



Mark Scheme (Results)

Summer 2024

Pearson Edexcel International Advanced Level
In Decision Mathematics (WDM11) Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

The total number of marks for the paper is 75.

The Edexcel Mathematics mark schemes use the following types of marks:

M marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.

A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.

B marks are unconditional accuracy marks (independent of M marks)

Marks should not be subdivided.

Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:

bod – benefit of doubt

ft – follow through

the symbol \surd will be used for correct ft

cao – correct answer only

cso – correct solution only. There must be no errors in this part of the question to obtain this mark

isw – ignore subsequent working

awrt – answers which round to

SC – special case

oe – or equivalent (and appropriate)

d... or dep – dependent

indep – independent

dp – decimal places

sf – significant figures

* – The answer is printed on the paper or ag- answer given

\square or d... – The second mark is dependent on gaining the first mark

All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through.

For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

If a candidate makes more than one attempt at any question:

- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
- If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

Ignore wrong working or incorrect statements following a correct answer.

Question Number	Scheme	Marks																																																																		
1.(a)	Bin 1: 5.2 4.7 3.1 Bin 2: 6.5 4.5 1.8 1.2 Bin 3: <u>5.1</u> <u>2.9</u> 3.4 Bin 4: 3.8	M1 A1 A1(3)																																																																		
(b)	e.g. middle right <div style="display: flex; justify-content: space-between;"> <table style="border-collapse: collapse;"> <tr><td>5.2</td><td>4.7</td><td>6.5</td><td>4.5</td><td>3.1</td><td>5.1</td><td>1.8</td><td>2.9</td><td>3.4</td><td>3.8</td><td>1.2</td></tr> <tr><td>4.7</td><td>4.5</td><td>3.1</td><td>1.8</td><td>2.9</td><td>3.4</td><td>3.8</td><td>1.2</td><td><u>5.1</u></td><td>5.2</td><td>6.5</td></tr> <tr><td>1.8</td><td>1.2</td><td><u>2.9</u></td><td>4.7</td><td>4.5</td><td>3.1</td><td>3.4</td><td>3.8</td><td><u>5.1</u></td><td>5.2</td><td><u>6.5</u></td></tr> <tr><td><u>1.2</u></td><td>1.8</td><td><u>2.9</u></td><td><u>3.1</u></td><td>4.7</td><td>4.5</td><td>3.4</td><td>3.8</td><td><u>5.1</u></td><td>5.2</td><td><u>6.5</u></td></tr> <tr><td><u>1.2</u></td><td>1.8</td><td><u>2.9</u></td><td><u>3.1</u></td><td><u>3.4</u></td><td>4.7</td><td>4.5</td><td>3.8</td><td><u>5.1</u></td><td>5.2</td><td><u>6.5</u></td></tr> <tr><td><u>1.2</u></td><td>1.8</td><td><u>2.9</u></td><td><u>3.1</u></td><td><u>3.4</u></td><td>3.8</td><td><u>4.5</u></td><td>4.7</td><td><u>5.1</u></td><td>5.2</td><td><u>6.5</u></td></tr> </table> <div style="text-align: right;"> Pivots 5.1 2.9, 6.5 1.2, 3.1, (5.2) (1.8), 3.4 4.5 (Sort Complete) </div> </div>	5.2	4.7	6.5	4.5	3.1	5.1	1.8	2.9	3.4	3.8	1.2	4.7	4.5	3.1	1.8	2.9	3.4	3.8	1.2	<u>5.1</u>	5.2	6.5	1.8	1.2	<u>2.9</u>	4.7	4.5	3.1	3.4	3.8	<u>5.1</u>	5.2	<u>6.5</u>	<u>1.2</u>	1.8	<u>2.9</u>	<u>3.1</u>	4.7	4.5	3.4	3.8	<u>5.1</u>	5.2	<u>6.5</u>	<u>1.2</u>	1.8	<u>2.9</u>	<u>3.1</u>	<u>3.4</u>	4.7	4.5	3.8	<u>5.1</u>	5.2	<u>6.5</u>	<u>1.2</u>	1.8	<u>2.9</u>	<u>3.1</u>	<u>3.4</u>	3.8	<u>4.5</u>	4.7	<u>5.1</u>	5.2	<u>6.5</u>	M1 A1 A1ft A1 (4)
5.2	4.7	6.5	4.5	3.1	5.1	1.8	2.9	3.4	3.8	1.2																																																										
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5.2	4.7	6.5	4.5	3.1	5.1	1.8	2.9	3.4	3.8	1.2																																																										
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(c)	Bin 1: 6.5 5.2 1.8 Bin 2: 5.1 4.7 3.8 Bin 3: 4.5 3.4 3.1 2.9 Bin 4: 1.2	M1 A1ft A1(3)																																																																		
(d)	$\frac{1.2+1.8+\dots+6.5}{14} = \frac{42.2}{14} = 3.01\dots$ so a minimum of 4 bins is required and (c) is optimal	B1 (1)																																																																		
(e)	$\left[\frac{1+11}{2} \right] = 6 \quad 3.8 > 3.0 \text{ so reject } 3.8 \text{ to } 6.5 \text{ (or reject } 6^{\text{th}} \text{ to } 11^{\text{th}} \text{ items)}$ $\left[\frac{1+5}{2} \right] = 3 \quad 2.9 < 3.0 \text{ so reject } 1.2 \text{ to } 2.9 \text{ (or reject } 1^{\text{st}} \text{ to } 3^{\text{rd}} \text{ item)}$ $\left[\frac{4+5}{2} \right] = 5 \quad 3.4 > 3.0 \text{ so reject } 3.4 \text{ (or reject } 5^{\text{th}} \text{ item)}$ $[4] = 4 \quad 3.1 \text{ is not } 3.0 \text{ so } 3.0 \text{ not found}$	M1 A1ft A1 (3)																																																																		
		14 marks																																																																		

