GradeMax

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

CANDIDATE NUMBER | | | | |

2330973605

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **May/June 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

© UCLES 2023

[Turn over

1 The equation of a curve is such that $\frac{dy}{dx} = \frac{4}{(x-3)^3}$ for $x > 3$. The curve passes through the point (4, 5).

Find the equation of the curve. [3]

© UCLES 2023

GradeMax

2 The coefficient of x^4 in the expansion of $(x + a)^6$ is p and the coefficient of x^2 in the expansion of $(ax + 3)^4$ is q. It is given that $p + q = 276$.

Find the possible values of the constant a. [4]

© UCLES 2023

[Turn over

GradeMax

3 **(a)** Express $4x^2 - 24x + p$ in the form $a(x + b)^2 + c$, where a and b are integers and c is to be given in terms of the constant p. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Hence or otherwise find the set of values of p for which the equation $4x^2 - 24x + p = 0$ has no real roots. [1]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

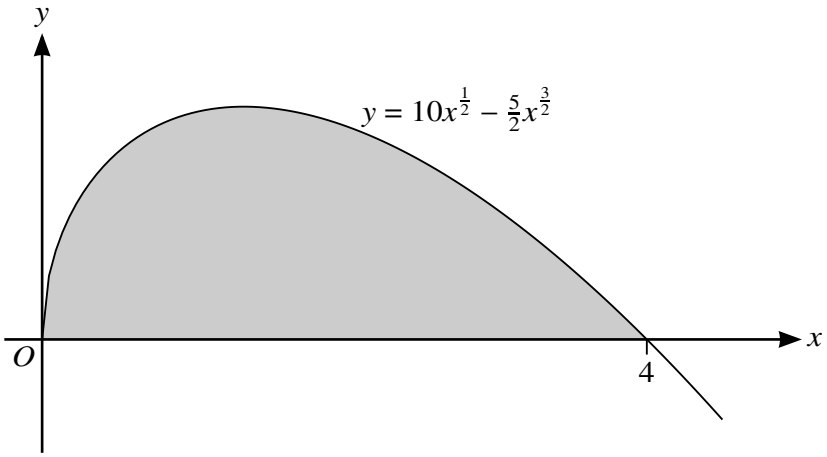
.....

© UCLES 2023

GradeMax

4 Solve the equation $8x^6 + 215x^3 - 27 = 0$. [3]

5



The diagram shows the curve with equation $y = 10x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}}$ for $x > 0$. The curve meets the x-axis at the points (0, 0) and (4, 0).

Find the area of the shaded region. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

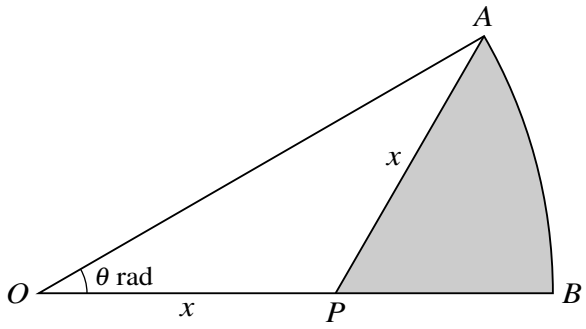
.....

.....

.....

GradeMax

6



The diagram shows a sector OAB of a circle with centre O. Angle $AOB = \theta$ radians and $OP = AP = x$.

(a) Show that the arc length AB is $2x\theta\cos\theta$. [2]

.....

.....

.....

.....

.....

(b) Find the area of the shaded region APB in terms of x and θ. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2023

[Turn over

GradeMax

7 **(a)** **(i)** By first expanding $(\cos\theta + \sin\theta)^2$, find the three solutions of the equation

$$(\cos\theta + \sin\theta)^2 = 1$$

for $0 \leqslant \theta \leqslant \pi$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(ii) Hence verify that the only solutions of the equation $\cos\theta + \sin\theta = 1$ for $0 \leqslant \theta \leqslant \pi$ are 0 and $\frac{1}{2}\pi$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2023

GradeMax

(b) Prove the identity $\dfrac{\sin\theta}{\cos\theta+\sin\theta}+\dfrac{1-\cos\theta}{\cos\theta-\sin\theta}\equiv\dfrac{\cos\theta+\sin\theta-1}{1-2\sin^2\theta}$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Using the results of **(a)(ii)** and **(b)**, solve the equation

$$\frac{\sin\theta}{\cos\theta+\sin\theta}+\frac{1-\cos\theta}{\cos\theta-\sin\theta}=2(\cos\theta+\sin\theta-1)$$

for $0\leqslant\theta\leqslant\pi$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

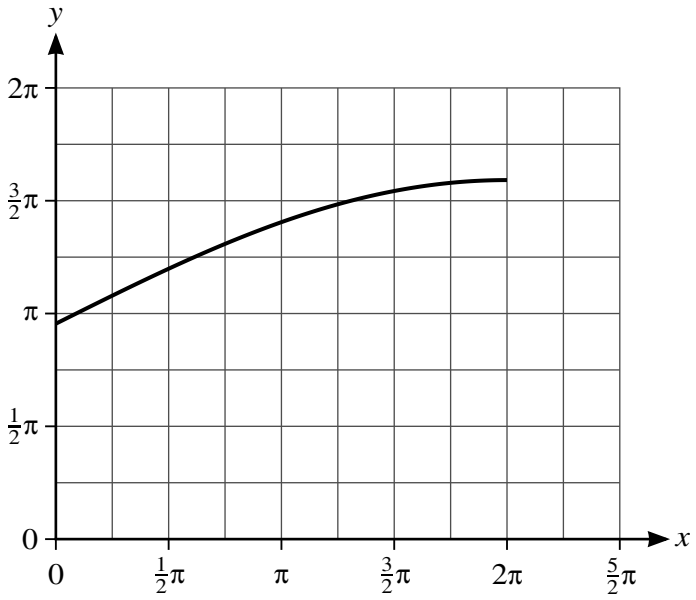
.....

