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* 0000800000001 * DFD

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

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MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **October/November 2025**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 Find the exact value of $\int_1^2 \ln 3x \, dx$. Give your answer in the form $a + \ln b$, where a and b are integers. [4]

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2 **(a)** Show that the equation $\log_4(2x+1) = 2\log_4(3x-1) - 2$ can be written as a quadratic equation in x. [3]

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(b) Hence solve the equation $\log_4(2x+1) = 2\log_4(3x-1) - 2$. [2]

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3 **(a)** Express $3\sqrt{2}\sin(x+45°)+\cos x$ in the form $R\cos(x-\alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [4]

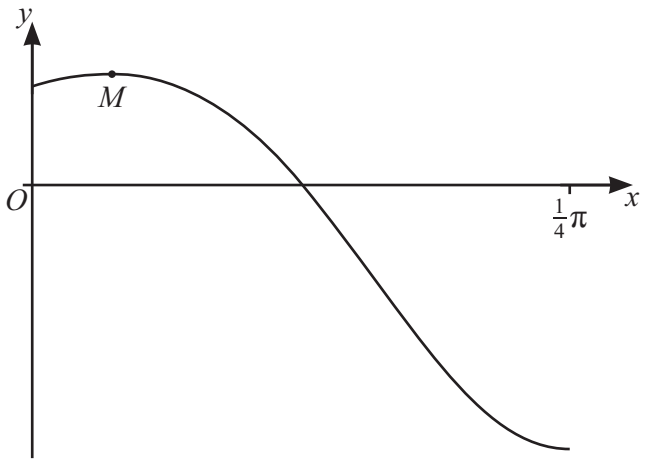
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(b) Hence solve the equation $3\sqrt{2}\sin(3\theta+45°)+\cos 3\theta=-4$ for $0° < \theta < 180°$. [4]

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The diagram shows the graph of $y = e^{\sin 2x}\cos 4x$ for $0 \leqslant x \leqslant \frac{1}{4}\pi$, and its maximum point M.

Find the x-coordinate of M. [5]

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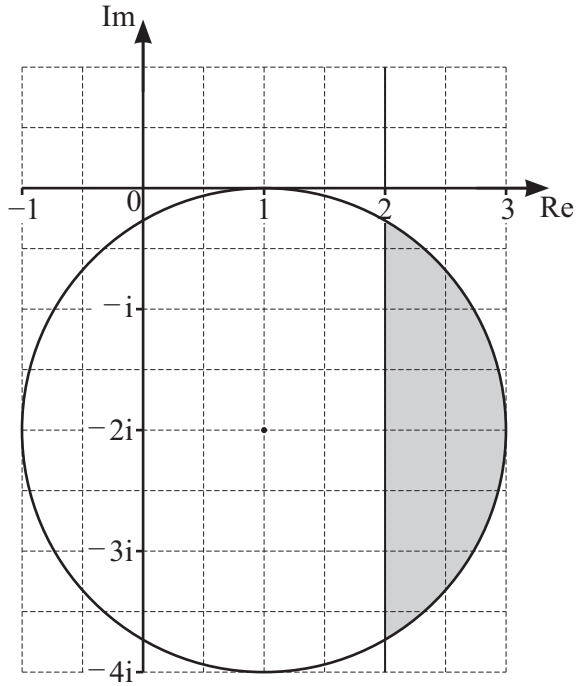


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The shaded region on the Argand diagram shows points representing complex numbers z defined by two inequalities. The shaded region is bounded by a circle and a line parallel to the imaginary axis. The boundaries of the region are included in the shaded region.

(a) Find two inequalities in terms of z that define the shaded region. [3]

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(b) Calculate the least value of $\arg z$ for points in this region. [3]

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6 Solve the quadratic equation $(2+\mathrm{i})w^2+4w+2-\mathrm{i}=0$. Give your answers in the form $x+\mathrm{i}y$, where x and y are real. [5]

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7 The parametric equations of a curve are

$$x = t^2 - \ln(2t+1), \quad y = \frac{t}{2t+1}.$$

Obtain a simplified expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. [5]

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8 The variables x and y satisfy the differential equation

$$\left(x^2+1\right)\frac{\mathrm{d}y}{\mathrm{d}x} = kx\mathrm{e}^{2y},$$

where k is a constant. It is given that $y = 0$ when $x = 0$ and that $y = -\frac{1}{2}$ when $x = 1$.

Solve the differential equation and find the exact value of y when $x = \sqrt{3}$. [8]

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9 **(a)** By sketching a suitable pair of graphs, show that the equation

$$\sec 2x = -e^{x}$$

has only one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(b) Show by calculation that this root lies between 0.9 and 1. [2]

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(c) Show that if a sequence of values given by the iterative formula

$$x_{n+1} = \tfrac{1}{2}\cos^{-1}\left(-\mathrm{e}^{-x_n}\right)$$

converges, then it converges to the root of the equation in part **(a)**. [1]

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(d) Use the iterative formula given in part **(c)** to calculate x correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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10 Let $f(x) = \dfrac{x^3 + 2x - 11}{(3+x)(2+x^2)}$.

(a) Express $f(x)$ in partial fractions. [6]

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(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

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11 With respect to the origin O, the points A, B, C and D have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix}, \qquad \overrightarrow{OB} = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix}, \qquad \overrightarrow{OC} = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} \qquad \text{and} \qquad \overrightarrow{OD} = \begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix}.$$

The line m passes through the points A and B.

(a) Find a vector equation for m. [2]

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(b) Find the position vector of the point of intersection of m and the line passing through the points C and D. [4]

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(c) Find the position vector of the foot of the perpendicular from C to m. [4]

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Additional page

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