

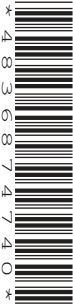


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CAMBRIDGE INTERNATIONAL MATHEMATICS

0607/61

Paper 6 Investigation and Modelling (Extended)

May/June 2024

1 hour 40 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer both part **A** (Questions 1 to 7) and part **B** (Questions 8 to 13).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 60.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Answer **both** parts **A** and **B**.

A INVESTIGATION (QUESTIONS 1 to 7)

SUMS OF POWERS (30 marks)

You are advised to spend no more than 50 minutes on this part.

This investigation looks at connections between the sum of the positive integers, $1 + 2 + 3 + \dots$, the sum of their squares, the sum of their cubes and the sum of their 5th powers.

Example $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 = 78$

The numbers can also be added using this method.

Step 1	Write down the first half of the numbers in a row.	1 2 3 ... 6
Step 2	Write down the second half of the numbers underneath the first half but in reverse order.	12 11 10 ... 7
Step 3	Add each column of two numbers to make a third row.	13 13 13 ... 13
Step 4	Find the total of the numbers in the third row by writing the calculation as a multiplication.	$6 \times 13 = 78$

- 1** Use the method to complete the sum of the first 60 positive integers.
That is $1 + 2 + 3 + \dots + 60 = 1830$.

1

2

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1830

[3]

3

- 2 Use the method to calculate the sum of the first 128 positive integers.
That is $1 + 2 + 3 + \dots + 128$.

..... [3]

- 3 Complete the table.
Use **Question 1**, **Question 2** and any patterns you notice.

	Number of positive integers, starting at 1	Multiplication	Sum
	12	6×13	78
	26	13×27	351
Question 1	60		1830
Question 2	128		
	204		20910

[1]

- 4 $1 + 2 + 3 + \dots + n$ has n positive integers and its sum is T .
Find a formula for T in terms of n .

..... [2]

5 (a) Complete the table.

n	Sum of first n positive integers		Sum of first n square numbers		$\frac{S}{T}$ written as a fraction with denominator 3
	Calculation	Sum (T)	Calculation	Sum (S)	
1	1	1	1^2	1	$\frac{3}{3}$
2	$1 + 2$	3	$1^2 + 2^2$	5	$\frac{5}{3}$
3	$1 + 2 + 3$	6	$1^2 + 2^2 + 3^2$	14	$\frac{14}{3}$
4	$1 + 2 + 3 + 4$	10	$1^2 + 2^2 + 3^2 + 4^2$		
5	$1 + 2 + 3 + 4 + 5$	15	$1^2 + 2^2 + 3^2 + 4^2 + 5^2$	55	

[3]

(b) Find an expression for $\frac{S}{T}$ in terms of n .

..... [3]

(c) The sum of the first 60 positive integers is 1830.

Find the sum of the first 60 square numbers.

..... [2]

5

(d) Use **Question 4** and **Question 5(b)** to help you find a formula for S in terms of n .
Write your answer as a single fraction.

..... [1]

6 (a) Complete the table.

Sum of first n positive integers		Sum of first n cube numbers	
Calculation	Sum (T)	Calculation	Sum (C)
$1 + 2$	3	$1^3 + 2^3$	9
$1 + 2 + 3$	6	$1^3 + 2^3 + 3^3$	36
$1 + 2 + 3 + 4$	10	$1^3 + 2^3 + 3^3 + 4^3$	100
$1 + 2 + 3 + 4 + 5$	15	$1^3 + 2^3 + 3^3 + 4^3 + 5^3$	225
$1 + 2 + 3 + 4 + 5 + 6$		$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3$	

[1]

(b) Write a formula for C in terms of T .

..... [1]

(c) $1 + 2 + 3 + \dots + 60 = 1830$

Calculate $1^3 + 2^3 + 3^3 + \dots + 60^3$.

..... [2]

7 (a) Complete the table.
Use **Question 6(a)** to help you.

