GradeMax

Cambridge Assessment International Education

# Cambridge International AS & A Level

**MATHEMATICS** **9709/41**

Paper 4 Mechanics **May/June 2023**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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**[Turn over**

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## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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| | Mathematics Specific Marking Principles |
|---|---|
| 1 | Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing. |
| 2 | Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected. |
| 3 | Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points. |
| 4 | Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw). |
| 5 | Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread. |
| 6 | Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear. |

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## Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

### Types of mark

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

**DM** or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

**FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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**Abbreviations**

| | |
|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed) |
| CWO | Correct Working Only |
| ISW | Ignore Subsequent Working |
| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | $m \times 5 + 0 = m \times 2 + 0.3v$ | **M1** | Attempt at conservation of momentum; 3 non-zero terms (with $m$ appearing in two terms); allow sign errors. |
| | Speed $= 10m$ $(\text{m s}^{-1})$ | **A1** | M1A0 if using $g$ in momentum terms.<br>$v = -10m$ is A0. |
| | | **2** | |
| 1(b) | $0.3 \times 10m + 0 = 0 + 0.6 \times 1.5$ $[3m = 0.9]$ | **M1** | Attempt at conservation of momentum between $Q$ and $R$ (so must be using correct masses of 0.3 and 0.6) to form a linear equation in $m$ using their answer from **(a)**; 2 non-zero terms; allow sign errors. |
| | $m = 0.3$ | **A1FT** | FT $\dfrac{3}{\textit{their}\ \text{+ve coefficient of}\ m\ \text{from}\ \textbf{(a)}}$<br><br>Condone including kg in answer. |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(a) | $0 = 10^2 + 2(-g)s \Rightarrow s = ...$ | **M1** | For use of $v^2 = u^2 + 2as$ with $v = 0$, $u = \pm 10$ and $a = \pm g$ or $\pm 10$ and solve for $s$ (or any other complete SUVAT method). Or using an energy method: $0.4gh = \frac{1}{2}(0.4)(10)^2$ and solve for $h$ (with two terms using correct given values – condone lack of masses in conservation of energy equation). |
| | Max. height = 5 (m) | **A1** | |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(b) | KE before impact $=\frac{1}{2}\times 0.4\times 10^2\,[=20]$ | **B1FT** | Or loss of PE = $\pm\ 0.4\times g\times 5$ using their maximum height from **(a)**. |
| | $\frac{1}{2}\times 0.4\times v^2 = 20-7.2\,[v=8]$<br>or $0.4gh = 20-7.2\,[h=3.2]$ | ***M1** | M1 for $\frac{1}{2}\times 0.4v^2$ = KE/PE before impact – 7.2 (must be correct method of subtracting 7.2 (OE)). Or, for finding the maximum height after first impact. Need not solve for $v$ (or $h$) for this mark. |
| | $-8 = 8+(-g)t \Rightarrow t = ...$<br>or $0 = 8t+\frac{1}{2}\times -g\times t^2 \Rightarrow t = ...$ | **DM1** | For use of a complete method to find $t$. Condone sign errors but $a=\pm g$. If calculating the time to the maximum height between the first and second impacts, then candidates must double this answer (OE). E.g., $0=8+(-g)T,\ t=2T=...$ or $3.2=0-0.5\times -g\times T^2 \Rightarrow t=2T=...$. |
