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Cambridge Assessment International Education

# Cambridge International AS & A Level

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*5794103083*

**MATHEMATICS** **9709/31**

Paper 3 Pure Mathematics 3 **May/June 2022**

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.


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**1** Solve the equation $2(3^{2x-1}) = 4^{x+1}$, giving your answer correct to 2 decimal places. [4]

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**2** **(a)** Expand $(2 - x^2)^{-2}$ in ascending powers of $x$, up to and including the term in $x^4$, simplifying the coefficients. [4]

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**(b)** State the set of values of $x$ for which the expansion is valid. [1]

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**3** Solve the equation $2\cot 2x + 3\cot x = 5$, for $0° < x < 180°$. [6]

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**4** The variables $x$ and $y$ satisfy the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{xy}{1+x^2},$$

and $y = 2$ when $x = 0$.

Solve the differential equation, obtaining a simplified expression for $y$ in terms of $x$. [7]

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**5** The polynomial $ax^3 - 10x^2 + bx + 8$, where $a$ and $b$ are constants, is denoted by $\mathrm{p}(x)$. It is given that $(x - 2)$ is a factor of both $\mathrm{p}(x)$ and $\mathrm{p}'(x)$.

**(a)** Find the values of $a$ and $b$. [5]

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**(b)** When $a$ and $b$ have these values, factorise $\mathrm{p}(x)$ completely. [3]

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**6** Let $I = \int_0^3 \frac{27}{(9+x^2)^2}\,\mathrm{d}x$.

**(a)** Using the substitution $x = 3\tan\theta$, show that $I = \int_0^{\frac{1}{4}\pi} \cos^2\theta\,\mathrm{d}\theta$. [4]

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**(b)** Hence find the exact value of $I$. [4]

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**7** The complex number $u$ is defined by $u = \dfrac{\sqrt{2} - a\sqrt{2}\mathrm{i}}{1 + 2\mathrm{i}}$, where $a$ is a positive integer.

**(a)** Express $u$ in terms of $a$, in the form $x + \mathrm{i}y$, where $x$ and $y$ are real and exact. [3]

It is now given that $a = 3$.

**(b)** Express $u$ in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$, giving the exact values of $r$ and $\theta$. [2]

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**(c)** Using your answer to part **(b)**, find the two square roots of $u$. Give your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$, giving the exact values of $r$ and $\theta$. [3]

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**8** The equation of a curve is $x^3 + y^3 + 2xy + 8 = 0$.

**(a)** Express $\frac{dy}{dx}$ in terms of $x$ and $y$. [4]

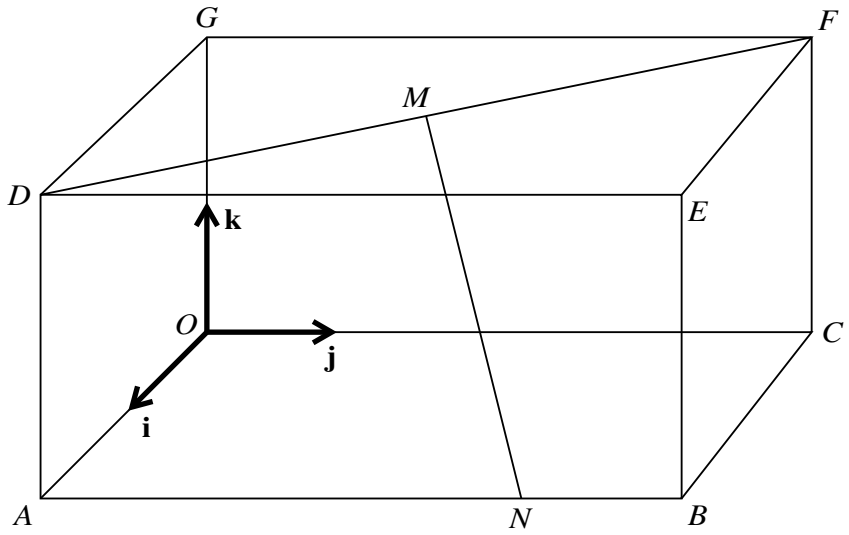
The tangent to the curve at the point where $x = 0$ and the tangent at the point where $y = 0$ intersect at the acute angle $\alpha$.

**(b)** Find the exact value of $\tan \alpha$. [5]

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**9**




In the diagram, $OABCDEFG$ is a cuboid in which $OA = 2$ units, $OC = 4$ units and $OG = 2$ units. Unit vectors **i**, **j** and **k** are parallel to $OA$, $OC$ and $OG$ respectively. The point $M$ is the midpoint of $DF$. The point $N$ on $AB$ is such that $AN = 3NB$.

**(a)** Express the vectors $\overrightarrow{OM}$ and $\overrightarrow{MN}$ in terms of **i**, **j** and **k**. [3]

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**(b)** Find a vector equation for the line through $M$ and $N$. [2]

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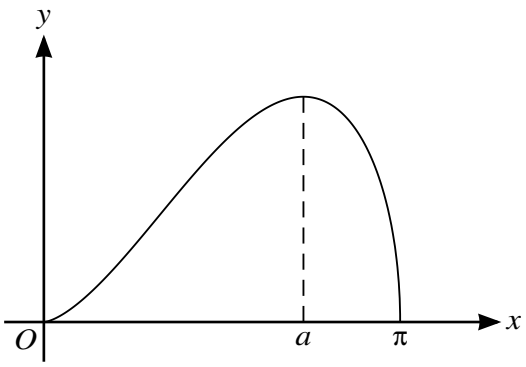
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**(c)** Show that the length of the perpendicular from $O$ to the line through $M$ and $N$ is $\sqrt{\frac{53}{6}}$. [4]

**10**




The curve $y = x\sqrt{\sin x}$ has one stationary point in the interval $0 < x < \pi$, where $x = a$ (see diagram).

**(a)** Show that $\tan a = -\frac{1}{2}a$. [4]

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**(b)** Verify by calculation that $a$ lies between 2 and 2.5. [2]

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**(c)** Show that if a sequence of values in the interval $0 < x < \pi$ given by the iterative formula $x_{n+1} = \pi - \tan^{-1}\left(\frac{1}{2}x_n\right)$ converges, then it converges to $a$, the root of the equation in part **(a)**. [2]

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**(d)** Use the iterative formula given in part **(c)** to determine $a$ correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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## Additional Page

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