GradeMax

Cambridge Assessment International Education

# Cambridge International AS & A Level

| | |
|---|---|
| CANDIDATE NAME | |
| CENTRE NUMBER | |
| CANDIDATE NUMBER | |

* 5 8 9 6 4 2 1 5 1 7 *

**MATHEMATICS** **9709/13**

Paper 1 Pure Mathematics 1 **October/November 2021**

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.


© UCLES 2021

**[Turn over**

GradeMax



**1** The graph of $y = f(x)$ is transformed to the graph of $y = 3 - f(x)$.

Describe fully, in the correct order, the two transformations that have been combined. [4]

© UCLES 2021

GradeMax



**2** **(a)** Find the first three terms, in ascending powers of $x$, in the expansion of $(1 + ax)^6$. [1]

.....

.....

.....

.....

.....

**(b)** Given that the coefficient of $x^2$ in the expansion of $(1 - 3x)(1 + ax)^6$ is $-3$, find the possible values of the constant $a$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

**3** **(a)** Express $5y^2 - 30y + 50$ in the form $5(y + a)^2 + b$, where $a$ and $b$ are constants. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

**(b)** The function f is defined by $f(x) = x^5 - 10x^3 + 50x$ for $x \in \mathbb{R}$.

Determine whether f is an increasing function, a decreasing function or neither. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2021

**4** The first term of an arithmetic progression is 84 and the common difference is −3.

**(a)** Find the smallest value of $n$ for which the $n$th term is negative. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

It is given that the sum of the first $2k$ terms of this progression is equal to the sum of the first $k$ terms.

**(b)** Find the value of $k$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

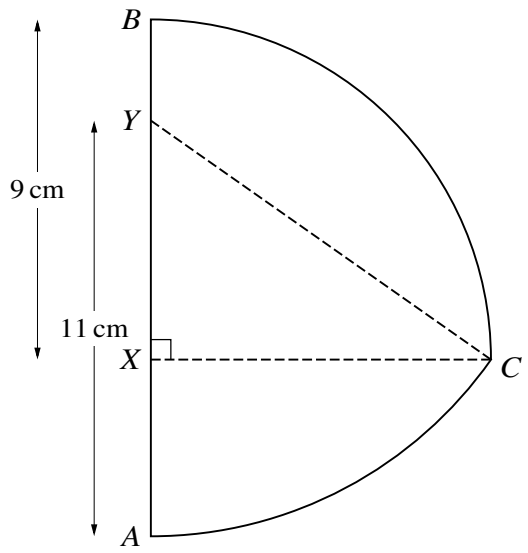
.....

.....

GradeMax



**5**




In the diagram, $X$ and $Y$ are points on the line $AB$ such that $BX = 9$ cm and $AY = 11$ cm. Arc $BC$ is part of a circle with centre $X$ and radius 9 cm, where $CX$ is perpendicular to $AB$. Arc $AC$ is part of a circle with centre $Y$ and radius 11 cm.

**(a)** Show that angle $XYC = 0.9582$ radians, correct to 4 significant figures. [1]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

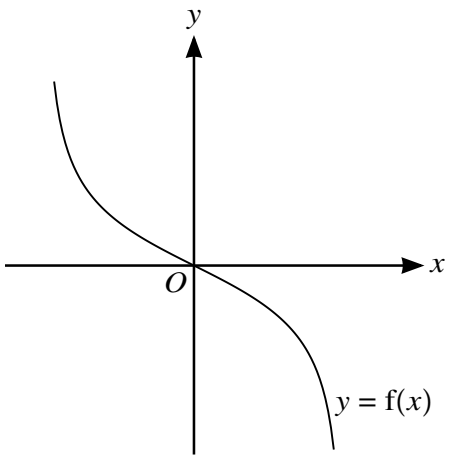
© UCLES 2021

GradeMax



**(b)** Find the perimeter of $ABC$. [6]

**6**




The diagram shows the graph of $y = \mathrm{f}(x)$.