Notes for Question 1	
a1M1	The correct first five items placed correctly (the bold values) and at least eight values placed in bins (allow repeated values). Condone cumulative totals for M1 only
a1A1	First eight values placed correctly (the bold and underlined values) with all eleven correct values only placed in bins. This mark cannot be awarded if any repeated values or incorrect values are seen (even if the first eight values have been placed correctly)
a2A1	CSO – no additional or repeated values (dependent on both previous marks)
b1M1	Quick sort using all 11 numbers (condone one slip), pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is M0). After the first pass the list must read (values less than the pivot), pivot, (values greater than the pivot). If only choosing one pivot per iteration then M1 only. Bubble sort is M0.
b1A1	First and second passes correct and pivots correctly chosen for third pass
b2A1ft	Third and fourth passes correct following through from their second pass and choice of pivots for the third pass. The pivots for the third pass must be consistent (if appropriate either both middle left or both middle right)
b3A1	CSO (correct solution only – all previous marks in this part must have been awarded) including a fifth pass (we do not need to see “sort complete” oe)
	Special Case for (b) If the candidate sorts into descending order they can score M1 as per the main scheme (but with the values either side of the pivot reversed), A1 for a fully correct sort then A0 A0 even if the list is reversed at the end (so 2 marks max.). See below
c1M1	Their six largest items placed correctly and at least eight values placed in bins (if correct this will be the bold items but must check their packing if any of their six largest values are incorrect – note that the maximum weight of a bin is 14). Condone cumulative totals for M1 only. First-fit increasing scores no marks in this part. If no sort seen in (b) then mark (c) assuming the correct ordered list is being used
c1A1ft	Their first nine values placed correctly (the bold and underlined values) with all their eleven values only placed in bins. This mark cannot be awarded if any repeated values are seen (even if their first nine values have been placed correctly)
c2A1	CSO – must be the correct values with no additional or repeated values (dependent on both previous marks)
d1B1	CAO using either a correct lower bound calculation (accept awrt 3.01 without seeing the full calculation) or spare capacity in the first 3 bins (e.g. total spare in first three bins is $0.5 + 0.4 + 0.1 = 1 < 1.2$ so not possible to pack into 3 bins oe e.g. $42.2 > 42$ hence cannot fit in 3 bins) and conclusion of optimal or states using the minimum number of bins (dependent on scoring at least M1 in (c))
e1M1	Must be using their sorted list. Choosing their middle pivot (3.8) and an attempt at discarding/retaining half the list (condone if retaining the wrong half of the list) (If pivot is retained at any stage M1 only). The actual rejections may be shown as values from the list or the positions in the list.
e1A1ft	First and second passes correct for their list i.e. selecting the 6 th item in the first pass and using 1 st to 5 th items in the second pass (must not be using the 6 th item for the second pass) and then correctly selecting the 3 rd item (the 2.9) in the second pass and rejecting the 1 st to 3 rd items. Candidates may either compare 3.0 with the actual values or the positions of items in a correctly sorted list. Comparisons may be implied by the choice of the correct item and corresponding rejection. If a candidate renumbers the list after a pass check the position numbers and values carefully.
e2A1	CSO – must be the correct values with the search completed correctly (so rejecting the 3.4 in the third pass) together with ‘not found’.

