GradeMax

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

CANDIDATE NUMBER | | | | |

3281101778

MATHEMATICS **9709/23**

Paper 2 Pure Mathematics 2 **May/June 2022**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

© UCLES 2022

[Turn over

GradeMax

1 Given that $y = \dfrac{\ln x}{x^2}$, find the exact value of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ when $x = \mathrm{e}$. [3]

© UCLES 2022

2 **(a)** Sketch, on the same diagram, the graphs of $y = |2x - 9|$ and $y = 5x - 3$. [2]

(b) Solve the equation $|2x - 9| = 5x - 3$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2022

GradeMax

3 A curve has equation $e^{2x} \cos 2y + \sin y = 1$.

Find the exact gradient of the curve at the point $(0, \frac{1}{6}\pi)$. [5]

GradeMax

4 **(a)** Use the trapezium rule with three intervals to show that the value of $\int_1^4 \ln x \, \mathrm{d}x$ is approximately $\ln 12$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Use a graph of $y = \ln x$ to show that $\ln 12$ is an under-estimate of the true value of $\int_1^4 \ln x \, \mathrm{d}x$. [2]

.....

.....

.....

© UCLES 2022

[Turn over

GradeMax

5 The polynomial $p(x)$ is defined by

$$p(x) = 2x^3 + ax^2 - 3x - 4,$$

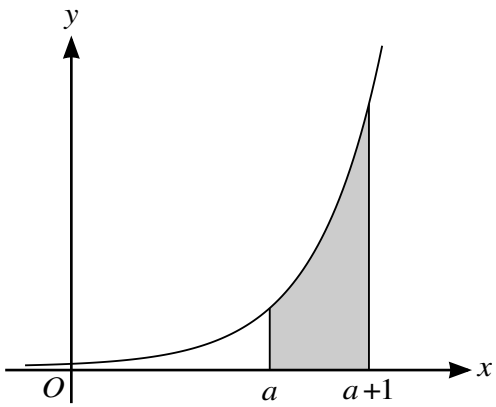
where a is a constant. It is given that $(x - 4)$ is a factor of $p(x)$.

(a) Find the value of a and hence factorise $p(x)$. [4]

GradeMax

(b) Show that the equation $p(e^{3y}) = 0$ has only one real root and find its exact value. [3]

6



The diagram shows the curve $y = 3e^{2x-1}$. The shaded region is bounded by the curve and the lines $x = a$, $x = a + 1$ and $y = 0$, where a is a constant. It is given that the area of the shaded region is 120 square units.

(a) Show that $a = \frac{1}{2}\ln(80 + e^{2a-1}) - \frac{1}{2}$. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2022

GradeMax

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Use an iterative formula, based on the equation in part **(a)**, to find the value of a correct to 3 significant figures. Give the result of each iteration to 5 significant figures. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

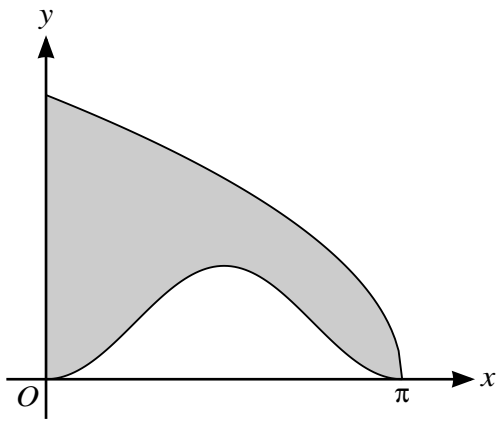
.....

.....

.....

.....

7



The diagram shows the curves $y = \sqrt{2\pi - 2x}$ and $y = \sin^2 x$ for $0 \leqslant x \leqslant \pi$. The shaded region is bounded by the two curves and the line $x = 0$.

Find the exact area of the shaded region. [8]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2022

GradeMax

GradeMax

8 **(a)** Express $3\sin 2\theta \sec\theta + 10\cos(\theta - 30°)$ in the form $R\sin(\theta + \alpha)$ where $R > 0$ and $0° < \alpha < 90°$. Give the value of α correct to 2 decimal places. [6]

GradeMax

(b) Hence solve the equation $3\sin 4\beta \sec 2\beta + 10\cos(2\beta - 30°) = 2$ for $0° < \beta < 90°$. [3]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2022

BLANK PAGE

GradeMax

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

© UCLES 2022