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Cambridge Assessment International Education

# Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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*2923681341*

**MATHEMATICS** **9709/12**

Paper 1 Pure Mathematics 1 **February/March 2024**

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

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**1** Find the exact value of $\int_3^\infty \frac{2}{x^2}\mathrm{d}x$. [3]

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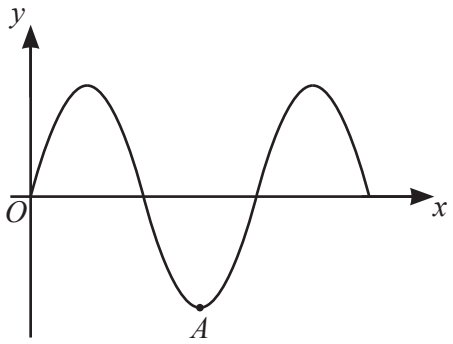
**2**




The diagram shows part of the curve with equation $y = k \sin \frac{1}{2}x$, where $k$ is a positive constant and $x$ is measured in radians. The curve has a minimum point $A$.

**(a)** State the coordinates of $A$. [1]

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**(b)** A sequence of transformations is applied to the curve in the following order.

Translation of 2 units in the negative $y$-direction

Reflection in the $x$-axis

Find the equation of the new curve and determine the coordinates of the point on the new curve corresponding to $A$. [3]

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**3** A curve is such that $\frac{dy}{dx} = 3(4x+5)^{\frac{1}{2}}$. It is given that the points $(1, 9)$ and $(5, a)$ lie on the curve.

Find the value of $a$. [5]

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**4** **(a)** Prove that $\dfrac{(\sin\theta+\cos\theta)^2-1}{\cos^2\theta} \equiv 2\tan\theta$. [3]

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**(b)** Hence solve the equation $\dfrac{(\sin\theta+\cos\theta)^2-1}{\cos^2\theta} = 5\tan^3\theta$ for $-90° < \theta < 90°$. [3]

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**5** A curve has the equation $y = \dfrac{3}{2x^2 - 5}$.

Find the equation of the normal to the curve at the point $(2, 1)$, giving your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers. [6]

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**6** It is given that the coefficient of $x^3$ in the expansion of

$$(2+ax)^4(5-ax)$$

is 432.

Find the value of the constant $a$. [5]

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**7** The straight line $y = x + 5$ meets the curve $2x^2 + 3y^2 = k$ at a single point $P$.

**(a)** Find the value of the constant $k$. [4]

**(b)** Find the coordinates of $P$. [2]

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**8** **(a)** An arithmetic progression is such that its first term is 6 and its tenth term is 19.5 .

Find the sum of the first 100 terms of this arithmetic progression. [4]

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**(b)** A geometric progression $a_1, a_2, a_3, ...$ is such that $a_1 = 24$ and the common ratio is $\frac{1}{2}$.

The sum to infinity of this geometric progression is denoted by $S$. The sum to infinity of the even-numbered terms (i.e. $a_2, a_4, a_6, ...$) is denoted by $S_E$.

Find the values of $S$ and $S_E$. [4]

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**9** The functions f and g are defined for all real values of $x$ by

$$\mathrm{f}(x) = (3x-2)^2 + k \quad \text{and} \quad \mathrm{g}(x) = 5x - 1,$$

where $k$ is a constant.

**(a)** Given that the range of the function gf is $\mathrm{gf}(x) \geqslant 39$, find the value of $k$. [4]

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**(b)** For this value of $k$, determine the range of the function fg. [2]

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**(c)** The function h is defined for all real values of $x$ and is such that $\mathrm{gh}(x) = 35x + 19$.

Find an expression for $\mathrm{g}^{-1}(x)$ and hence, or otherwise, find an expression for $\mathrm{h}(x)$. [3]

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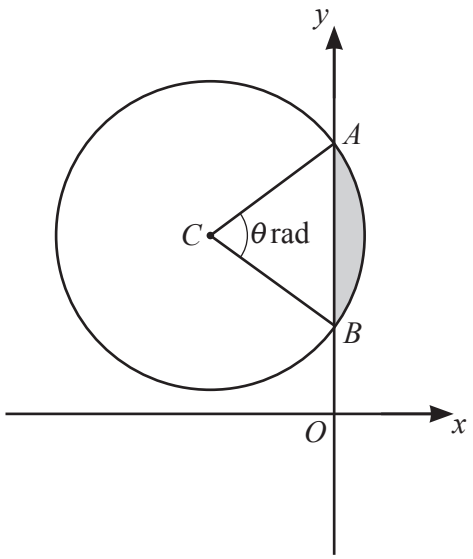


The diagram shows the circle with centre $C(-4, 5)$ and radius $\sqrt{20}$ units. The circle intersects the $y$-axis at the points $A$ and $B$. The size of angle $ACB$ is $\theta$ radians.

**(a)** Find the equation of the tangent to the circle at the point $(-6, 9)$. [3]

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**(b)** Find the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$. [2]

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**(c)** Find the value of $\theta$ correct to 4 significant figures. [3]

**(d)** Find the perimeter and area of the segment shaded in the diagram. [4]

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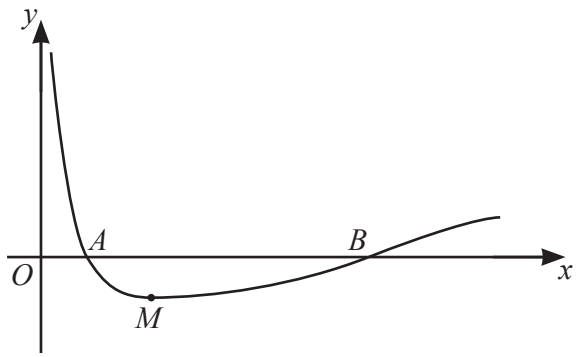
**11**




The diagram shows the curve with equation $y = 2x^{-\frac{2}{3}} - 3x^{-\frac{1}{3}} + 1$ for $x > 0$. The curve crosses the $x$-axis at points $A$ and $B$ and has a minimum point $M$.

**(a)** Find the exact coordinates of $M$. [4]

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**(b)** Find the area of the region bounded by the curve and the line segment $AB$. [7]

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## Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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