In (e) if candidates use the list sorted into descending order, they can score full marks. The scheme above applies in the same way. **Note there are only 3 comparisons needed.**

6.5 5.2 5.1 4.7 4.5 3.8 3.4 3.1 2.9 1.8 1.2

$$\left[\frac{1+11}{2} \right] = 6 \quad 3.8 > 3.0 \text{ so reject } 6.5 \text{ to } 3.8$$

$$\left[\frac{7+11}{2} \right] = 9 \quad 2.9 < 3.0 \text{ so reject } 2.9 \text{ to } 1.2$$

$$\left[\frac{7+8}{2} \right] = 8 \quad 3.1 > 3.0 \text{ so reject } 3.4 \text{ to } 3.1$$

3.0 is not in the list

For reference – descending sort

Middle right

5.2	4.7	6.5	4.5	3.1	5.1	1.8	2.9	3.4	3.8	1.2	Pivots
5.2	6.5	<u>5.1</u>	4.7	4.5	3.1	1.8	2.9	3.4	3.8	1.2	5.1
<u>6.5</u>	5.2	<u>5.1</u>	4.7	4.5	3.1	3.4	3.8	<u>2.9</u>	1.8	1.2	(5.2), 3.1, 1.2
<u>6.5</u>	<u>5.2</u>	<u>5.1</u>	4.7	4.5	3.4	3.8	<u>3.1</u>	<u>2.9</u>	1.8	<u>1.2</u>	3.4 (1.8)
<u>6.5</u>	<u>5.2</u>	<u>5.1</u>	4.7	4.5	3.8	<u>3.4</u>	<u>3.1</u>	<u>2.9</u>	<u>1.8</u>	<u>1.2</u>	4.5
<u>6.5</u>	<u>5.2</u>	<u>5.1</u>	4.7	<u>4.5</u>	3.8	<u>3.4</u>	<u>3.1</u>	<u>2.9</u>	<u>1.8</u>	<u>1.2</u>	(Sort Complete)

Middle left

5.2	4.7	6.5	4.5	3.1	5.1	1.8	2.9	3.4	3.8	1.2	Pivots
5.2	6.5	<u>5.1</u>	4.7	4.5	3.1	1.8	2.9	3.4	3.8	1.2	5.1
6.5	<u>5.2</u>	<u>5.1</u>	4.7	4.5	3.1	2.9	3.4	3.8	<u>1.8</u>	1.2	(6.5), 3.1, (1.2)
<u>6.5</u>	<u>5.2</u>	<u>5.1</u>	4.7	4.5	3.4	3.8	<u>3.1</u>	2.9	<u>1.8</u>	<u>1.2</u>	4.5, (2.9)
<u>6.5</u>	<u>5.2</u>	<u>5.1</u>	4.7	<u>4.5</u>	3.4	3.8	<u>3.1</u>	<u>2.9</u>	<u>1.8</u>	<u>1.2</u>	(4.7), 3.4
<u>6.5</u>	<u>5.2</u>	<u>5.1</u>	4.7	<u>4.5</u>	3.8	<u>3.4</u>	<u>3.1</u>	<u>2.9</u>	<u>1.8</u>	<u>1.2</u>	(Sort Complete)

