Mark Scheme (Final)

Summer 2018

Pearson Edexcel GCE
In Further Pure Mathematics FP1 (6667/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.

- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.

- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.

- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.

- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should **also be prepared to award zero marks if the candidate's** response is not worthy of credit according to the mark scheme.

- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.

- When examiners are in doubt regarding the application **of the mark scheme to a candidate's response, the team leader** must be consulted.

- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

Question Number	Scheme	Notes	Marks
1.	$\mathrm{f}(z) = 2z^3 - 4z^2 + 15z - 13 \equiv (z-1)(2z^2 + az + b)$		
(a)	$a = -2, b = 13$	At least one of either $a = -2$ or $b = 13$ or seen as their coefficients.	B1
		Both $a = -2$ and $b = 13$ or seen as their coefficients.	B1
			[2]
(b)	$\{z =\}$ 1 is a root	1 is a root, seen anywhere.	B1
	$\left\{2z^2 - 2z + 13 = 0 \Rightarrow z^2 - z + \dfrac{13}{2} = 0\right\}$		
	Either $\quad \bullet \quad z = \dfrac{2 \pm \sqrt{4 - 4(2)(13)}}{2(2)}$	Correct method for solving a 3-term quadratic equation. Do not allow M1 here for an attempt at factorising.	M1
	or $\quad \bullet \quad \left(z - \dfrac{1}{2}\right)^2 - \dfrac{1}{4} + \dfrac{13}{2} = 0 \ $ and $z = ...$		
	or $\quad \bullet \quad (2z-1)^2 - 1 + +13 = 0 \ $ and $z = ...$		
	So, $\{z =\} \ \dfrac{1}{2} + \dfrac{5}{2}\mathrm{i}, \ \dfrac{1}{2} - \dfrac{5}{2}\mathrm{i}$	At least one of either $\dfrac{1}{2} + \dfrac{5}{2}\mathrm{i}$ or $\dfrac{1}{2} - \dfrac{5}{2}\mathrm{i}$ or any equivalent form.	A1
		For conjugate of first complex root	A1ft
			[4]
			Total 6

Question Number	Scheme	Notes	Marks
2. (a)	$f(-3) = \quad 2.05555555...$ $f(-2.5) = -1.15833333...$	Attempt **both** of $f(-3) =$ awrt 2.1 or trunc 2 or 2.0 or $\dfrac{37}{18}$ **and** $f(-2.5) =$ awrt -1.2 or trunc -1.1 or $-\dfrac{139}{120}$	M1
	Sign change oe (and $f(x)$ is continuous) therefore a root α {exists in the interval $[-3, -2.5]$.}	Both $f(-3) =$ awrt 2.1 and $f(-2.5) =$ awrt -1.2, sign change and 'root' or 'α'. Any errors award A0.	A1
			[2]
(b)	$f'(x) = 3x - \dfrac{4}{3x^2} + 2$	$\dfrac{3}{2}x^2 \to \pm Ax$ or $\dfrac{4}{3x} \to \pm Bx^{-2}$ or $2x - 5 \to 2$ Calculus must be seen for this to be awarded.	M1
		At least two terms differentiated correctly	A1
		Correct derivative.	A1
	$\alpha = -3 - \left(\dfrac{"\,2.055...\,"}{"-7.148...\,"} \right)$	Correct application of Newton-Raphson **using their values from calculus**.	M1
	$= -2.71243523...$ or $-\dfrac{1047}{386}$ or $-2\dfrac{275}{386}$	Exact value or awrt -2.712	A1
			[5]
(c)	$\dfrac{-2.5 - \alpha}{"1.158...\,"} = \dfrac{\alpha - -3}{"2.055...\,"}$ or $\dfrac{\alpha - -3}{"2.055...\,"} = \dfrac{-2.5 - -3}{"\,2.055...\," + "1.158...\,"}$	A correct linear interpolation statement with correct signs. $\dfrac{-2.5 + \alpha}{"1.158...\,"} = \dfrac{-\alpha - -3}{"2.055...\,"}$ provided α sign changed at the end. Do not award until α is seen.	M1
	$\alpha = -3 + \left(\dfrac{"\,2.055...\,"}{"\,2.055...\," + "1.158...\,"} \right)(0.5)$ or $\alpha = -3 + \left(\dfrac{"2.055...\,"}{"3.213...\,"} \right)(0.5)$ or $\alpha = \left(\dfrac{(-2.5)("\,2.055...\,") - 3("1.158...\,")}{"\,2.055...\," + "1.158...\,"} \right)$	Achieves a correct linear interpolation statement with correct signs for $\alpha = ...$ dependent on the previous method mark.	dM1
	$= -2.68020743...$ or $-\dfrac{3101}{1157}$ or $-2\dfrac{787}{1157}$		
	$= -2.680$ (3 dp)	-2.680 : only penalise accuracy once in (b) and (c), but must be to at least 3sf.	A1 **cao**