<i>n</i>	Sum of first <i>n</i> cube numbers		Sum of first <i>n</i> 5th powers		$\frac{F}{C}$ as a fraction with denominator 3
	Calculation	Sum (<i>C</i>)	Calculation	Sum (<i>F</i>)	
1	1^3	1	1^5	1	$1 = \frac{3}{3}$
2	$1^3 + 2^3$	9	$1^5 + 2^5$	33	$\frac{33}{9} = \frac{11}{3}$
3	$1^3 + 2^3 + 3^3$	36	$1^5 + 2^5 + 3^5$	276	$\frac{276}{36} = \frac{23}{3}$
4	$1^3 + 2^3 + 3^3 + 4^3$	100	$1^5 + 2^5 + 3^5 + 4^5$	1300	
5	$1^3 + 2^3 + 3^3 + 4^3 + 5^3$	225	$1^5 + 2^5 + 3^5 + 4^5 + 5^5$	4425	$\frac{4425}{225} = \frac{59}{3}$
6	$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3$		$1^5 + 2^5 + 3^5 + 4^5 + 5^5 + 6^5$		

[2]

7

- (b) The fraction $\frac{F}{C}$ is written with a denominator of 3.

Find an expression for the numerator in terms of n .

..... [4]

- (c) $1 + 2 + 3 + \dots + 60 = 1830$

Calculate $1^5 + 2^5 + 3^5 + \dots + 60^5$.

Write down all the numbers on your calculator display.

..... [2]

The modelling starts on the next page.

B MODELLING (QUESTIONS 8 to 13)**INCOME INEQUALITY (30 marks)**

You are advised to spend no more than 50 minutes on this part.

This task looks at a model to measure the spread of income within the population of a country.

In this task, x is the decimal fraction of the population of a country
and y is the decimal fraction of the total income for the country.

Examples

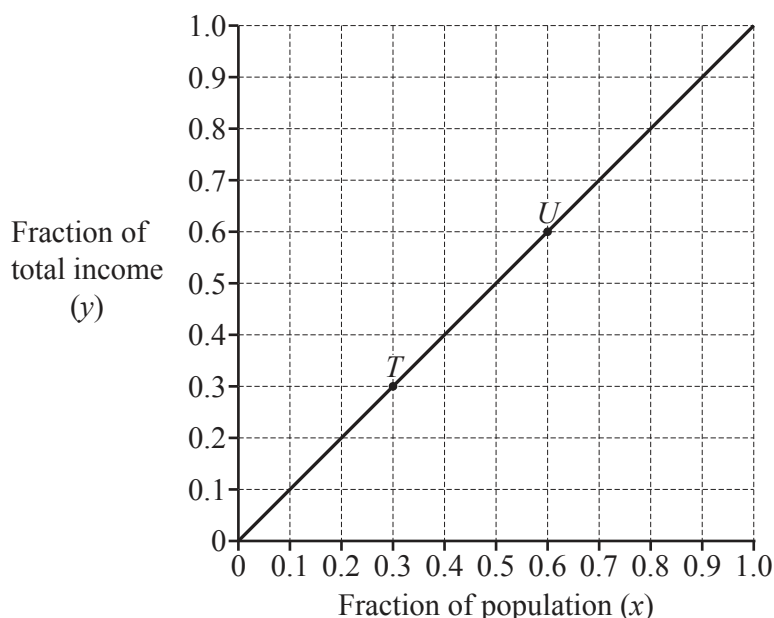
When $x = 1$ this is the total population of a country.

When $x = 0.5$ this is half of the population of a country.

When $y = 1$ this is the total income for the country.

When $y = 0.5$ this is half of the total income for the country.

The graph shows how the total income of a country is shared among the population.



The point T shows that 0.3 of the population of a country earn 0.3 of the total income.

The point U shows that 0.6 of the population of a country earn 0.6 of the total income.

- 8 (a)** Mark a point on the line and label it Z .
Complete this statement for your point.

The point Z shows that of the population of a country earn of the total income.

[1]

- (b)** For this graph there is perfect equality because the income is shared equally among the population of a country.

The graph shows the *line of perfect equality*.