Question Number	Scheme	Marks
2. (a)(i)		<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>
(a)(ii)	Minimum completion time is 20 (days)	A1ft (5)
(b)	Lower bound = $\frac{59}{20} = 2.95 = 3$ workers	B1 (1)
(c)		<p>M1</p> <p>A1</p> <p>A1</p> <p>A1 (4)</p>
		10 marks

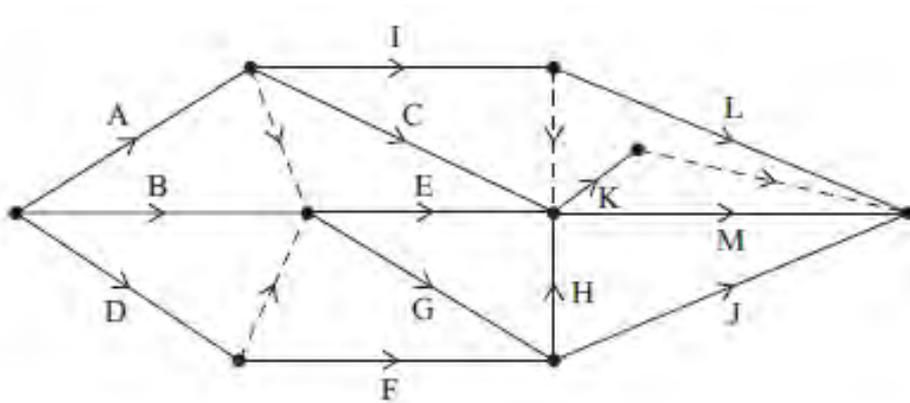
Notes for Question 2																																																													
ai1M1	All top boxes complete, values generally increasing in the direction of the arrows ('left to right'), condone one rogue																																																												
ai1A1	CAO (top boxes)																																																												
ai2M1	All bottom boxes complete, values generally decreasing in the opposite direction of the arrows ('right to left'), condone one rogue. Condone missing 0 and/or their 20 (at the end event) for the M mark only																																																												
ai2A1	CAO (bottom boxes)																																																												
aii3A1ft	Follow through the top value at the end node – dependent on both M marks – if labelled must be answered in the correct part of the question																																																												
b1B1	CSO – requires both a correct calculation or 2.95 seen and 3. An answer of 3 with no working scores no marks																																																												
c1M1	Not a cascade (Gantt) chart. 5 'workers' used at most and at least 8 activities placed. End time must be the same as their end time from the diagram in (a)(i)																																																												
c1A1	3 or 4 workers. All 14 activities present (just once). Condone at most three errors. An activity can give rise to at most three errors; one on duration, one on time interval and only one on IPA																																																												
c2A1	3 or 4 workers. All 14 activities present (just once). Condone one error either precedence or time interval or activity length; An activity can give rise to at most three errors; one on duration, one on time interval and only one on IPA																																																												
c3A1	3 workers. All 14 activities present (just once). No errors.																																																												
For (c) the following may be useful in checking their schedule Note J and K are interchangeable																																																													
<table border="1"> <thead> <tr> <th>Activity</th> <th>Duration</th> <th>Time interval</th> <th>IPA</th> </tr> </thead> <tbody> <tr><td>A</td><td>5</td><td>0 – 5</td><td>-</td></tr> <tr><td>B</td><td>4</td><td>0 – 12</td><td>-</td></tr> <tr><td>C</td><td>7</td><td>0 – 11</td><td>-</td></tr> <tr><td>D</td><td>4</td><td>5 – 9</td><td>A</td></tr> <tr><td>E</td><td>3</td><td>5 – 12</td><td>A</td></tr> <tr><td>F</td><td>3</td><td>9 – 12</td><td>D</td></tr> <tr><td>G</td><td>2</td><td>9 – 17</td><td>D</td></tr> <tr><td>H</td><td>4</td><td>9 – 18</td><td>D</td></tr> <tr><td>I</td><td>6</td><td>12 – 