ALT (c)	The gradient of the line between (-3, 2.055…) and (-2.5, -1.158…) is $\dfrac{2.055... - -1.158...}{-3 - -2.5} = -6.427...$		
	Equation of the line joining the points $y - 2.055... = -6.427...(x - -3)$	Correct attempt to find the equation of a line between the two points.	M1
	At $y = 0$, $0 - 2.055... = -6.427...(x - -3)$	Subs $y = 0$ in their line and achieves $x = ...$	dM1
	$\Rightarrow x = -2.680$	-2.680 : only penalise accuracy once in (b) and (c), but must be to at least 3sf.	A1 cao
			[3]
			Total 10

Question Number	Scheme	Notes	Marks
3. (i) (a)	$\mathbf{A}^{-1} = \dfrac{1}{-2-3}\begin{pmatrix} 1 & -3 \\ -1 & -2 \end{pmatrix}$	Either $\dfrac{1}{-2-3}$ or $-\dfrac{1}{5}$ or $\begin{pmatrix} 1 & -3 \\ -1 & -2 \end{pmatrix}$	M1
		Correct expression for $\mathbf{A}^{-1}$	A1
			[2]
(b)	$\{\mathbf{B} = \mathbf{A}^{-1}(\mathbf{AB})\}$		
	$\mathbf{B} = -\dfrac{1}{5}\begin{pmatrix} 1 & -3 \\ -1 & -2 \end{pmatrix}\begin{pmatrix} -1 & 5 & 12 \\ 3 & -5 & -1 \end{pmatrix}$	Writing down their $\mathbf{A}^{-1}$ multiplied by $\mathbf{AB}$	M1
	$= \left\{-\dfrac{1}{5}\right\}\begin{pmatrix} -10 & 20 & 15 \\ -5 & 5 & -10 \end{pmatrix}$	At least one correct row or at least two correct columns of $\begin{pmatrix} \cdots \\ \cdots \end{pmatrix}$. (Ignore $-\dfrac{1}{5}$).	A1
	$= \begin{pmatrix} 2 & -4 & -3 \\ 1 & -1 & 2 \end{pmatrix}$	Correct simplified matrix for $\mathbf{B}$	A1
			[3]
ALT (b)	Let $\mathbf{B} = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$		
	$\begin{array}{ll} -2a + 3d = -1 & -2b + 3e = 5 \\ a + d = 3 & b + e = -5 \\ -2c + 3f = 12 & \\ c + f = -1 & \end{array}$	Writes down at least 2 correct sets of simultaneous equations	M1
	$\{a = 2, d = 1, b = -4, e = -1, c = -3, f = 2\}$		
	$\mathbf{B} = \begin{pmatrix} 2 & -4 & -3 \\ 1 & -1 & 2 \end{pmatrix}$	At least one correct row or at least two correct columns for the matrix $\mathbf{B}$	A1
		Correct matrix for $\mathbf{B}$	A1
			[3]
(ii) (a)	Rotation	Rotation only.	M1
	90° clockwise about the origin	**90°** $\left(\text{or } \dfrac{\pi}{2}\right)$ **clockwise** about the **origin** or **270°** $\left(\text{or } \dfrac{3\pi}{2}\right)$ (anti-clockwise) about the **origin.** **-90°** $\left(\text{or } -\dfrac{\pi}{2}\right)$ (**anticlockwise**) about the **origin.** Origin can be written as $(0, 0)$ or O.	A1
			[2]
(b)	$\{\mathbf{C}^{39}\} = \mathbf{C}^{-1}$ or $\mathbf{C}^3 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	For stating $\mathbf{C}^{-1}$ or $\mathbf{C}^3$ or 'rotation of **270°** clockwise o.e. about the **origin.** Can be implied by correct matrix.	M1
		$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ Correct answer with no working award M1A1	A1
			[2]
			Total 9