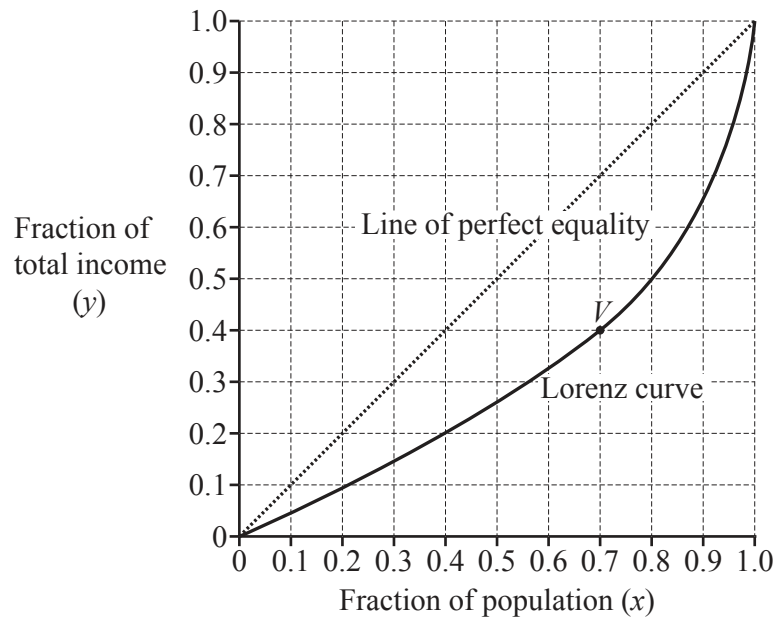
Write down the equation of the line.

..... [1]

10

- 9 In reality, the total income of a country is not shared equally, so there is income inequality. In 1905 the American economist Max Lorenz invented the *Lorenz curve* to show income inequality. A Lorenz curve is always on or below the line of perfect equality.

The graph shows the Lorenz curve for one country.



The point V on this Lorenz curve shows that the poorest 0.7 of the population only earn 0.4 of the total income.

- (a) Use another point on the Lorenz curve to make a similar statement.

.....

..... [1]

- (b) From the statement for point V we can also say that the richest 0.3 of the population earn 0.6 of the total income.

- (i) Write calculations to show why the values in this statement are correct.

.....

..... [1]

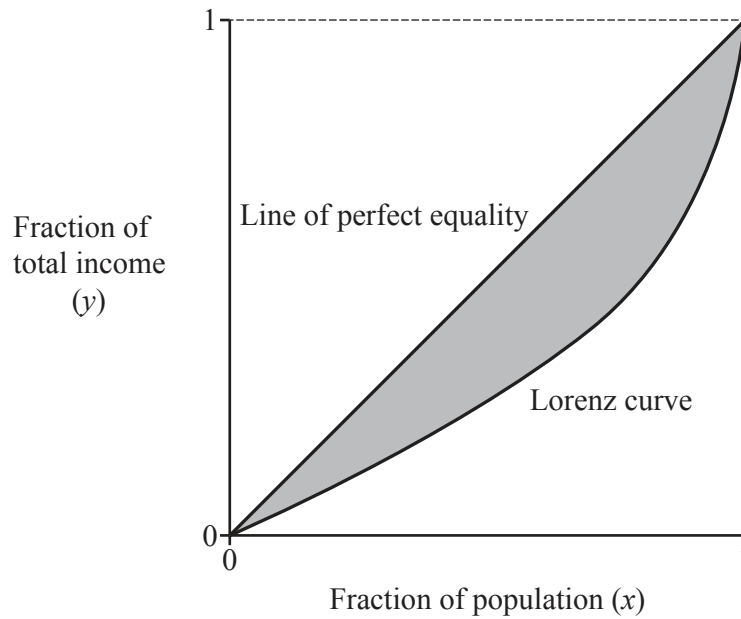
- (ii) Write a similar statement for the point that you chose in **part (a)**.

.....

..... [1]

11

- 10** In 1912 the Italian statistician Corrado Gini invented the *Gini coefficient*.
The Gini coefficient measures how much income inequality there is in a country.



The Gini coefficient is **two times** the shaded area between the line of perfect equality and the Lorenz curve.

This means that the greater the shaded area, the greater the income inequality.

- (a)** When there is perfect equality of income write down the Gini coefficient.

..... [1]

- (b)** When the income inequality is a maximum, the shaded area will be as large as possible.

Find the Gini coefficient when there is maximum inequality of income.

