Mark Scheme (Results)

October 2021

Pearson Edexcel International A Level
In Decision Mathematics (WDM11) Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

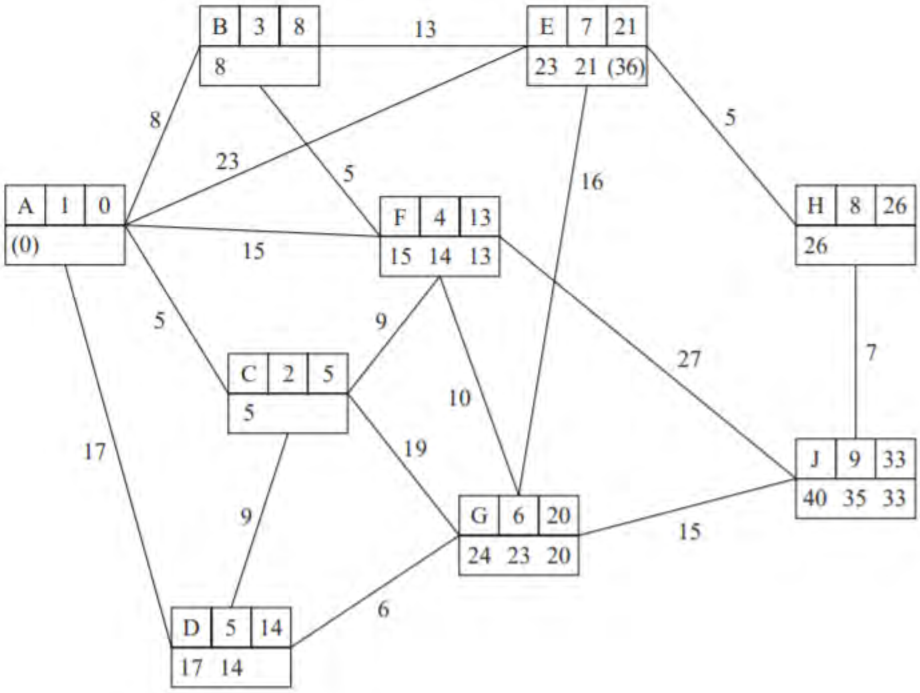
- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- ⌊ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

Question Number	Scheme	Marks
1.(a)	A path is a (i) finite sequence of edges, such that (ii) the end vertex of one edge in the sequence is the start vertex of the next, and in which (iii) no vertex appears more than once	B2, 1, 0 **(2)**
(b)	 A 1 0 (0) B 3 8 8 E 7 21 23 21 (36) H 8 26 26 F 4 13 15 14 13 C 2 5 5 G 6 20 24 23 20 J 9 33 40 35 33 D 5 14 17 14 13 8 23 5 5 16 15 5 9 10 27 7 17 19 9 15 6 Shortest path: A B E H J Length: 33 (km)	M1 A1 (ACBFD) A1 (GE) A1ft (HJ) A1 A1ft **(6)**
(c)	Shortest path from J to A via G: J G D C A Length: 20 + 15 = 35 (km)	B1 B1ft **(2)**
		10 marks

Question Number	Scheme	Marks
	Notes for Question 1	

a1B1: One of the three points made clearly ('finite, edges', 'end vertex of one edge is the start vertex of the next', 'no vertex appears more than once')
a2B1: All three points made clearly. Candidates who state that a path is a walk in which no vertex appears more than once can score B1B0 only

In (b) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at F the working values must be 15 14 13 in that order (so 15 13 14 is incorrect). It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling

b1M1: Working values - a larger value replaced by a smaller value for at least two of the five activities D, E, F, G, J
b1A1: All values at A, C, B, F and D correct and the working values in the correct order
b2A1: All values at G and E correct and the working values in the correct order
b3A1ft: All values in H and J correct on the follow through and the working values in the correct order
b4A1: cao (A B E H J only)
b5A1ft: Follow through on their final value at J **only** (condone lack of units)
c1B1: cao (J G D C A only)
c2B1ft: 35 or follow through their final value at G + 15

