GradeMax

Cambridge Assessment International Education

# Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

CANDIDATE NUMBER | | | | |

*7036013722*

**MATHEMATICS** **9709/13**

Paper 1 Pure Mathematics 1 **May/June 2022**

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.


© UCLES 2022

**[Turn over**

**BLANK PAGE**

© UCLES 2022

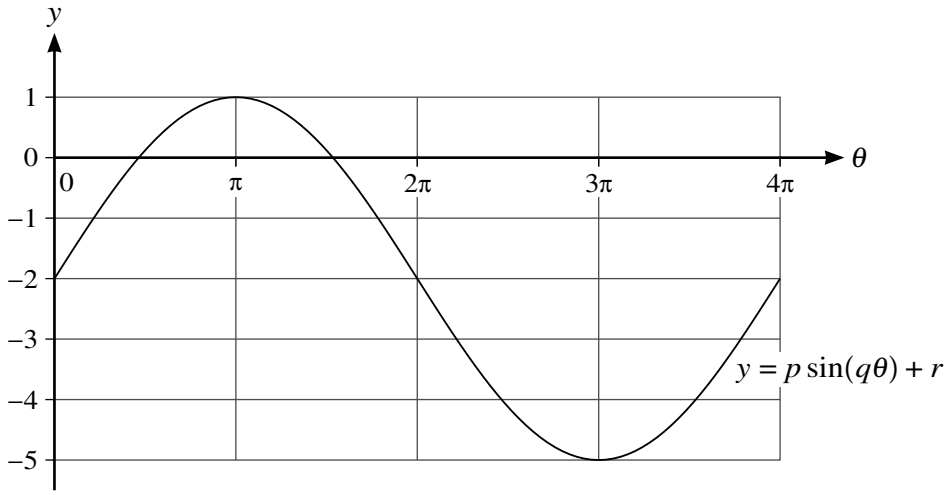
GradeMax



**1** The coefficient of $x^3$ in the expansion of $\left(p + \frac{1}{p}x\right)^4$ is 144.

Find the possible values of the constant $p$. [4]

**2**




The diagram shows part of the curve with equation $y = p\sin(q\theta) + r$, where $p$, $q$ and $r$ are constants.

**(a)** State the value of $p$. [1]

.....

.....

.....

.....

**(b)** State the value of $q$. [1]

.....

.....

.....

.....

.....

**(c)** State the value of $r$. [1]

.....

.....

.....

.....

© UCLES 2022

GradeMax



**3** An arithmetic progression has first term 4 and common difference $d$. The sum of the first $n$ terms of the progression is 5863.

**(a)** Show that $(n-1)d = \frac{11\,726}{n} - 8$. [1]

.....

.....

.....

.....

.....

.....

.....

.....

**(b)** Given that the $n$th term is 139, find the values of $n$ and $d$, giving the value of $d$ as a fraction. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

GradeMax



**4** **(a)** The curve with equation $y = x^2 + 2x - 5$ is translated by $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$.

Find the equation of the translated curve, giving your answer in the form $y = ax^2 + bx + c$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

**(b)** The curve with equation $y = x^2 + 2x - 5$ is transformed to a curve with equation $y = 4x^2 + 4x - 5$.

Describe fully the single transformation that has been applied. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2022

GradeMax



**5** **(a)** Solve the equation $6\sqrt{y} + \frac{2}{\sqrt{y}} - 7 = 0$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

**(b)** Hence solve the equation $6\sqrt{\tan x} + \frac{2}{\sqrt{\tan x}} - 7 = 0$ for $0° \leqslant x \leqslant 360°$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

GradeMax



**6** The function f is defined by $f(x) = 2x^2 - 16x + 23$ for $x < 3$.

**(a)** Express $f(x)$ in the form $2(x + a)^2 + b$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

**(b)** Find the range of f. [1]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2022

**(c)** Find an expression for $f^{-1}(x)$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The function g is defined by $g(x) = 2x + 4$ for $x < -1$.

**(d)** Find and simplify an expression for $fg(x)$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

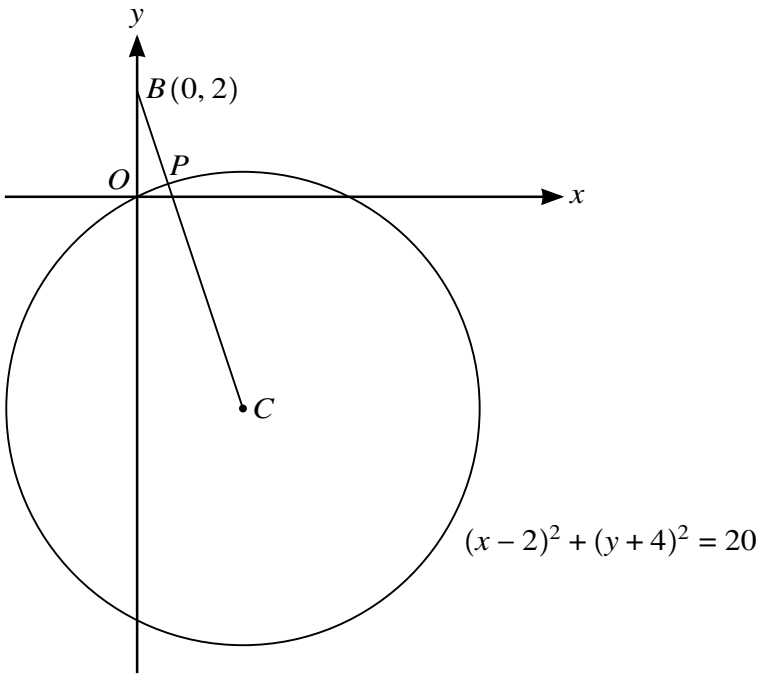
.....