..... [2]

12

11 Mei uses these steps to model the Gini coefficient for the curve in **Question 9**.

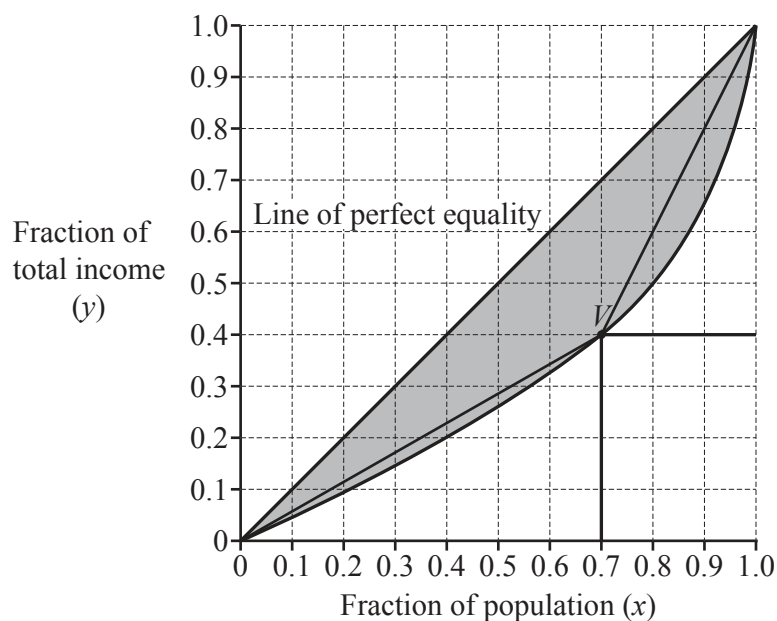
Step 1: Plot point V from **Question 9** on the Lorenz curve.

Step 2: Approximate the area below the curve with a rectangle and two triangles.

Step 3: Calculate the total area of the rectangle and the two triangles. This is T .

Step 4: Calculate the shaded area by subtracting T from the area of the large triangle with vertices $(0, 0)$, $(1, 1)$ and $(1, 0)$.

Step 5: Multiply the result by 2.

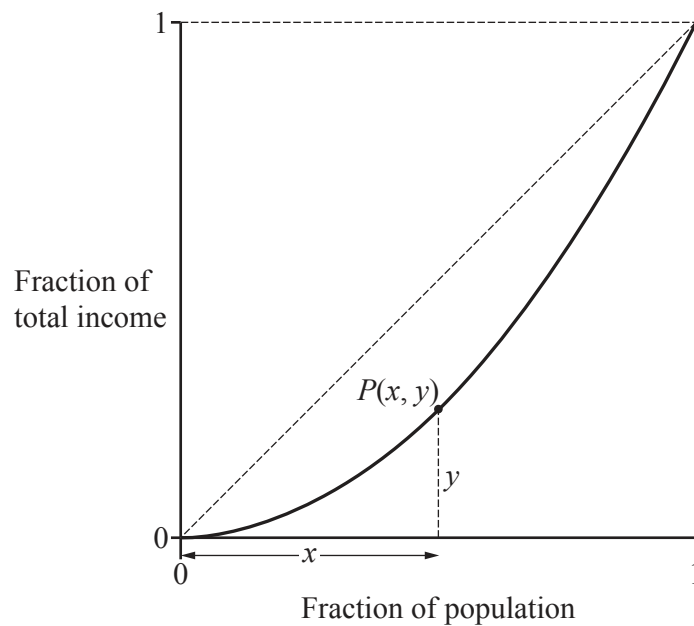


Steps 1 and 2 in Mei's model have been done for you on the graph.

Do Steps 3, 4 and 5 to calculate her approximation of the Gini coefficient.

13

- 12 Mei decides to use a point $P(x, y)$ in Step 1 of her model.



- (a) Find the area in Step 3 of her model as an expression without brackets in terms of x and y .

..... [5]

- (b) Do steps 4 and 5 in her model to show that her approximation, G , for the Gini coefficient is $x - y$.

[2]

- (c) Give a reason why G will be smaller than the actual Gini coefficient.

..... [1]

14

- 13** Mei knows that her approximation, $G = x - y$, is always smaller than the actual Gini coefficient. So, to make her model as accurate as possible, she takes the maximum value of $G = x - y$.

(a) For Country A, the equation of the Lorenz curve is $y = x^2$.

Find Mei's most accurate estimate for the Gini coefficient by finding the maximum of $G = x - x^2$.

..... [2]

15

(b) (i) Write down the coordinates of the two points that must be on every Lorenz curve.

..... [1]

(ii) For Country B, the equation of the Lorenz curve is $y = a - \sqrt{1 - x}$ where a is a constant.

Use **part (i)** to find the value of a .

..... [1]

(iii) Sketch the graph of $G = x - y$ for Country B.



[2]

(c) Use the most accurate estimates in Mei’s model of the Gini coefficient to compare income inequality in Country A and Country B.

..... [2]

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