Question Number	Scheme	Marks
2.	$y \geq 3x$	B1
	$z - x \geq 50$	B1
	$y \leq 120$	B1
	Sub. $x + y + z = 180$	M1
	$2x + y \leq 130$	A1
	Maximise $(P =) x + y$	B1 **(6)**
		6 marks
	Notes for Question 2	

1B1: cao ($y \geq 3x$) oe (two terms only with integer coefficients)

2B1: cao ($z - x \geq 50$) – may be implied by later working oe (three terms only with integer coefficients)

3B1: $y \leq 120$ oe

1M1: Eliminating *z* by substituting $x + y + z = 180$ into an inequality that involves *z* and *x* only

1A1: $2x + y \leq 130$ oe (three terms only with integer coefficients)

4B1: correct objective with ‘maximise’ or ‘max’ but not ‘maximum’ – either the expression *x* + *y* or any other letter for *P* except *x*, *y* or *z*

Question Number	Scheme	Marks
3. (a)	Prim: AE, EG, CE; DG, CF; DH, BF	M1 A1 A1 **(3)**
(b)	Weight of MST = 197	B1 **(1)**
(c)	Initial upper bound = 2(197) = 394	B1ft **(1)**
(d)	A – E – G – D – H – B – F – C – A	M1
	23 + 24 + 26 + 33 + 38 + 34 + 32 + 38 = 248	A1
	A – E – G – D – H – F – C – B – A	A1
	23 + 24 + 26 + 33 + 38 + 32 + 35 + 36 = 247	A1 **(4)**
(e)	247	B1ft **(1)**
(f)	Weight of RMST is 174	B1ft
	Lower bound = 174 + 23 + 35 = 232	M1 A1 **(3)**
(g)	232 □ optimal value □ 247	M1 A1 **(2)**
		15 marks
	Notes for Question 3	

a1M1: Prim's – first three arcs correctly chosen in order (AE, EG, CE, …) **or** first four nodes {A, E, G, C, …} correctly chosen in order. If any explicit rejections seen at some point then M1 (max) only. Order of nodes may be seen at the top of a matrix/table {1, -, 4, - , 2 , -, 3, -}. Starting at any other node can score M1 only for first three arcs chosen correctly

a1A1: First five arcs correctly chosen in order (AE, EG, CE, DG, CF, …) **or** all eight nodes {A, E, G, C, D, F, H, B} correctly chosen in order. Order of nodes may be seen at the top of a matrix so for the first two marks accept {1, 8, 4, 5, 2, 6, 3, 7} (**no** missing numbers)

a2A1: cso – all **arcs** correctly **stated** and chosen in the correct order (with no additional arcs). They must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen)

b1B1: cao (197 – ignore units) should come from 23 + 24 + 25 + 26 + 32 + 33 + 34

c1B1ft: Follow through double their answer to **(b)**

Mark (d) and (e) together

d1M1: Nearest neighbour starting at A with first five nodes correct (A – E – G – D – H –)

d1A1: One correct route (must return to A)

d2A1: One correct value **or** both correct routes

d3A1: Both correct values (do not isw if values doubled) **and** both correct routes (must both return to A)

SC in (d) correct Hamiltonian paths and corresponding weights (AEGDHBFC (210) and AEGDHFCB (211)) scores M1A1A0A0

e1B1ft: Follow through their least weight route from **(d) –** must have or imply two Hamiltonian cycles in **(d)** or **(e)**

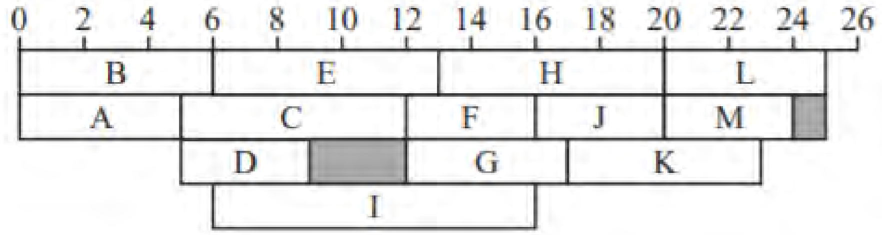
f1B1ft: Either 174 **or** 24 + 25 + 26 + 32 + 33 + 34 **or** 197 – 23 **or** the weight of their MST from **(b)** – 23

f1M1: Weight of RMST + 23 + 35 (two smallest arcs incident to A) with 151£ RMST£ 197 (if clearly not six arcs in RMST then M0)

f1A1: cao (232) – if correct answer with no working then awarded B0M1A1 – as a minimum for full marks accept 174 + 23 + 35 = 232 but 174 + 58 = 232 scores B1M1A0

g1M1: Any indication of an interval from their answer to **(f)** to their answer to **(e)** with one value correct

g1A1: cao (either 232£ optimal value £ 247 or 232 < optimal value £ 247)