| | Time = 1.6(s) | **A1** | |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | For an attempt at differentiation. | *M1 | Decrease power by 1 and a change in coefficient in at least one term. $v=\frac{s}{t}$ is M0. |
| | $(v=)\frac{5}{2}t^{\frac{3}{2}}-\frac{45}{8}t^{\frac{1}{2}}$ | A1 | Allow unsimplified, including indices (including a $+c$ is A0). |
| | $v=0\Rightarrow\frac{1}{8}t^{\frac{1}{2}}(20t-25)=0\Rightarrow t=...$ | DM1 | Attempting to find $t$ by equating $v$ to 0 and attempt to solve a linear equation for $t$ (if correct $t = 2.25$). Must be of the form $t = \ldots$ . |
| | $s=\frac{15}{16}[=0.9375]$ | A1 | Condone 0.938 . |
| | | 4 | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | Distance $=\frac{1}{2}(6+10)\times 0.9$ | M1 | Completely correct method for finding the total area underneath the velocity-time graph from $t = 0$ to 10 only. Can be done as two triangles and a rectangle e.g., $\frac{1}{2}\times 3\times 0.9+(9-3)\times 0.9+\frac{1}{2}\times 1\times 0.9$ (allow a slip in one value); need not see all three components added together. |
| | Distance = 7.2(m) | A1 | |
| | | 2 | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(b) | $\frac{1}{2}(12-10)v_{\min} = -7.2$ | **M1** | Setting $\frac{1}{2}(12-10)v_{\min}$ equal to $\pm$ their **(a)**. |
| | Minimum velocity = $-7.2$ $(\text{m s}^{-1})$ | **A1** | Must be negative – allow those who solve $\frac{1}{2}(12-10)v_{\min} = 7.2$ and obtain $v_{\min} = 7.2$ and then change to $-7.2$ without justification.<br>**SC B1** for assuming the triangle is isosceles. |
| | | **2** | |
| 4(c) | $\frac{1}{2}\times(T-10)\times 3 = 7.2$ or $\frac{1}{2}\times t\times 3 = 7.2$ | ***M1** | Correct method for finding $T$ or $t$. Condone sign errors but must equate to their answer to **(a)**. |
| | $T = 14.8$ | **A1** | OE (e.g. from $t + 10 = 4.8 + 10 = 14.8$). |
| | $\frac{14.4}{14.8}$ | **DM1** | **M1** for $\frac{2(\textit{their}\ \textbf{(a)})}{\textit{their}\ T}$ or $\frac{2(\textit{their}\ \textbf{(a)})}{10+\textit{their}\ T}$. |
| | Average speed $= \frac{36}{37}$ $(\text{m s}^{-1})$ | **A1** | OE 0.973 [For reference: 0.97297…].<br>**SC *B1** (for $T = 14.8$). DM1A1 for assuming the triangle is isosceles. |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | Resolving either direction. | **M1** | 3 terms; allow sign errors and allow sin/cos mix. Must be an equation with either = 0 or with an attempt to balance forces. |
| | Vertical: $F\sin\theta + 40\sin 60 - 50 = 0$ | **A1** | $\left[F\sin\theta = 50 - 20\sqrt{3} = 15.358...\right]$ |
| | Horizontal: $F\cos\theta + 10 - 40\cos 60 = 0$ | **A1** | $\left[F\cos\theta = 10\right]$ |
| | $\theta = \tan^{-1}(5 - 2\quad)$ | **M1** | Attempt to solve for $\theta$; one missing term in total $\theta = \tan^{-1} 1.535898...$. |
| | $F = \sqrt{15.358...^2 + 10^2}$ | **M1** | Attempt to solve for $F$: one missing term in total. |
| | $\theta = 56.9,\quad F = 18.3$ | **A1** | Both correct (18.327530…, 56.932462…). |
| | | **6** | |
| 5(b) | $(Y =) \pm (10\quad \sin 45 + 40\sin 60 - 50)\ [= \pm(20\sqrt{3} - 40)]$ | **B1** | Allow non-exact values for $\sqrt{2}$ etc. in correct expression. |
| | $(X =) \pm (10\quad \cos 45 + 10 - 40\cos 60)\ [= 0]$ | **B1** | Allow non-exact values for $\sqrt{2}$ etc. in correct expression. Could be implied by correct answer. |
| | Resultant force is $40 - 20\quad$ (N) in the same direction as the 50 (N) force. | **B1** | Allow vertically downwards, south, 180°, negative $y$-direction.<br>Resultant force must be exact and positive (so $20\sqrt{3} - 40$ is B0). |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | $R = 0.1g\cos 60 [= 0.5]$ | **B1** | Correctly resolving perpendicular to the plane for $Q$. |
