

FP1 2017 (MA)

Q1a) $f(6) = -0.8888\dots$
 $f(7) = 1.414966\dots$ } Change in sign
 between $x=6$ and $x=7$
 \therefore a root lies between
 these values.

b) $f(x) = \frac{1}{3}x^2 + 4x^{-2} - 2x - 1$

$$f'(x) = \frac{2}{3}x - 8x^{-3} - 2$$

$$f(6) = -0.8888\dots$$

$$f'(6) = \frac{2}{3}(6) - 8(6)^{-3} - 2 = 1.96296\dots$$

$$\therefore x_1 = 6 - \frac{-0.8888\dots}{1.96296\dots} = 6.45283\dots$$

$$= \boxed{6.45}$$

Q2a) $\det A = 6 + 4 = 10$

$$\therefore A^{-1} = \frac{1}{10} \begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix}$$

b) $P = AB$

$$A^{-1}P = A^{-1}AB$$

$$\therefore A^{-1}P = B = \frac{1}{10} \begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix} \begin{pmatrix} 3 & 6 \\ 11 & -8 \end{pmatrix}$$

$$= \frac{1}{10} \begin{pmatrix} 9+11 & 18-8 \\ 22-12 & -16-24 \end{pmatrix} = \frac{1}{10} \begin{pmatrix} 20 & 10 \\ 10 & -40 \end{pmatrix}$$

$$B = \begin{pmatrix} 2 & 1 \\ 1 & -4 \end{pmatrix}$$

Q3a) P (1, 16) Q (8, 2)

$$m_{PQ} = \frac{16-2}{1-8} = \frac{14}{-7} = -2 //$$

$\therefore y = \left(\frac{1}{2}\right)x$ (passes through 0)

b) $xy = 4t \times \frac{4}{t}$

$$xy = 16$$

c) $y = \left(\frac{1}{2}\right)x$

$$\hookrightarrow x \left(\frac{1}{2}x\right) = 16$$

$$\Rightarrow x^2 = 32 \quad \therefore x = \pm 4\sqrt{2} (= \sqrt{32})$$

$\therefore y = \pm 2\sqrt{2}$ so points are: $(4\sqrt{2}, 2\sqrt{2})$
and $(-4\sqrt{2}, -2\sqrt{2})$

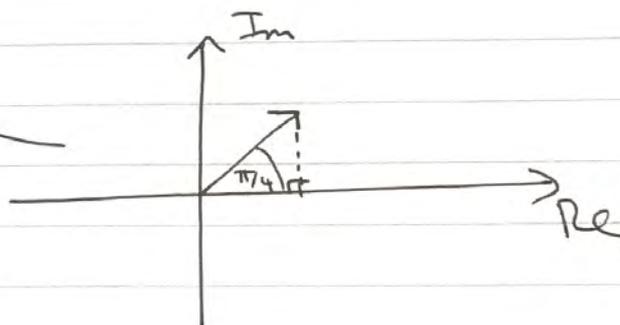
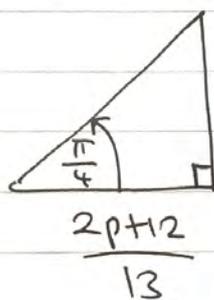
$$Q4ia) \quad w = \frac{(p-4i) \times (2+3i)}{(2-3i) \times (2+3i)}$$

$$= \frac{(p-4i)(2+3i)}{4 - 6i + 6i + 9} = \frac{2p + 3pi - 8i - 12i^2}{13}$$

$$= \frac{(2p+12) + (3p-8)i}{13}$$

$$= \left(\frac{2p+12}{13} \right) + \left(\frac{3p-8}{13} \right) i$$

b)



$$\Rightarrow \tan \frac{\pi}{4} = \frac{\frac{3p-8}{13}}{\frac{2p+12}{13}} = \frac{3p-8}{2p+12} = 1$$

