - 8x + 7$. The curve has no stationary points in the interval $a < x < b$. Find the least possible value of a and the greatest possible value of b. [4]

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The diagram shows a semicircle ACB with centre O and radius r. Arc OC is part of a circle with centre A.

(i) Express angle CAO in radians in terms of π. [1]

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(ii) Find the area of the shaded region in terms of r, π and $\sqrt{3}$, simplifying your answer. [4]

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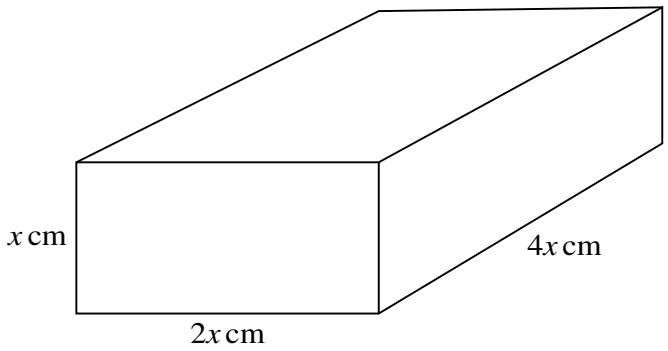
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[Turn over

5



The dimensions of a cuboid are x cm, $2x$ cm and $4x$ cm, as shown in the diagram.

(i) Show that the surface area S cm^2 and the volume V cm^3 are connected by the relation

$$S = 7V^{\frac{2}{3}}.$$ [3]

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(ii) When the volume of the cuboid is $1000\,\text{cm}^3$ the surface area is increasing at $2\,\text{cm}^2\,\text{s}^{-1}$. Find the rate of increase of the volume at this instant. [4]

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6 A line has equation $y = 3kx - 2k$ and a curve has equation $y = x^2 - kx + 2$, where k is a constant.

(i) Find the set of values of k for which the line and curve meet at two distinct points. [4]

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(ii) For each of two particular values of k, the line is a tangent to the curve. Show that these two tangents meet on the x-axis. [3]

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7 **(i)** Show that the equation $3\cos^4\theta + 4\sin^2\theta - 3 = 0$ can be expressed as $3x^2 - 4x + 1 = 0$, where $x = \cos^2\theta$. [2]

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(ii) Hence solve the equation $3\cos^4\theta + 4\sin^2\theta - 3 = 0$ for $0° \leqslant \theta \leqslant 180°$. [5]

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8 A function f is defined for $x > \frac{1}{2}$ and is such that $f'(x) = 3(2x - 1)^{\frac{1}{2}} - 6$.

(i) Find the set of values of x for which f is decreasing. [4]

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(ii) It is now given that $f(1) = -3$. Find $f(x)$. [4]

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9 The first, second and third terms of a geometric progression are $3k$, $5k - 6$ and $6k - 4$, respectively.

(i) Show that k satisfies the equation $7k^2 - 48k + 36 = 0$. [2]

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(ii) Find, showing all necessary working, the exact values of the common ratio corresponding to each of the possible values of k. [4]

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(iii) One of these ratios gives a progression which is convergent. Find the sum to infinity. [2]

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10 Relative to an origin O, the position vectors of the points A, B and X are given by

$$\overrightarrow{OA} = \begin{pmatrix} -8 \\ -4 \\ 2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 10 \\ 2 \\ 11 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OX} = \begin{pmatrix} -2 \\ -2 \\ 5 \end{pmatrix}.$$

(i) Find $\overrightarrow{AX}$ and show that AXB is a straight line. [3]

The position vector of a point C is given by $\overrightarrow{OC} = \begin{pmatrix} 1 \\ -8 \\ 3 \end{pmatrix}$.

(ii) Show that CX is perpendicular to AX. [3]

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(iii) Find the area of triangle ABC. [3]

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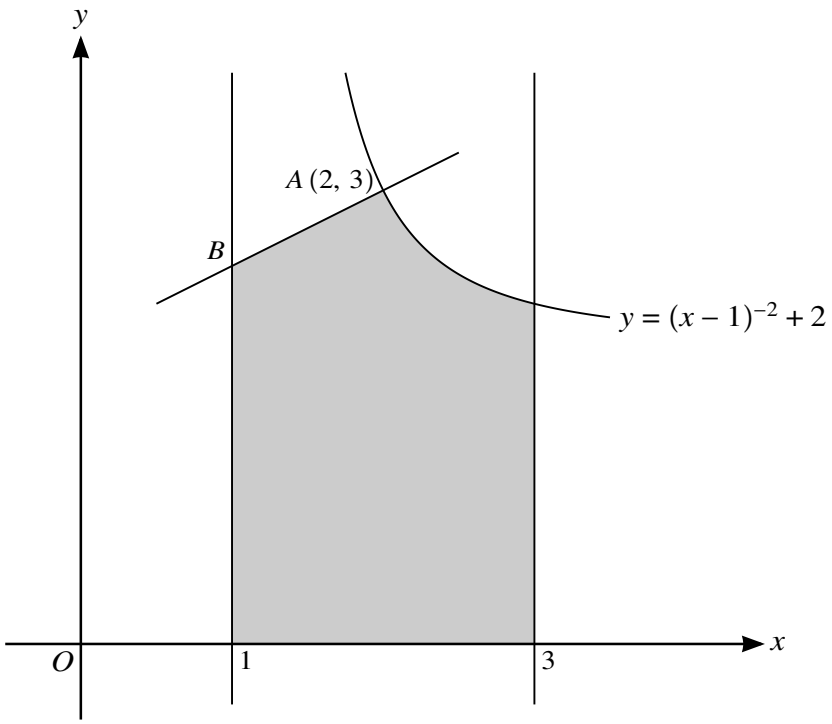
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The diagram shows part of the curve $y = (x - 1)^{-2} + 2$, and the lines $x = 1$ and $x = 3$. The point A on the curve has coordinates (2, 3). The normal to the curve at A crosses the line $x = 1$ at B.

(i) Show that the normal AB has equation $y = \frac{1}{2}x + 2$. [3]

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(ii) Find, showing all necessary working, the volume of revolution obtained when the shaded region is rotated through 360° about the x-axis. [8]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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