

FP1 Jan 2014 (IAL) (MA)

Q1a) $f(1) = -0.7015115 \dots$
 $f(1.4) = 1.950164 \dots$ } change in sign between $x=1$ and $x=1.4$ \therefore a root lies between $x=1$ and 1.4 .

b)	a	$f(a)$	b	$f(b)$	$\frac{a+b}{2}$	$f\left(\frac{a+b}{2}\right)$
	1	-0.7015...	1.4	1.95016...	1.2	0.588...
	1	-0.7015	1.2	0.588...	1.1	-0.06798...

\therefore interval is $\boxed{[1.1, 1.2]}$

Q2a) Area image = Area object \times $|\det A|$.

$$2x = x \cdot |\det A|$$

$$\det A = -4u + 30 //$$

$$\therefore 2x = x |30 - 4u|$$

$$2 = |30 - 4u|$$

$$2 = 30 - 4u \quad \text{or} \quad 2 = -(30 - 4u)$$

$$4u = 28$$

$$\boxed{u = 7}$$

$$4u = 32$$

$$\boxed{u = 8}$$

ii) $BC = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 5 & 1 \end{pmatrix} \begin{pmatrix} 2 & 8 \\ 0 & 2 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} 2+3 & 8-4-6 \\ -4+1 & 10-18 \end{pmatrix}$

$\begin{matrix} \textcircled{2} \times 3 & & 3 \times \textcircled{2} & & 2 \times 2 \end{matrix}$

$$= \begin{pmatrix} 5 & -2 \\ -3 & -8 \end{pmatrix}$$

$$Q3) \quad x = 2t \quad y = \frac{2}{t}$$

$$P(1, 4) \quad Q(8, \frac{1}{2})$$

$$m_{PQ} = \frac{\frac{1}{2} - 4}{8 - 1} = -\frac{1}{2} //$$

$$\therefore m_L = 2 // \quad \left(-\frac{1}{2} \times 2 = -1 \right)$$

↳ L is perp to PQ.

$$\Rightarrow y - 0 = 2(x - 0)$$

$$\Rightarrow \boxed{y = 2x}$$

$$Q4) \quad f(x) = 2x^{\frac{1}{2}} - 6x^{-2} - 3$$

$$f'(x) = x^{-\frac{1}{2}} + 12x^{-3}$$

$$f(3.5) = 0.25186\dots$$

$$f'(3.5) = 0.81441\dots$$

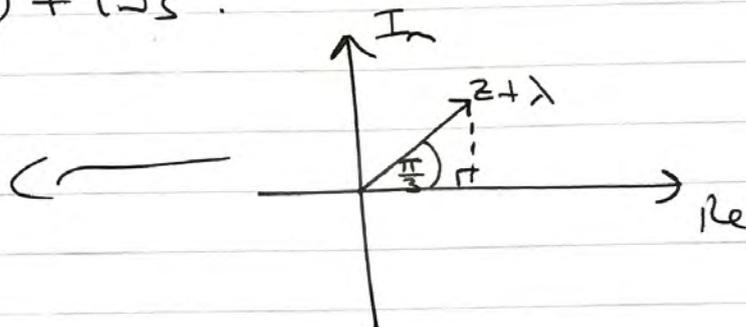
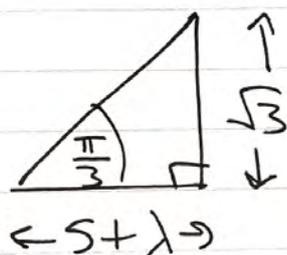
$$\Rightarrow x_1 = 3.5 - \frac{0.25186\dots}{0.81441\dots} = \boxed{3.191} \text{ 3dp.}$$

$$Q5a) |w| = \sqrt{(\sqrt{3})^2 + (4)^2} = \boxed{2}$$

$$b) zw = (5 + \sqrt{3}i)(\sqrt{3} - i) = 5\sqrt{3} - 5i + 3i + \sqrt{3} \\ = \boxed{6\sqrt{3} - 2i}$$

$$c) \frac{z}{w} = \frac{5 + i\sqrt{3}}{\sqrt{3} - i} = \frac{(5 + i\sqrt{3})(\sqrt{3} + i)}{(\sqrt{3} - i)(\sqrt{3} + i)} \\ = \frac{5\sqrt{3} + 5i + 3i - \sqrt{3}}{3 + 1} = \frac{4\sqrt{3} + 8i}{4} \\ = \boxed{\sqrt{3} + 2i}$$

$$d) z + \lambda = (5 + \lambda) + i\sqrt{3}$$



$$\tan \frac{\pi}{3} = \frac{\sqrt{3}}{5 + \lambda} = \sqrt{3} \quad (\div \sqrt{3}) \quad \frac{1}{5 + \lambda} = 1$$

$$\Rightarrow 5 + \lambda = 1$$

$$\Rightarrow \boxed{\lambda = -4}$$

$$\text{Q6a)} \quad \sum_1^n r(r+1)(r-1) = \sum_1^n r(r^2-1) = \sum_1^n r^3 - r$$