18</td><td>B, C, E, F</td></tr> <tr><td>J</td><td>7</td><td>7 – 18</td><td>C</td></tr> <tr><td>K</td><td>7</td><td>7 – 18</td><td>C</td></tr> <tr><td>L</td><td>3</td><td>11 – 20</td><td>G</td></tr> <tr><td>M</td><td>2</td><td>18 – 20</td><td>G, H, I, J, K</td></tr> <tr><td>N</td><td>2</td><td>14 – 20</td><td>K</td></tr> </tbody> </table>		Activity	Duration	Time interval	IPA	A	5	0 – 5	-	B	4	0 – 12	-	C	7	0 – 11	-	D	4	5 – 9	A	E	3	5 – 12	A	F	3	9 – 12	D	G	2	9 – 17	D	H	4	9 – 18	D	I	6	12 – 18	B, C, E, F	J	7	7 – 18	C	K	7	7 – 18	C	L	3	11 – 20	G	M	2	18 – 20	G, H, I, J, K	N	2	14 – 20	K
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Question Number	Scheme	Marks
3.(a)		<p>M1</p> <p>A1 (ACBFG)</p> <p>A1 (DE)</p> <p>A1ft (HJ)</p>
	Shortest time to travel from A to J is 74 (mins)	A1ft
	Quickest route is ACBFGDEHJ	A1 (6)
b)	As the quickest route found in (a) is a path through all vertices in the network the weight of this path is equal to the weight of the MST for the network	M1
	Weight of MST is 74 (mins)	A1ft (2)
		8 marks
Notes for Question 3		
<p>In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at E the working values must be 71 61 52 in that order (so 71 52 61 is incorrect)</p> <p>It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, ... will be penalised once (see notes below) but 1, 2, 3, 5, 6, ... is fine.</p> <p>Errors in the final values and working values are penalised before errors in the order of labelling</p>		
a1M1	A larger value replaced by a smaller value at least twice in the working values at either B, D, E, F, H, J	
a1A1	All values at A, C, B, F and G correct and the working values in the correct order	
a2A1	All values at D and E correct and the working values in the correct order	
a3A1ft	All values in H and J correct on the follow through and the working values in the correct order. To follow through H check that the working values at H follow from the candidate's final values for the nodes that are directly attached to H (which are D, E and G (and J)). For example, if correct then the order of labelling of nodes D, E and G are 6, 7 and 5 respectively so the working values at H should come from G, D and E in that order. The first working value at H should be their 32 (the Final value at G) + 51 (the weight of the arc GH), the second working value at H should be their 45 (the Final value at D) + 18 (the weight of the arc DH) and the third working value should be their 52 (the Final value at E) + 9 (the weight of the arc EH) . Repeat the process for J (which will have working values from D, G and H with the order of these nodes determined by the candidate's order of labelling at D, G and H)	
a4A1ft	Follow through their final value at J only – if answer is 74 but this is not the Final Value at J then A0	
a5A1	CAO (ACBFGDEHJ)	
b1M1	A statement that the route found in (a) is the MST, either stated directly, or an indication that the route has visited all 9 nodes.	
b1A1ft	Follow through their answer to (a) provided that the route from (a) passed through all nine nodes in the network (a weight of 74 with no reason scores M0)	