**(a)** On this diagram sketch the graph of $y = \mathrm{f}^{-1}(x)$. [1]

It is now given that $\mathrm{f}(x) = -\dfrac{x}{\sqrt{4 - x^2}}$ where $-2 < x < 2$.

**(b)** Find an expression for $\mathrm{f}^{-1}(x)$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2021

.....

.....

.....

.....

.....

.....

.....

The function g is defined by $g(x) = 2x$ for $-a < x < a$, where $a$ is a constant.

**(c)** State the maximum possible value of $a$ for which fg can be formed. [1]

.....

.....

.....

.....

.....

**(d)** Assuming that fg can be formed, find and simplify an expression for $fg(x)$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

GradeMax

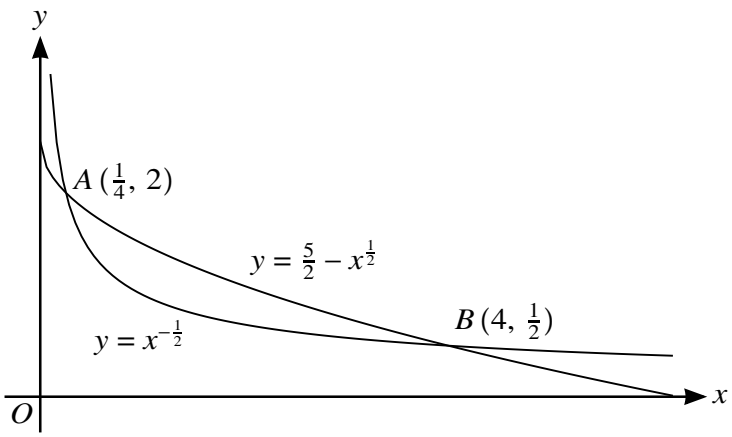


**7** **(a)** Show that the equation $\dfrac{\tan x + \cos x}{\tan x - \cos x} = k$, where $k$ is a constant, can be expressed as

$$(k + 1)\sin^2 x + (k - 1)\sin x - (k + 1) = 0.$$ [4]

**(b)** Hence solve the equation $\frac{\tan x + \cos x}{\tan x - \cos x} = 4$ for $0° \leqslant x \leqslant 360°$. [4]

**8**




The diagram shows the curves with equations $y = x^{-\frac{1}{2}}$ and $y = \frac{5}{2} - x^{\frac{1}{2}}$. The curves intersect at the points $A\left(\frac{1}{4}, 2\right)$ and $B\left(4, \frac{1}{2}\right)$.

**(a)** Find the area of the region between the two curves. [6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2021

GradeMax



**(b)** The normal to the curve $y = x^{-\frac{1}{2}}$ at the point (1, 1) intersects the $y$-axis at the point $(0, p)$.

Find the value of $p$. [4]

GradeMax



**9** The line $y = 2x + 5$ intersects the circle with equation $x^2 + y^2 = 20$ at $A$ and $B$.

**(a)** Find the coordinates of $A$ and $B$ in surd form and hence find the exact length of the chord $AB$. [7]

A straight line through the point (10, 0) with gradient $m$ is a tangent to the circle.

**(b)** Find the two possible values of $m$. [5]

GradeMax



**10** A curve has equation $y = \mathrm{f}(x)$ and it is given that

$$\mathrm{f}'(x) = \left(\tfrac{1}{2}x + k\right)^{-2} - (1 + k)^{-2},$$

where $k$ is a constant. The curve has a minimum point at $x = 2$.

**(a)** Find $\mathrm{f}''(x)$ in terms of $k$ and $x$, and hence find the set of possible values of $k$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

It is now given that $k = -3$ and the minimum point is at $(2, 3\frac{1}{2})$.

**(b)** Find $\mathrm{f}(x)$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2021

.....

.....

.....

.....

.....

.....

**(c)** Find the coordinates of the other stationary point and determine its nature. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2021

## Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2021

GradeMax



**BLANK PAGE**

GradeMax



**BLANK PAGE**

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.