$$\Rightarrow 2p + 12 = 3p - 8$$

$$\Rightarrow \boxed{p = 20}$$

$$\text{ii) } z = 4 + 3i - 4\lambda i + 3\lambda$$

$$z = (4 + 3\lambda) + (3 - 4\lambda)i$$

$$|z| = \sqrt{(4 + 3\lambda)^2 + (3 - 4\lambda)^2} = 45$$

$$\therefore (4 + 3\lambda)^2 + (3 - 4\lambda)^2 = 45^2$$

$$\Rightarrow 16 + 24\lambda + 9\lambda^2 + 9 - 24\lambda + 16\lambda^2 = 2025$$

$$\Rightarrow 25\lambda^2 + 25 = 2025$$

$$\Rightarrow 25\lambda^2 = 2000$$

$$\Rightarrow \lambda^2 = 80$$

$$\Rightarrow \lambda = \boxed{4\sqrt{5}}, \lambda = \boxed{-4\sqrt{5}}$$

$$Q5ia) AB = \begin{pmatrix} p & 2 \\ 3 & p \end{pmatrix} \begin{pmatrix} -5 & 4 \\ 6 & -5 \end{pmatrix} = \boxed{\begin{pmatrix} -5p+12 & 4p-10 \\ -15+6p & -5p+12 \end{pmatrix}}$$

$$b) AB + 2A = kI$$

$$\begin{pmatrix} 12-5p & 4p-10 \\ 6p-15 & 12-5p \end{pmatrix} + \begin{pmatrix} 2p & 4 \\ 6 & 2p \end{pmatrix} = \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$$

$$\Rightarrow 6p - 15 + 6 = 0$$

$$\therefore 6p = 9 \quad \therefore p = \boxed{\frac{3}{2}}$$

$$\Rightarrow 12 - 5p + 2p = k$$

$$12 - 3\left(\frac{3}{2}\right) = k = \boxed{\frac{15}{2}}$$

$$ii) \text{Area}_{\text{image}} = \text{Area}_{\text{object}} \times |\det M|.$$

$$270 = 15 \det M$$

$$\therefore \frac{270}{15} = |\det M| = 18 //$$

$$|\det M| = |2a + 9| = 18$$

$$2a + 9 = 18 \quad \text{or} \quad -2a - 9 = 18$$

$$\boxed{a = 9/2}$$

$$\boxed{a = -27/2}$$

Q6a) $-3-2i$ (complex conjugate pair)

b) $(x - (-3-2i))(x - (-3+2i))(x-4) = 0$

$$\Rightarrow (x^2 - x(-6) + (9+4))(x-4) = 0$$

$$\Rightarrow (x^2 + 6x + 13)(x-4) = 0$$

$$\Rightarrow x^3 + 6x^2 + 13x - 4x^2 - 24x - 52 = 0$$

$$\Rightarrow x^3 + 2x^2 - 11x - 52 = 0$$

$$\therefore \boxed{a=2, b=-11}$$

Q7a) $y^2 = 4ax$ ← [Implicit differentiation]

$$2y \frac{dy}{dx} = 4a \quad \therefore \frac{dy}{dx} = \frac{2a}{y} = \frac{2a}{2aq} = \frac{1}{q}$$

$$\Rightarrow y - 2aq = \frac{1}{q}(x - aq^2)$$

$$\Rightarrow y = \frac{1}{q}x - aq + 2aq$$

$$\xrightarrow{\times q} \quad qy = x + aq^2$$



b) substitute X into tangent:

$$q(0) = -\frac{1}{4}a + aq^2$$

$$\frac{a}{4} = aq^2$$

$$1 = 4q^2$$

$$\therefore q^2 = \frac{1}{4} \text{ so } q = \frac{1}{2} //$$

directrix : $x = -a$: $qy = -a + aq^2$

$$\frac{1}{2}y = -a + \frac{1}{4}a$$

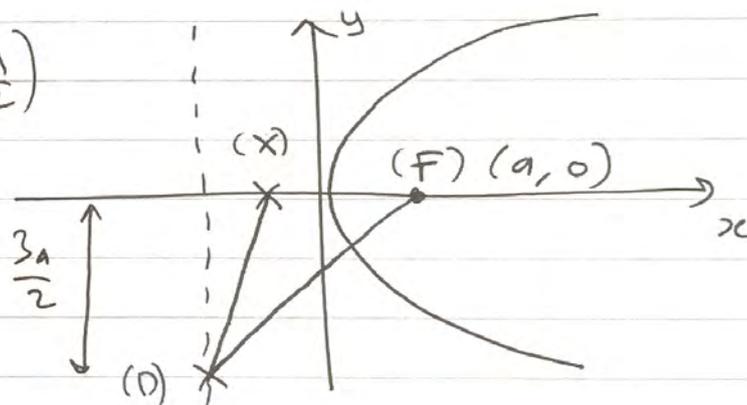
$$y = -\frac{3a}{2} //$$

$$\therefore D \left(-a, -\frac{3a}{2} \right)$$

XF \rightarrow

$$c) \text{ Area} = \frac{1}{2} \left(a + \frac{a}{4} \right) \left(\frac{3a}{2} \right)$$