.....

© UCLES 2023

[Turn over

8



The diagram shows the graph of $y = \mathrm{f}(x)$ where the function f is defined by

$$\mathrm{f}(x) = 3 + 2\sin\tfrac{1}{4}x \text{ for } 0 \leqslant x \leqslant 2\pi.$$

(a) On the diagram above, sketch the graph of $y = \mathrm{f}^{-1}(x)$. [2]

(b) Find an expression for $\mathrm{f}^{-1}(x)$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

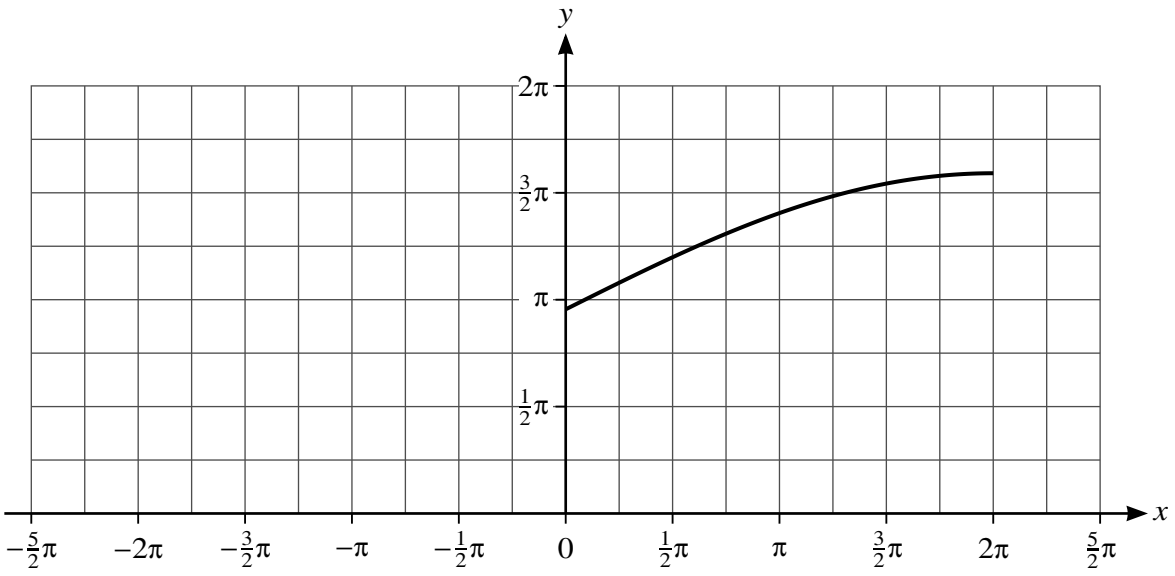
.....

.....

.....

© UCLES 2023

(c)



The diagram above shows part of the graph of the function $g(x) = 3 + 2\sin\frac{1}{4}x$ for $-2\pi \leqslant x \leqslant 2\pi$.

Complete the sketch of the graph of $g(x)$ on the diagram above and hence explain whether the function g has an inverse. [2]

.....

.....

.....

(d) Describe fully a sequence of three transformations which can be combined to transform the graph of $y = \sin x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$ to the graph of $y = f(x)$, making clear the order in which the transformations are applied. [6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2023

GradeMax

9 The second term of a geometric progression is 16 and the sum to infinity is 100.

(a) Find the two possible values of the first term. [4]

GradeMax

(b) Show that the nth term of one of the two possible geometric progressions is equal to 4^{n-2} multiplied by the nth term of the other geometric progression. [4]

GradeMax

10 The equation of a circle is $(x - a)^2 + (y - 3)^2 = 20$. The line $y = \frac{1}{2}x + 6$ is a tangent to the circle at the point P.

(a) Show that one possible value of a is 4 and find the other possible value. [5]

(b) For $a = 4$, find the equation of the normal to the circle at P. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) For $a = 4$, find the equations of the two tangents to the circle which are parallel to the normal found in **(b)**. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

GradeMax

11 The equation of a curve is

$$y = k\sqrt{4x + 1} - x + 5,$$

where k is a positive constant.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the x-coordinate of the stationary point in terms of k. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2023

GradeMax

(c) Given that $k = 10.5$, find the equation of the normal to the curve at the point where the tangent to the curve makes an angle of $\tan^{-1}(2)$ with the positive x-axis. [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2023

BLANK PAGE

© UCLES 2023

GradeMax

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

© UCLES 2023