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>4. (a)</td><td>
<table><tr><th>Activity</th><th>IPA</th></tr><tr><td>A</td><td>-</td></tr><tr><td>B</td><td>-</td></tr><tr><td>C</td><td>A</td></tr><tr><td>D</td><td>A</td></tr><tr><td>E</td><td>A, B</td></tr></table>
<table><tr><th>Activity</th><th>IPA</th></tr><tr><td>F</td><td>C</td></tr><tr><td>G</td><td>C</td></tr><tr><td>H</td><td>C, D, E</td></tr><tr><td>I</td><td>A, B</td></tr><tr><td>J</td><td>A, B</td></tr></table>
<table><tr><th>Activity</th><th>IPA</th></tr><tr><td>K</td><td>F</td></tr><tr><td>L</td><td>F, G, H, I</td></tr><tr><td>M</td><td>F, G, H, I, J</td></tr></table>
</td><td>B2, 1, 0 (2)</td></tr>
<tr><td>(b)</td><td>$v = 7$, $w = 4$, $x = 6$, $y = 16$, $z = 19$</td><td>B3, 2, 1, 0 (3)</td></tr>
<tr><td>(c)</td><td>$\frac{74}{25} = 2.96$ so 3 workers</td><td>B1 (1)</td></tr>
<tr><td>(d)</td><td>e.g.

 </td><td>M1
A1
A1 (3)</td></tr>
<tr><td>(e)</td><td>New minimum project completion time: 27 (days)</td><td>B1</td></tr>
<tr><td></td><td>Critical path: ACFK</td><td>B1 (2)</td></tr>
<tr><td></td><td></td><td>11 marks</td></tr>
<tr><td></td><td>Notes for Question 4</td><td></td></tr>
</table>

a1B1: Any 6 rows correct (not including A and B)
a2B1: cao (condone blank rows for A and B)

b1B1: Any 2 correct values
b2B1: Any 3 correct values
b3B1: All 5 values correct

c1B1: cao (3 from correct working) – as a minimum for correct working accept either 2.96 or

$$\frac{5+6+7+4+7+4+5+7+10+4+6+5+4}{25} \text{ or } \frac{74}{25}$$

d1M1: Not a cascade chart. 4 workers used at most, at least 9 activities placed
d1A1: 4 workers. All 13 activities present (just once). Condone **two** errors **either** precedence **or** time interval **or** activity length. An activity can give rise to at most three errors; one on duration, one on time interval and only one on IPA
d2A1: 4 workers. All 13 activities present (just once). No errors

e1B1: cao (27 only)
e2B1: cao (ACFK or KFCA only)