Question Number	Scheme	Marks
4.(a)	Prim: AC, AB, AE; DE, DF, BJ; HJ, GH or AC, AE, DE; AB, DF, BJ; HJ, GH	M1 A1 A1 (3)
(b)	127 (km)	B1 (1)
(c)	If starting at E and finishing at F then the shortest path between C and H needs to be traversed twice. So the length of the route is $58 + 494 = 552$ (km)	M1 A1 (2)
(d)	Route must start at C and finish at A therefore need to consider pairings of the nodes A, E, F and H	M1
	AE + FH = $15 + 53 = 68$ AF + EH = $42 + 66 = 108$ AH + EF = $51 + 27 = 78$	A1 A1
	Repeat roads: AE, FG, GH	A1 (4)
ALT	Special Case – considers C, E, F and H – mark as misread removing final 2 A marks earned in this section	M1
	CE + FH = $22 + 53 = 75$ CF + EH = $49 + 66 = 115$ CH + EF = $58 + 27 = 85$	A1 A1
	Repeat roads: CA, AE, FG, GH	A1 (4)
(e)	B – A – C – E – D – F – G – H – J – B $15 + 7 + 22 + 10 + 17 + 40 + 13 + 19 + 31 = 174$ (km)	M1 A1 (2)
(f)	$(127 - 7) + 7 + 22 = 149$ (km)	M1 A1 (2)
		14 marks

Notes for Question 4	
a1M1:	Prim's – first three arcs correctly chosen in order (AC, AB, AE, ... or AC, AE, DE) or first four nodes {A, C, B, E, ... or A, C, E, D,...} correctly chosen in order. If any explicit rejections seen at some point then M1 (max) only. Order of nodes may be seen at the top of a matrix/table {1, 3, 2, -, 4, -, -, -, - or 1, -, 2, 4, 3, -, -, -, -}. Starting at any other node can score M1 only for first three arcs chosen correctly
a1A1:	First six arcs correctly chosen in order {AC, AB, AE, DE, DF, BJ, ...} or all nine nodes {A, C, B, E, D, F, J, H, G} correctly chosen in order. Order of nodes may be seen at the top of a matrix so for the first two marks accept {1, 3, 2, 5, 4, 6, 9, 8, 7} (no missing numbers). Or the alternative six arcs {AC, AE, DE, AB, DF, BJ,...}, nine nodes {A, C, E, D, B, F, J, H, G} or numbers {1, 5, 2, 4, 3, 6, 9, 8, 7}
a2A1:	CSO – all arcs correctly stated and chosen in the correct order (with no additional arcs). They must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen)
b1B1:	CAO (127)
c1M1:	Indication that the shortest path between C and H needs to be traversed twice (correct answer can imply this mark)
c1A1:	CAO (552)
d1M1:	The correct three pairings of the correct four nodes (A, E, F and H)
d1A1:	Two rows correct including pairings and totals
d2A1:	All three rows correct including pairings and totals
d3A1:	CAO (AE, FG, GH only but in any order) – must be stated as edges so A0 for AE, FH or AE F(G)H
	Special Case – considers C, E, F and H – mark as misread removing last two A marks earned in this section, so max 2/4
d1M1:	The correct three pairings of the four nodes (C, E, F and H)
d1A1:	Two rows correct including pairings and totals
d2A1:	All three rows correct including pairings and totals
d3A1:	CAO (CA, AE, FG, GH only but in any order) – must be stated as edges so A0 for CE, FH or C(A)E F(G)H
e1M1:	Nearest neighbour starting at B with first five nodes correct (B – A – C – E – D –) Accept arcs so BA, AC, CE, ED,
e1A1:	Correct nearest neighbour route (must return to B) and correct length (174) Accept arcs so BA, AC, CE, ED, DF, FG, GH, HJ, JB
f1M1:	(weight of their MST from (b) or 127 only) – $7 + 7(AC) + 22(CE \text{ or } CB)$ (oe so may not see the $-7 + 7$). A correct answer of 149 with no working can imply this (and the next) mark. If (b) is incorrect a value of their “127” from (b) plus 22 scores M1 (Note new calculation to find RMST $120 + 7 + 22 = 149$ may be seen)
f1A1:	CAO (149)