Question Number	Scheme	Notes	Marks
4. (a)	$\displaystyle\sum_{r=1}^{n}\left(r^2 - r - 8\right)$		
	$= \dfrac{1}{6}n(n+1)(2n+1) - \dfrac{1}{2}n(n+1) - 8n$	At least one of the first two terms is correct.	M1
		Correct expression	A1
	$= \dfrac{1}{6}n\big((2n+1)(n+1) - 3(n+1) - 48\big)$	An attempt to factorise out at least n.	M1
	$= \dfrac{1}{6}n\left(2n^2 + 3n + 1 - 3n - 3 - 48\right)$		
	$= \dfrac{1}{6}n\left(2n^2 - 50\right)$		
	$= \dfrac{2}{6}n\left(n^2 - 25\right)$		
	$= \dfrac{1}{3}n(n-5)(n+5)$	Achieves the correct answer.	A1
			[4]
(b)	$n = 5$	5. Give B0 for 2 or more possible values of n.	B1 cao
			[1]
(c)	$\left(\dfrac{k}{4}(17^2)(18^2) - \dfrac{k}{4}(3^2)(2^2)\right) + \left(\dfrac{1}{3}(17)(22)(12) - \dfrac{1}{3}(2)(-3)(7)\right)$	Applying *at least one of* $n = 17$ **or** $n = 2$ to **both** $\dfrac{1}{4}n^2(n+1)^2$ and their $\dfrac{1}{3}n(n-5)(n+5)$	M1
		Applying $n = 17$ **and** $n = 2$ **only** to **both** $\dfrac{1}{4}n^2(n+1)^2$ and their $\dfrac{1}{3}n(n-5)(n+5)$. Require differences only for both brackets.	M1
	$\{\Sigma = 6710 \Rightarrow\}\ 23409k - 9k + 1496 + 14 = 6710 \Rightarrow k = \dfrac{2}{9}$	Sets their sum to 6710 and solves to give $k = ...$	ddM1
		$k = \dfrac{2}{9}$ or $0.\dot{2}$	A1 **cso**
			[4]
			Total 9