Question Number	Scheme	Marks

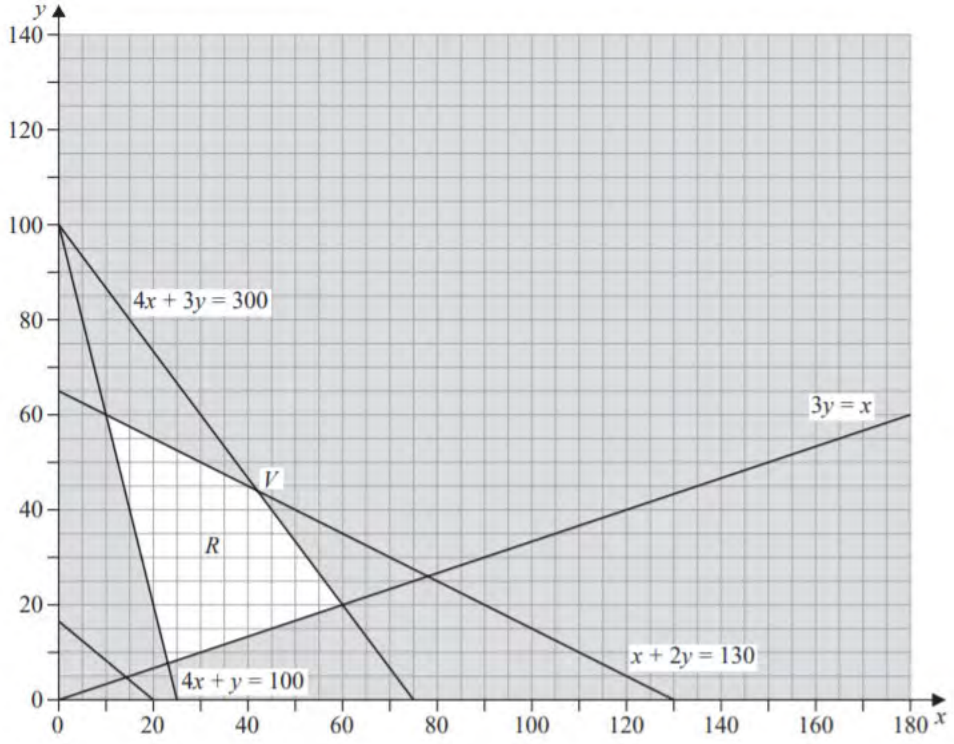
Activity	**Duration**	**Time**	**IPA**
A	5	0 – 6	-
B	6	0 – 6	-
C	7	5 – 13	A
D	4	5 – 13	A
E	7	6 – 13	A, B
F	4	12 – 19	C
G	5	12 – 20	C
H	7	13 – 20	C, D, E
I	10	6 – 20	A, B
J	4	6 – 21	A, B
K	6	16 – 25	F
L	5	20 – 25	F, G, H, I
M	4	20 – 25	F, G, H, I, J

Question Number	Scheme	Marks
5.(a)	A(DG)C + D(GH)E = 12 + 9 = 21	M1 A1
	AD + C(GH)E = 5 + 10 = 15*	A1
	A(DGH)E + C(G)D = 14 + 7 = 21	A1
	Repeated arcs: AD, CG, GH, EH	A1
	Length of route: 166 + 15 = 181 (km)	A1ft **(6)**
(b)	Vertex C: 3 times	B1 **(1)**
(c)	CD (7) is the shortest path between two odd nodes excluding A	M1
	Repeat CGD (7) since this is the shortest path excluding A The route finishes at E	A1
	Length of route = 166 + 7 = 173 (km)	A1 **(3)**
		10 marks
	Notes for Question 5	

a1M1: Three distinct pairings of the correct four odd nodes (A, C, D, E)
a1A1: One row correct including pairings **and** totals
a2A1: Two rows correct including pairings **and** totals
a3A1: All three rows correct including pairings **and** totals
a4A1: The smallest repeat **arcs** (accept AD, CG, GH, EH only)
a5A1ft: Correct answer of 181 **or** 166 + their least

b1B1: cao (3)

c1M1: Identifies the need to repeat one path of the three (DE, CE, CD) which does not include A (this maybe implicit) **or** listing of only these three possible repeats. This mark is dependent on either scoring the M mark in **(a)** or stating all three possible paths in this part. As a minimum accept the stating of one of these three paths
c1A1: Identifies C(G)D as the least **and** E as the finishing point. They have to explicitly state that C(G)D is the least path of those that do not include A (this can be done by stating that CD is the least of CD, CE, DE only (so with no others) **or** stating that CD is the least of those that don't include A but not for just 'CD is the least')
c2A1: cao (173)

Question Number	Scheme	Marks
6. (a)		B1 B1 B1 B1 **(4)**
(b)(i)	A correct objective line	B1
	Correct optimal vertex labelled as *V*	B1 **(2)**
(b)(ii)	Solve correct set of simultaneous equations consistent with either their *V* or objective line	M1
	$(42, 44)$	A1
	77.6	A1 **(3)**
(c)	$(10, 60) > V \Rightarrow 10k + 60 > 42k + 44$ **or** $(60, 20) > V \Rightarrow 60k + 20 > 42k + 44$ $-\frac{1}{2} < -k$ **or** $-\frac{4}{3} > -k$	M1
	$k < \frac{1}{2}$ **or** $k > \frac{4}{3}$	A1
	$(10, 60) > V \Rightarrow 10k + 60 > 42k + 44$ **and** $(60, 20) > V \Rightarrow 60k + 20 > 42k + 44$ $-\frac{1}{2} < -k$ **and** $-\frac{4}{3} > -k$	M1dep
	$k < \frac{1}{2}$ **and** $k > \frac{4}{3}$ only	A1 **(4)**
		13 marks