.....

.....

© UCLES 2022

**7**




The diagram shows the circle with equation $(x - 2)^2 + (y + 4)^2 = 20$ and with centre $C$. The point $B$ has coordinates (0, 2) and the line segment $BC$ intersects the circle at $P$.

**(a)** Find the equation of $BC$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

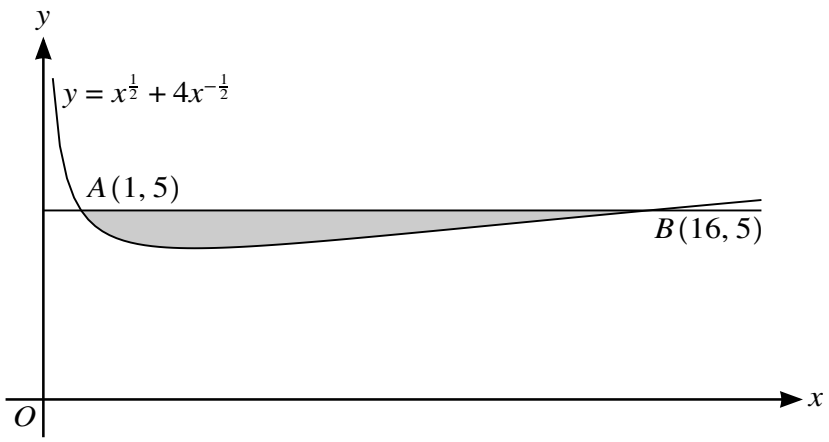
© UCLES 2022

GradeMax



**(b)** Hence find the coordinates of $P$, giving your answer in exact form. [5]

**8**




The diagram shows the curve with equation $y = x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}$. The line $y = 5$ intersects the curve at the points $A\,(1,\,5)$ and $B\,(16,\,5)$.

**(a)** Find the equation of the tangent to the curve at the point $A$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

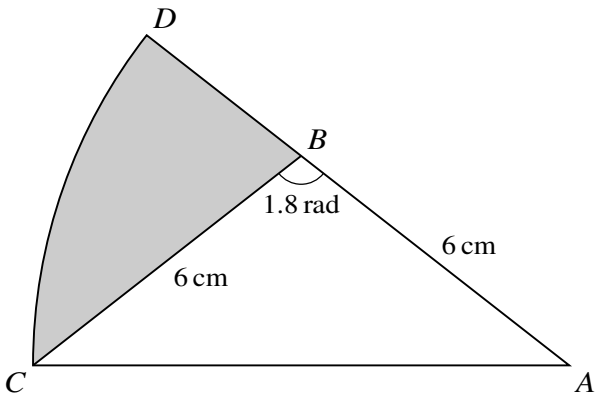
.....

.....

© UCLES 2022

**(b)** Calculate the area of the shaded region. [4]

**9**




The diagram shows triangle $ABC$ with $AB = BC = 6$ cm and angle $ABC = 1.8$ radians. The arc $CD$ is part of a circle with centre $A$ and $ABD$ is a straight line.

**(a)** Find the perimeter of the shaded region. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2022

**(b)** Find the area of the shaded region. [3]

GradeMax



**10** The function f is defined by $f(x) = (4x + 2)^{-2}$ for $x > -\frac{1}{2}$.

**(a)** Find $\int_1^\infty f(x)\,dx$. [4]

A point is moving along the curve $y = f(x)$ in such a way that, as it passes through the point $A$, its $y$-coordinate is **decreasing** at the rate of $k$ units per second and its $x$-coordinate is **increasing** at the rate of $k$ units per second.

**(b)** Find the coordinates of $A$. [6]

GradeMax



**11** The point $P$ lies on the line with equation $y = mx + c$, where $m$ and $c$ are positive constants. A curve has equation $y = -\frac{m}{x}$. There is a single point $P$ on the curve such that the straight line is a tangent to the curve at $P$.

**(a)** Find the coordinates of $P$, giving the $y$-coordinate in terms of $m$. [6]

GradeMax



The normal to the curve at $P$ intersects the curve again at the point $Q$.

**(b)** Find the coordinates of $Q$ in terms of $m$. [4]

GradeMax



## Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

© UCLES 2022