$$= \boxed{\frac{15a^2}{16}}$$



$$Q8a) \quad 3 \sum_1^n (r^2) + 8 \sum_1^n (r) + 3 \sum_1^n (1)$$

$$= \frac{3n}{6} (n+1)(2n+1) + \frac{8n}{2} (n+1) + 3n$$

$$= \frac{n}{2} (2n^2 + 3n + 1) + 4n^2 + 4n + 3n$$

$$= n^3 + \frac{3n^2}{2} + \frac{n}{2} + 4n^2 + 7n$$

$$= n^3 + \frac{11}{2}n^2 + \frac{15n}{2} = \frac{1}{2} (2n^3 + 11n^2 + 15n)$$

$$= \frac{n}{2} (2n^2 + 11n + 15) = \frac{n}{2} (2n+5)(n+3)$$

$$b) \quad \sum_1^{12} (3r^2 + 8r + 3) + k \sum_1^{12} 2^{r-1} = 3520$$

$$\frac{12}{2} (24+5)(15) + k \sum_1^{12} \frac{2^r}{2} = 3520$$

$$\frac{k}{2} \sum_1^{12} 2^r = 910$$

$$k \sum_1^{12} 2^r = 1820$$

$$\sum_1^{12} 2^r \sim \text{Geometric series from } (2, \dots)$$

$$2 + 2(2) + 2(2)^2 + \dots$$

$$a=2, r=2$$

$$\therefore \sum_1^{12} 2^r = \frac{a(1-r^n)}{1-r} = \frac{2(1-(2)^{12})}{1-2} = 8190 //$$

$$\Rightarrow 8190(u) = 1820$$

$$\Rightarrow u = \frac{1820}{8190} = \boxed{\frac{2}{9}}$$

Q9:)

$$u_1 = 6 \quad u_{n+2} = 6u_{n+1} - 9u_n$$

$$u_2 = 27 \quad \text{prove: } u_n = 3^n(n+1)$$

$$\underline{n=1}: u_1 = 3^1(2) = 6 // \text{ as given.}$$

$$\underline{n=2}: u_2 = 3^2(3) = 27 // \text{ as given.}$$

\therefore true for $n=1, n=2$

now assume $[u_n = 3^n(n+1)]$ is true,

and $[u_{n+1} = 3^{n+1}(n+2)]$ is true

consider $n=k+2,$

$$\Rightarrow u_{k+2} = 6u_{k+1} - 9u_k$$

$$= 6 \cdot 3^{k+1}(k+2) - 3^2 \cdot 3^k(k+1)$$

$$= 2 \cdot 3^{k+2}(k+2) - 3^{k+2}(k+1)$$

$$= 2 \cdot 3^{k+2}(k+1) + 2 \cdot 3^{k+2} - 3^{k+2}(k+1)$$

$$= (2-1)3^{k+2}(k+1) + 2(3^{k+2})$$

$$= 3^{k+2}(k+1) + 2 \cdot 3^{k+2}$$

$$= \frac{3^{k+2}(k+3)}{\quad} \therefore \text{true for } n=k+2$$

so, true for $n=1, 2$.

true for $n=k+2$ when true for $n=k, n=k+1$.

\therefore By Mathematical Induction the relationship is true for all $n \in \mathbb{Z}^+$

ii) $f(n) = 3^{3n-2} + 2^{3n+1}$

$n=1$: $f(1) = 3 + 2^4 = 19 = (19) \times 1$
 \therefore true for $n=1$.

assume $f(k)$ is divisible by 19,

consider $f(k+1)$,

$$f(k+1) = 3^{3(k+1)-2} + 2^{3(k+1)+1}$$

$$= 3^{3k+1} + 2^{3k+4}$$

$$= 3^{3k-2+3} + 2^{3k+1+3}$$

$$= 3^3 (3^{3k-2}) + 8 (2^{3k+1})$$

$$= 27 (3^{3k-2}) + 8 (2^{3k+1})$$

[factoring out $f(k)$!]

$$= 27 [3^{3k-2} + 2^{3k+1}] - 19(2^{3k+1})$$

$$= 27f(k) - 19(2^{3k+1}) //$$

\therefore true for $n = k+1$.

So, true for $n=1$.
true for $n=k+1$ when assumed true
for $n=k$.

\therefore By Mathematical Induction true for all
 $n \in \mathbb{Z}^+$