Question Number	Scheme	Marks
	Notes for Question 6	

The lines in **(a)** must define the correct FR and pass within half a square of the points stated:

$4x + 3y = 300$ with points (0, 100) and (75, 0)

$4x + y = 100$ with points (0, 100) and (25, 0)

$x + 2y = 130$ with points (0, 65) and (130, 0)

$3y = x$ with points (0, 0) and (60, 20)

a1B1: Any two lines correctly drawn
a2B1: Any three lines correctly drawn
a3B1: All four lines correctly drawn
a4B1: Correct *R* labelled – dependent on all three previous B marks
bi1B1: A correct objective line drawn on the graph with a gradient of –0.8 – intersections points with each axes given below

x	y
10	8
20	16
30	24
40	32
50	40
60	48
70	56
80	64
90	72
100	80

x	y
12.5	10
25	20
37.5	30
50	40
62.5	50
75	60
87.5	70
100	80
112.5	90
125	100

bi2B1: cao *V* labelled – dependent on first three B marks in **(a)** and the first B mark in **(b)**
bii1M1: Solving correct pair of simultaneous equations for their *V* (or if not labelled then the vertex consistent with their objective line) – this mark can be implied by (42, 44) but in all cases they must have drawn four lines with at least two correct **and** an attempt at an objective line
bii1A1: cao ((42, 44) only) – dependent on first three B marks in **(a)** and the first B mark in **(b)**
bii2A1: cao (77.6 only) – dependent on first three B marks in **(a)** and the first B mark in **(b)**

SC in (b) if no objective line drawn then can score in (b) B0B0M1A1A0 for both (42, 44) and 77.6 only provided that the first three B marks earned in (a)
Marks in part (c) are not dependent on the marks in (a) and/or (b)
c1M1: Point testing method: $10k + 60 \square kx_1 + y_1$ **or** $60k + 20 \square kx_1 + y_1$ **or** $42k + 44 \square kx_1 + y_1$ where $\square$ is any inequality sign or the equals sign and (x_1, y_1) is their numerical *V* or (42, 44). Objective line method: $-\frac{1}{2} \square - k$ or $-\frac{4}{3} \square - k$ or $\frac{1}{3} \square - k$ where $\square$ is any inequality or equals. Or one correct answer stated
c1A1: One correct answer $(k < \frac{1}{2}, k \text{ £ } \frac{1}{2}, k > \frac{4}{3}, k \text{ ³ } \frac{4}{3})$ - if no method or working (as shown above) then **A0**
c2M1dep: Point testing: $10k + 60 \square kx_1 + y_1$ **and** $60k + 20 \square kx_1 + y_1$ where $\square$ is any inequality sign or the equals sign and (x_1, y_1) is (42, 44) or their *V* (but not ((10, 60) or (60, 20)) (so *V* must now be the intersection of the two lines 4*x* + 3*y* = 300 **and** *x* + 2*y* = 130). Objective line: $-\frac{1}{2} \square - k$ **and** $-\frac{4}{3} \square - k$ where $\square$ is any inequality or equals. Or both correct answers stated with no working
c2A1: Both correct answers **only** $(k < \frac{1}{2}$ or $k \text{ £ } \frac{1}{2}$ **and** $k > \frac{4}{3}$ or $k \text{ ³ } \frac{4}{3})$ with working as shown above