Question Number	Scheme	Notes	Marks
5. (a)	$y = c^2 x^{-1} \Rightarrow \dfrac{dy}{dx} = -c^2 x^{-2}$ or (implicitly) $y + x\dfrac{dy}{dx} = 0$ or (chain rule) $\dfrac{dy}{dx} = -ct^{-2} \times \dfrac{1}{c}$	$\dfrac{dy}{dx} = \pm k\, x^{-2}$ or $y + x\dfrac{dy}{dx} = 0$ or $\dfrac{\text{their } \frac{dy}{dt}}{\text{their } \frac{dx}{dt}}$	M1
	When $x = ct$, $m_T = \dfrac{dy}{dx} = \dfrac{-c^2}{(ct)^2} = -\dfrac{1}{t^2}$ or at $P\left(ct, \dfrac{c}{t}\right)$, $m_T = \dfrac{dy}{dx} = -\dfrac{y}{x} = -\dfrac{ct^{-1}}{ct} = -\dfrac{1}{t^2}$	$\dfrac{dy}{dx} = -\dfrac{1}{t^2}$	A1
	T: $y - \dfrac{c}{t} = -\dfrac{1}{t^2}(x - ct)$	Applies $y - \dfrac{c}{t} = \left(\text{their } m_T\right)(x - ct)$ where their m_T has come from calculus	M1
	T: $t^2 y - ct = -x + ct$	At least one line of working.	
	T: $t^2 y + x = 2ct$ *	Correct solution.	A1 **cso** *
			[4]
(b)	$t^2\left(\dfrac{3c}{5}\right) + \left(-\dfrac{8c}{5}\right) = 2ct$	Substitutes $\left(-\dfrac{8c}{5}, \dfrac{3c}{5}\right)$ into tangent.	M1
	$3t^2 - 8 = 10t$	Correct 3TQ in terms of t Can include uncancelled c.	A1
	$\left\{3t^2 - 10t - 8 = 0 \Rightarrow\right\} (t - 4)(3t + 2) = 0 \Rightarrow t = \ldots$	Attempt to solve their 3TQ for t	M1
	$t = 4, -\dfrac{2}{3} \Rightarrow A\left(4c, \dfrac{c}{4}\right), B\left(-\dfrac{2}{3}c, -\dfrac{3c}{2}\right)$	Uses one of their values of t to find A or B	M1
		Correct coordinates. Condone A and B swapped or missing.	A1
			[5]
			Total 9
ALT 1 (b)	$y - \dfrac{3c}{5} = -\dfrac{1}{t^2}\left(x - -\dfrac{8c}{5}\right)$ $\Rightarrow \dfrac{c}{t} - \dfrac{3c}{5} = -\dfrac{1}{t^2}\left(ct + \dfrac{8c}{5}\right)$	Substitutes $\left(ct, \dfrac{c}{t}\right)$ into their $y - \dfrac{3c}{5} = -\dfrac{1}{t^2}\left(x - -\dfrac{8c}{5}\right)$	M1
	$3t^2 - 10t = 8$	Correct 3TQ in terms of t. Can include uncancelled c.	A1
	then apply the original mark scheme.		
ALT 2 (b)	$A\left(ct_1, \dfrac{c}{t_1}\right), B\left(ct_2, \dfrac{c}{t_2}\right)$ $t_1^2 y + x = 2ct_1$ $t_2^2 y + x = 2ct_2$	Substitutes A and B into the equation of the tangent, solves for x and y	M1
	$t_1 + t_2 = \dfrac{10}{3}, \; t_1 t_2 = -\dfrac{8}{3}$		
	$3t^2 - 8 = 10t$	Correct 3TQ in terms of t_1 or t_2 Can include uncancelled c.	A1
	then apply original scheme		