Question Number	Scheme	Marks
5.(a)	$\frac{1}{2}(x + y + z) \leq y \quad (\Rightarrow x - y + z \leq 0)$	M1
	$\frac{3}{20}(x + y + z) \geq z \quad (\Rightarrow 3x + 3y - 17z \geq 0)$	M1
	$2x = 5z$	B1
	$5x + 12y + 15z \leq 834$	B1
	Eliminating z from the objective $x + y + z$ and at least one correct constraint or states the objective and eliminates z from at least two correct constraints	M1
	Maximise ($P =$) $1.4x + y$ subject to $11x + 12y \leq 834$ $7x - 5y \leq 0$ $19x - 15y \leq 0$ $(x \geq 0, y \geq 0)$	A1 A1 (7)
(b)(i)	Substitute $y = 42$ into LP gives $x \leq 30, x \leq 30, x \leq \frac{630}{19}$ which implies that $x = 30$	M1
	Total number of reams ordered is $1.4(30) + 42 = 84$	A1
(b)(ii)	12 reams of graph paper ordered	A1 (3)
		10 marks
Notes for Question 5		
a1M1	$\frac{1}{2}(x + y + z) \bullet y$ where \bullet is any inequality or equals sign (accept any equivalent form)	
a2M1	$\frac{3}{20}(x + y + z) \bullet z$ where \bullet is any inequality or equals sign (accept any equivalent form)	
a1B1	Correct relationship between x and z (accept any equivalent form)	
a2B1	CAO ($5x + 12y + 15z \leq 834$)	
a3M1	Either eliminating z (using either $2x = 5z$ or $2z = 5x$ oe) from the correct objective and at least one correct constraint or states objective in the form ($P =$) $x + y + z$ and eliminates z from at least two correct constraints	
a1A1	Either two correct inequalities in x and y only or one correct inequality and the correct objective (but do not need 'maximise' or 'max') in x and y only – note the correct inequality must have integer coefficients (accept equivalent forms for the objective e.g. $5P = 7x + 5y$ (must include $5P =$) if $P = 7x + 5y$ stated do not ISW)	
a2A1	Correct LP formulation in x and y including 'maximise' or 'max' with objective (condone if previously stated) – note that all constraints must have integer coefficients (condone lack of $x \geq 0, y \geq 0$). Do not ISW if any incorrect simplifications seen.	
bi1M1	Substituting $y = 42$ into at least two of their constraints (must only be in terms of x and y) leading to a value of x (if they over simplify and assume that 42 is exactly $\frac{1}{2}$ of the total number of reams, they must still substitute into at least 2 constraints)	
bi1A1	CAO (84) – must come from correct working	
bii1A1	CAO (12) – must come from correct working	

Question Number	Scheme	Marks
6.(a)	e.g. 	M1 A1 (A B D I C F) A1 (E G H J) A1 (K L M) A1 (5)
(b)	Activities B, G, H and M are critical	B1 (1)
(c)	At time 9.5, activities G, C, E and I must all be happening therefore a minimum of 4 workers is required	M1 A1 (2)
(d)	$2 \leq x \leq 6$	B2, 1, 0 (2)
10 marks		

Notes for Question 6

Condone lack of, or incorrect, numbered events throughout. ‘Dealt with correctly’ means that the activity starts from the correct event but need not necessarily finish at the correct event, e.g. ‘J dealt with correctly’ requires the correct precedences for this activity, i.e. F and G labelled correctly and leading into the same node and J starting from that node but do not consider the end event for J. **Activity on node is M0**

If an arc is not labelled, for example, if the arc for activity F is not labelled (but the arc is present) then this will lose the first A mark and the final (CSO) A mark – they can still earn the second A mark on the bod. If two or more arcs are not labelled then mark according to the scheme. Assume that a solid line is an activity which has not been labelled rather than a dummy (even if in the correct place for where a dummy should be). **Ignore missing arrows on the activities for the first four marks only**

a1M1	Nine activities (labelled on arc), one start and at least two dummies placed
a1A1	Activities A, B, D, 1 st two dummies (including correct arrows), I, C and F dealt with correctly
a2A1	Activities E, G, H and J dealt with correctly
a3A1	Activities K, L and M dealt with correctly (so dummy at the end of I required + arrow)
a4A1	CSO – all arrows correctly placed for each activity with one finish and exactly four dummies (so uniqueness dummy for K/M required)