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>**7. (a)**</td><td>$3 < \frac{228}{n} \leqslant 4$</td><td>M1</td></tr>
<tr><td></td><td>Critical value of 57 and 76 (or 57 and 75)</td><td>A1</td></tr>
<tr><td></td><td>$57 \leqslant n < 76$ (or $57 \leqslant n \leqslant 75$)</td><td>A1 **(3)**</td></tr>
<tr><td>**(b)**</td><td>e.g. middle right
<table>
<tr><td>14</td><td>20</td><td>23</td><td>17</td><td>15</td><td>**22**</td><td>19</td><td>25</td><td>13</td><td>28</td><td>32</td></tr>
<tr><td>23</td><td>25</td><td>**28**</td><td>32</td><td>22</td><td>14</td><td>20</td><td>17</td><td>**15**</td><td>19</td><td>13</td></tr>
<tr><td>32</td><td>28</td><td>23</td><td>**25**</td><td>22</td><td>20</td><td>**17**</td><td>19</td><td>15</td><td>14</td><td>**13**</td></tr>
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</table>
e.g. middle left
<table>
<tr><td>14</td><td>20</td><td>23</td><td>17</td><td>15</td><td>**22**</td><td>19</td><td>25</td><td>13</td><td>28</td><td>32</td></tr>
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<tr><td>32</td><td>28</td><td>25</td><td>23</td><td>22</td><td>20</td><td>19</td><td>17</td><td>15</td><td>14</td><td>13</td></tr>
</table>
</td><td>M1
A1
A1ft
A1 **(4)**</td></tr>
<tr><td>**(c)**</td><td>From first-fit Bin 1 could not fit the 17 so $n < 74$ (or $n \leqslant 73$) but could fit the 15 so n is either 72 (as the largest total is 72 in Bin 1 from first-fit) or 73</td><td>B1</td></tr>
<tr><td></td><td>From first-fit decreasing the 13 could not fit in Bin 1</td><td>B1</td></tr>
<tr><td></td><td>So $n = 72$</td><td>ddB1 **(3)**</td></tr>
<tr><td></td><td></td><td>**10 marks**</td></tr>
</table>

Question Number	Scheme	Marks
	Notes for Question 7	

a1M1: An equation or inequality linking the expression $\frac{228}{n}$ with either 3 or 4

a1A1: Correct critical values of 57 and 76 (or 57 and 75)
a2A1: $57 \leq n < 76$ or $57 \leq n \leq 75$

b1M1: Quick sort – pivots, p, selected and first pass gives >p, p, <p. If only choosing 1 pivot per iteration M1 only. If sorting into ascending order then mark as a misread
b1A1: First pass correct and next pivots chosen correctly/consistently for second pass
b2A1ft: Second and third passes correct (ft from their first pass and choice of pivots)
b3A1: cso (including a fourth pass with 19 used as a pivot if middle right or 14 if middle left)

c1B1: Correct deduction from first-fit that *n* is at least 72 **or** at most 73 (oe e.g. less than 74). For example, may see (14 + 20 + 23 + 15 =) 72 stated therefore $n \geq 72$ **or** 14 + 20 + 23 + 17 = 74 followed by $n < 74$ (so realising that the 17 did not fit in Bin 1). As a minimum accept the statement that $n < 74$ **or** $n \leq 73$ **or** $n \geq 72$
c2B1: Correct deduction from first-fit decreasing that the 13 was not placed in Bin 1. For example, may see 32 + 28 + 13 = 73 so therefore *n* < 73 or $n \leq 72$. As a minimum accept the statement that $n < 73$ **or** $n \leq 72$ **or** simply stating that ’13 did not fit in Bin 1’ (give bod here if not clear which Bin 1 they are considering)
c3ddB1: cao (dependent on both previous B marks) – must state that the largest total in any bin is 72 (**or** they need to say or show there exists a bin with 72) **and** that the 13 did not fit in Bin 1 in first-fit decreasing so *n* cannot be 73 and therefore *n* = 72 (not just ‘13 does not fit in Bin 1’ – must be clear that they are talking about first-fit decreasing)

No marks in **(c)** if *n* = 72 stated with no working or if all the candidate does is to sum the numbers in each bin

Note that the first B mark in **(c)** can be implied if the candidate considers the first-fit decreasing packing first or argues with first-fit decreasing before considering first-fit, e.g., ’The 13 does not fit in Bin 1 in the first-fit decreasing packing therefore *n* is at most 72 and the total of Bin 1 in first-fit is 72’ would imply the first two B marks in this part. Stating that therefore *n* must be 72 would then score all three marks

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