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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

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MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **October/November 2024**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 Expand $(9-3x)^{\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

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2 **(a)** By sketching a suitable pair of graphs, show that the equation $\cot 2x = \sec x$ has exactly one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(b) Show that if a sequence of real values given by the iterative formula

$$x_{n+1} = \frac{1}{2}\tan^{-1}(\cos x_n)$$

converges, then it converges to the root in part **(a)**. [1]

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3 The square roots of $6-8\mathrm{i}$ can be expressed in the Cartesian form $x+\mathrm{i}y$, where x and y are real and exact.

By first forming a quartic equation in x or y, find the square roots of $6-8\mathrm{i}$ in exact Cartesian form. [5]

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4 Solve the equation $5^x = 5^{x+2} - 10$. Give your answer correct to 3 decimal places. [3]

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5 **(a)** The complex number u is given by

$$u = \frac{(\cos\frac{1}{7}\pi + \mathrm{i}\sin\frac{1}{7}\pi)^4}{\cos\frac{1}{7}\pi - \mathrm{i}\sin\frac{1}{7}\pi}.$$

Find the exact value of arg u. [2]

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(b) The complex numbers u and u^* are plotted on an Argand diagram.

Describe the single geometrical transformation that maps u onto u^* and state the exact value of arg u^*. [2]

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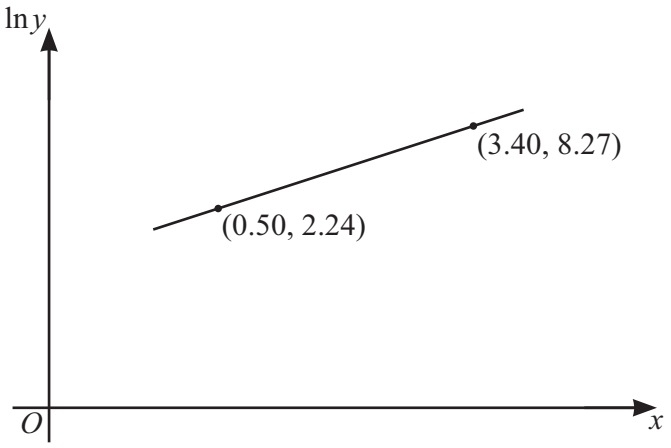
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The variables x and y satisfy the equation $ay = b^x$, where a and b are constants. The graph of $\ln y$ against x is a straight line passing through the points (0.50, 2.24) and (3.40, 8.27), as shown in the diagram.

Find the values of a and b. Give each value correct to 1 significant figure. [4]

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7 **(a)** Show that the equation $\tan^3 x + 2\tan 2x - \tan x = 0$ may be expressed as

$$\tan^4 x - 2\tan^2 x - 3 = 0$$

for $\tan x \neq 0$. [3]

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(b) Hence solve the equation $\tan^3 2\theta + 2\tan 4\theta - \tan 2\theta = 0$ for $0 < \theta < \pi$. Give your answers in exact form. [3]

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8 The parametric equations of a curve are

$$x = \tan^2 2t, \quad y = \cos 2t,$$

for $0 < t < \frac{1}{4}\pi$.

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{2}\cos^3 2t$. [4]

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(b) Hence find the equation of the normal to the curve at the point where $t = \frac{1}{8}\pi$. Give your answer in the form $y = mx + c$. [4]

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9 With respect to the origin O, the points A, B and C have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} -3 \\ -2 \\ 2 \end{pmatrix}.$$

(a) The point D is such that $ABCD$ is a trapezium with $\overrightarrow{DC} = 3\overrightarrow{AB}$.

Find the position vector of D. [2]

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(b) The diagonals of the trapezium intersect at the point P.

Find the position vector of P. [5]

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(c) Using a scalar product, calculate angle ABC. [4]

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10 A balloon in the shape of a sphere has volume V and radius r. Air is pumped into the balloon at a constant rate of 40π starting when time $t = 0$ and $r = 0$. At the same time, air begins to flow out of the balloon at a rate of $0.8\pi r$. The balloon remains a sphere at all times.

(a) Show that r and t satisfy the differential equation

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{50 - r}{5r^2}.$$ [3]

(b) Find the quotient and remainder when $5r^2$ is divided by $50 - r$. [3]

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(c) Solve the differential equation in part **(a)**, obtaining an expression for t in terms of r. [6]

(d) Find the value of t when the radius of the balloon is 12. [1]

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11 Let $f(x) = \dfrac{2e^{2x}}{e^{2x} - 3e^{x} + 2}$.

(a) Find $f'(x)$ and hence find the exact coordinates of the stationary point of the curve with equation $y = f(x)$. [5]

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(b) Use the substitution $u = \mathrm{e}^x$ and partial fractions to find the exact value of $\int_{\ln 3}^{\ln 5} \mathrm{f}(x)\,\mathrm{d}x$.

Give your answer in the form $\ln a$, where a is a rational number in its simplest form. [9]

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