Please check all arcs carefully for arrows – if there are no arrows on any dummies then M1 only. Note that additional (but unnecessary) ‘correct’ dummies that still maintain precedence for the network should only be penalised with the final A mark if earned. Note that this answer is not unique

b1B1	CAO (B, G, H and M and no others)
c1M1	Either a statement with the correct number of workers (4) and stating the correct activities (G, C, E and I) with any numerical time stated or the correct number of workers (4) and a time in the interval $9 \leq t \leq 10$ – mark the numerical value only not their use of the words ‘day/time’ (or equivalent)
c1A1	A completely correct statement with details of both time and activities. Candidates must give a time within the correct interval of $9 < t < 10$, e.g. 9.5 (or ‘on/during hour 10’) and state the correct activities (G, C, E and I). Please note the strict inequalities for the time interval (e.g. implying a time of 10 is incorrect). Answers given as an interval of time are acceptable provided the time interval stated is correct for all its possible values (e.g. time 9 – 10 or ‘between 9 and 10’ is A0). A completely correct statement with an additional incorrect statement scores A0 (so do not ignore subsequent working)
d1B1	For either $x \geq 2$ or $x \leq 6$ (must be x not L)
d2B1	CAO ($2 \leq x \leq 6$) – SC in (d) B1B0 for $2 < x < 6$

Question Number	Scheme	Marks
7. (a)	e.g. Equation of line through A and D is $\frac{y - \frac{21}{2}}{\frac{27}{2} - \frac{21}{2}} = \frac{x - \frac{9}{4}}{\frac{27}{4} - \frac{9}{4}}$	M1
	$-2x + 3y = 27$	A1
	$2x + y \geq 15, -2x + 5y \geq 15, -2x + 3y \leq 27, 2x + y \leq 27$	B1 A1 (4)
(b)	$A: P = 2\left(\frac{9}{4}\right) + \frac{21}{2}k, B: P = 2(5) + 5k,$ $C: P = 2(10) + 7k, D: P = 2\left(\frac{27}{4}\right) + \frac{27}{2}k$	B1
	First possibility is that min is A and max is C $2\left(\frac{9}{2} + \frac{21}{2}k\right) \geq 20 + 7k$	M1
	$k \geq \frac{11}{14}$	A1
	Second possibility is that min is B and max is D $2(10 + 5k) \geq \frac{27}{2} + \frac{27}{2}k$	M1
	$k \leq \frac{13}{7}$	A1 (5)
	Special Case Candidates may consider $k < 1$ and $k > 1$ separately and obtain two ranges for k $\frac{11}{14} \leq k < 1$ and $1 < k \leq \frac{13}{7}$ scores M1 A1 M1 A0 $\frac{11}{14} \leq k \leq 1$ and $1 \leq k \leq \frac{13}{7}$ scores M1 A1 M1 A1	
		9 marks
	Notes for Question 7	
a1M1	Correct method for finding the equation of the line through A and D (condone a slip in values for this mark)	
a1A1	Correct equation of the line through A and D (a correct 3 term equation, in any equivalent form, implies M1 A1) (may also be implied by any inequality with the correct 3 terms)	
a1B1	Any two correct inequalities (condone strict inequalities)	
a2A1	CAO (all four correct, accept any equivalent 3 term form or e.g. $2x + y - 15 \geq 0$)	
b1B1	Correct objective applied at any of A, B, C or D	
b1M1	Setting up a linear inequality using A and C – allow any inequality or equals and the 2 on the wrong side	
b1A1	CAO ($k \geq \frac{11}{14}$)	
b2M1	Setting up a linear inequality using B and D – allow any inequality or equals and the 2 on the wrong side	
b2A1	CAO ($k \leq \frac{13}{7}$) Note we do not need to see the inequalities combined to form a single range of values. If they do combine the two correct inequalities, they must give a correct range $\frac{11}{14} \leq k \leq \frac{13}{7}$ do not ISW, remove the final A mark	

