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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

3233379591

MATHEMATICS **9709/22**

Paper 2 Pure Mathematics 2 **October/November 2021**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

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[Turn over

1 The polynomial $\mathrm{p}(x)$ is defined by

$$\mathrm{p}(x) = ax^3 + bx - 10,$$

where a and b are constants. It is given that $(x + 2)$ is a factor of $\mathrm{p}(x)$ and that the remainder is -55 when $\mathrm{p}(x)$ is divided by $(x + 3)$.

(a) Find the values of a and b. [5]

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(b) Hence factorise p(x) completely. [3]

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2 **(a)** Sketch, on the same diagram, the graphs of $y = x + 3$ and $y = |2x - 1|$. [2]

(b) Solve the equation $x + 3 = |2x - 1|$. [3]

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(c) Find the value of y such that $5^{\frac{1}{2}y} + 3 = \left|2 \times 5^{\frac{1}{2}y} - 1\right|$. Give your answer correct to 3 significant figures. [2]

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3 The curve with equation

$$y = 5x - 2\tan 2x$$

has exactly one stationary point in the interval $0 \leqslant x < \frac{1}{4}\pi$.

Find the coordinates of this stationary point, giving each coordinate correct to 3 significant figures. [6]

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4 Given that $\int_{a}^{a+14} \frac{1}{3x}\,\mathrm{d}x = \ln 2$, find the value of the positive constant a. [5]

5 A curve has equation $x^2 + 4x\cos 3y = 6$.

Find the exact value of the gradient of the normal to the curve at the point $\left(\sqrt{2}, \frac{1}{12}\pi\right)$. [6]

6 **(a)** By sketching a suitable pair of graphs on the same diagram, show that the equation

$$\ln x = 2e^{-x}$$

has exactly one root. [2]

(b) Verify by calculation that the root lies between 1.5 and 1.6. [2]

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(c) Show that if a sequence of values given by the iterative formula

$$x_{n+1} = e^{2e^{-x_n}}$$

converges, then it converges to the root of the equation in part **(a)**. [1]

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(d) Use the iterative formula in part **(c)** to determine the root correct to 3 significant figures. Give the result of each iteration to 5 significant figures. [3]

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7 **(a)** Prove that $4\sin x \sin\left(x + \frac{1}{6}\pi\right) \equiv \sqrt{3} - \sqrt{3}\cos 2x + \sin 2x$. [3]

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(b) Find the exact value of $\int_0^{\frac{5}{6}\pi} 4\sin x \sin\left(x + \frac{1}{6}\pi\right) \mathrm{d}x$. [4]

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(c) Find the smallest positive value of y satisfying the equation

$$4\sin(2y)\sin\left(2y+\tfrac{1}{6}\pi\right)=\sqrt{3}.$$

Give your answer in an exact form. [3]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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