$$= \sum_1^n r^3 - \sum_1^n r$$

$$= \frac{n^2}{4}(n+1)^2 - \frac{n}{2}(n+1)$$

$$= \frac{n}{4}(n+1)[n(n+1) - 2]$$

$$= \frac{n}{4}(n+1)(n^2+n-2)$$

$$= \frac{n}{4}(n+1)(n+2)(n-1)$$

$$a=2$$

$$\text{b)} \quad 10 \sum_1^n r^2 = \frac{10}{6}n(n+1)(2n+1)$$

$$\Rightarrow \frac{10}{6}n(n+1)(2n+1) = \frac{1}{4}(n+1)(n+2)(n-1)$$

$$\Rightarrow \frac{5}{3}(2n+1) = \frac{1}{4}(n^2+n-2)$$

$\times 12$

$$\Rightarrow 20(2n+1) = 3n^2 + 3n - 6$$

$$\Rightarrow 3n^2 - 37n - 26 = 0$$

$$\Rightarrow (3n+2)(n-13) = 0 \quad n > 1 \quad \therefore \boxed{n=13}$$

$$\begin{aligned} \text{Q7a)} \quad \det P &= 3a(2b) - (2ab) = 6ab - 2ab \\ &= 4ab \end{aligned}$$

$$\therefore P^{-1} = \frac{1}{4ab} \begin{pmatrix} 2b & 2a \\ b & 3a \end{pmatrix} //$$

$$\text{b)} \quad M = PQ$$

$$P^{-1}M = P^{-1}PQ$$

$$P^{-1}M = Q //$$

$$P^{-1}M = \frac{1}{4ab} \begin{pmatrix} 2b & 2a \\ b & 3a \end{pmatrix} \begin{pmatrix} -6a & 7a \\ 2b & -b \end{pmatrix}$$

$$= \frac{1}{4ab} \begin{pmatrix} -12ab + 4ab & 14ab - 2ab \\ -6ab + 6ab & -3ab + 7ab \end{pmatrix} = \frac{1}{4ab} \begin{pmatrix} -8ab & 12ab \\ 0 & 4ab \end{pmatrix}$$

$$= \begin{pmatrix} -2 & 3 \\ 0 & 1 \end{pmatrix}$$

Q8a) $y^2 = 4ax$ [Implicit differentiation]

$$2y \frac{dy}{dx} = 4a \quad \therefore \frac{dy}{dx} = \frac{2a}{y} = \frac{2a}{2ap} = \frac{1}{p}$$

so at normal, $m = -p$

$$\Rightarrow y - 2ap = -p(x - ap^2)$$

$$\Rightarrow y = -px + ap^3 + 2ap$$

$$\Rightarrow y + px = ap^3 + 2ap$$

b) $(6a, 0) : 0 + p(6a) = ap^3 + 2ap$

$$6ap - 2ap = ap^3$$

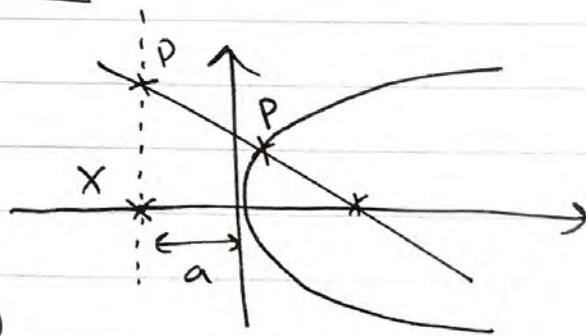
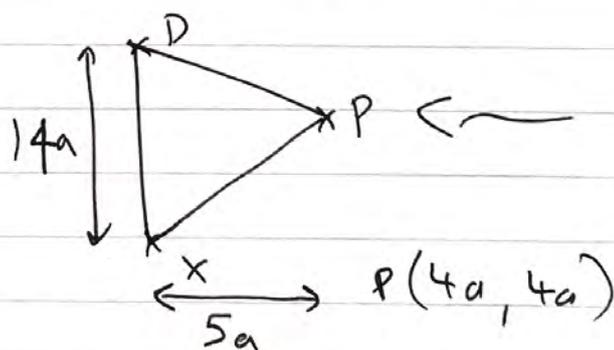
$$4 = p^2 \quad \therefore p = 2 \quad (p > 2)$$

at D, $x = -a : y - a(2) = a(8) + 2a(2)$

$$y = 8a + 4a + 2a = 14a$$

so $D(-a, 14a)$

c)



$$\therefore \text{Area } \triangle XPD = \frac{1}{2} \times 5a \times 14a = \boxed{35a^2}$$

$$(19) \quad (3-i)z^* + 2iz = 9-i$$

$$z^* = x-iy$$

$$z = x+iy$$

$$(3-i)(x-iy) + 2i(x+iy) = 9-i$$

$$3x - 3iy - ix - y + 2xi - 2y = 9-i$$

Gather all real / imaginary terms,

$$\Rightarrow (3x - y - 2y) + (2x - x - 3y)i = 9 - i$$

$$\Rightarrow (3x - 3y) + (x - 3y)i = (9) + (-1)i$$

compare real / imaginary terms,

$$3x - 3y = 9 \quad \sim \quad (1)$$

$$x - 3y = -1 \quad \sim \quad (2)$$

} solve simultaneously,

$$(1) : x - y = 3 \quad \therefore y - x = -3$$

$$(2) : x - 3y = -1$$

$$(1) + (2) : y + x - x - 3y = -4$$

$$\begin{aligned} \hookrightarrow -2y &= -4 \\ \therefore y &= 2 \\ \text{so } x &= y + 3 \\ x &= 5 \end{aligned}$$