| | $F = 0.7 \times 0.1g\cos 60\ [= 0.35]$ | **M1** | Use of $F = \mu R$ for $Q$ where $R$ is a component of weight but not mass; allow sin/cos mix. |
| | For whole system: LHS of Newton's second law:<br>$0.2g\sin 60 - 0.1g\sin 60 - F\ \ [= 0.866... - F]$<br><br>Or separately for $P$ and $Q$:<br>$0.2g\sin 60 - T (= 0.2a)$ and $T - 0.1g\sin 60 - F\,(= 0.1a)$, and eliminate $T$ to get $0.2g\sin 60 - 0.1g\sin 60 - F (= 0.3a)$ | **M1** | Complete method to determine the resultant force for the whole system. Allow sign errors and sin/cos mix, but must include all required terms and be dimensionally correct. If considering either the while system or $P$ and $Q$ separately then ignore the RHS of their Newton's second law equations. |
| | As $\frac{}{2} - 0.35 > 0$ the particles do move. | **A1** | Correct indication (with no incorrect working) that the resultant force is positive (e.g. $0.8660... - 0.35 > 0$ or 0.516… (to at least 1 sf) which is positive) together with a correct conclusion. Candidates may calculate the acceleration which is $\frac{10\sqrt{3} - 7}{6} = 1.72008...$ and then say that the particles are moving. |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(b) | Attempt to use Newton's second law for $P$:<br>$0.2g\sin 60 - (\quad -1) = 0.2a$ | **M1** | Allow sign errors, sin/cos mix. but must be dimensionally consistent. |
| | $a = 5\,(\text{m s}^{-2})$ | **A1** | |
| | Newton's second law for system: $0.2g\sin 60 - 0.1g\sin\theta = 0.3(5)$<br>or Newton's second law for $Q$: $(\sqrt{3}-1) - 0.1g\sin\theta = 0.1(5)$ | **M1** | Attempt Newton's second law for $Q$, or for the whole system. Allow sign errors, sin/cos mix, but must be dimensionally consistent. |
| | $\theta = 13.4$ | **A1** | 13.41784… |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | Driving force $F = \dfrac{16000}{20}[= 800]$ | **B1** | OE e.g. $16000 = 20 \times F$ |
| | $F - 500 = 1200a$ | **M1** | Use of Newton's second law; allow sign errors but must be 3 terms. Allow $F$ or any non-zero value for the driving force (allow 0.8 from using 16 rather than 16000) but not 16000, 16, 20 or 500 for $F$. |
| | $a = 0.25\,(\text{m s}^{-2})$ | **A1** | |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(b) | $\frac{16000}{v} - 500 = 0$ | **M1** | Allow sign errors but must be 2 terms. Condone $\frac{16}{v} - 500 = 0$ for M1. |
| | $v = 32$ $(\text{m s}^{-1})$ | **A1** | |
| | | **2** | |
| 7(c) | Work done by engine = $16000 \times 15 [= 240000]$ | **B1** | Or $16000 = \frac{\text{WD}}{15}$. |
| | KE change = $\pm\left(\frac{1}{2} \times 1200 \times v^2 - \frac{1}{2} \times 1200 \times 20^2\right)$ | **B1** | $\pm(600v^2 - 240000)$ |
| | PE change = $\pm\left(1200 \times g \times 316 \times \frac{1}{60}\right) [= \pm 63200]$ | **B1** | Allow $1200 \times g \times 316 \times \sin 0.955$ or $1200 \times g \times 316 \times \sin 0.95$ or $1200 \times g \times \frac{79}{15}$ or $1200 \times g \times 5.266...$ . |
| | Attempt at work-energy equation. | **M1** | Use of work-energy principle with 5 terms; dimensionally correct. Allow sign errors and sin/cos mix on PE term |
| | $16000 \times 15 - 128400 = \frac{1}{2} \times 1200 \times v^2 - \frac{1}{2} \times 1200 \times 20^2 + 1200g \times 316 \times \frac{1}{60}$ $(240000 - 128400 = 600v^2 - 240000 + 63200)$ | **A1** | Allow a value in the interval [62870,63600] for the PE term from using non-exact values for the given angle (but not if from incorrect working). |
| | $v = 21.9$ $(\text{m s}^{-1})$ | **A1** | 21.924111… |
| | | **6** | |