Question Number	Scheme		Marks
6. (a)	$\{\det \mathbf{M} = (8)(2) - (-1)(-4)\} \Rightarrow \det \mathbf{M} = 12$	12	B1
			[1]
(b)	Area $T = \dfrac{216}{12}\{=18\}$	Area $T = \dfrac{216}{\text{their "}\det \mathbf{M}\text{"}}$	M1
	$h = \pm(1-k)$	Uses $(k-1)$ or $(1-k)$ in their solution.	M1
	$\dfrac{1}{2}8(k-1) = 18$ or $\dfrac{1}{2}8(1-k) = 18$ or $(k-1) = \dfrac{18}{4}$ or $(1-k) = \dfrac{18}{4}$ or $\{\dfrac{1}{2}8h = 18\} \Rightarrow h = \dfrac{9}{2}, k = 1 \pm \dfrac{9}{2}$	**dependent on the two previous M marks** $\dfrac{1}{2}8(k-1)$ or $\dfrac{1}{2}8(1-k) = \dfrac{216}{\text{their "}\det \mathbf{M}\text{"}}$ or $(k-1)$ or $(1-k) = \dfrac{216}{4(\text{their "}\det \mathbf{M}\text{"})}$ or $h = \dfrac{216}{4(\text{their "}\det \mathbf{M}\text{"})}, k = 1 \pm \dfrac{216}{4(\text{their "}\det \mathbf{M}\text{"})}$	ddM1
	$\Rightarrow k = 5.5$ or $k = -3.5$	At least one of either $k = 5.5$ or $k = -3.5$	A1
		Both $k = 5.5$ and $k = -3.5$	A1
			[5]
ALT (b)	$\mathbf{T}' = \begin{pmatrix} 8 & -1 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} 4 & 6 & 12 \\ 1 & k & 1 \end{pmatrix}$		
	$\mathbf{T}' = \begin{pmatrix} 31 & 48-k & 95 \\ -14 & -24+2k & -46 \end{pmatrix}$ or 18 seen	At least 5 out of 6 elements are correct or 18 seen..	M1
	$\dfrac{1}{2}\begin{vmatrix} 31 & 48-k & 95 & 31 \\ -14 & -24+2k & -46 & -14 \end{vmatrix} = 216$ or $\dfrac{1}{2}\begin{vmatrix} 4 & 6 & 12 & 4 \\ 1 & k & 1 & 1 \end{vmatrix} = 18$	$\dfrac{1}{2}\left|\text{their } \mathbf{T}'\right| = 216$ or $\dfrac{1}{2}\begin{vmatrix} 4 & 6 & 12 & 4 \\ 1 & k & 1 & 1 \end{vmatrix} = 18$	M1
	$\dfrac{1}{2}\begin{vmatrix} -744 + 62k + 672 - 14k - 2208 + 46k \\ +2280 - 190k - 1330 + 1426 \end{vmatrix} = 216$ $\dfrac{1}{2}\left|4k - 6 + 6 - 12k + 12 - 4\right| = 18$	**Dependent on the two previous M marks.** Full method of evaluating a determinant.	ddM1
	$\dfrac{1}{2}\left|96 - 96k\right| = 216$ or $\dfrac{1}{2}\left|8 - 8k\right| = 18$		
	So, $1-k = 4.5$ or $k-1 = 4.5$		
	$\Rightarrow k = -3.5$ or $k = 5.5$	At least one of either $k = -3.5$ or $k = 5.5$	A1
		Both $k = -3.5$ and $k = 5.5$	A1
			[5]
			Total 6

Question Number	Scheme	Notes	Marks
7.	$y^2 = 4ax,\ S(a,0),\ D\left(-a, \dfrac{24a}{5}\right),\ P(ak^2, 2ak)$		
(a)	$m_l = \dfrac{\dfrac{24a}{5} - 0}{-a - a}\ \left\{= \dfrac{\dfrac{24a}{5} - 0}{-2a} = -\dfrac{12}{5}\right\}$ $\dfrac{y - \dfrac{24a}{5}}{0 - \dfrac{24a}{5}} = \dfrac{x - -a}{a - -a}$ or $\dfrac{y - 0}{\dfrac{24a}{5} - 0} = \dfrac{x - a}{-a - a}$	Uses $S(a, 0)$ and $D\left(\text{their "}-a\text{"}, \dfrac{24a}{5}\right)$ to find an expression for the gradient of l or applies the formula $\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$ Can be un-simplified or simplified.	M1
	$l:\ y - 0 = -\dfrac{12}{5}(x - a) \Rightarrow 5y = -12x + 12a$ $l:\ 12x + 5y = 12a$ (*)	Correct solution only leading to $12x + 5y = 12a$ No errors seen.	A1 *
			[2]
ALT (a)	$y = mx + c$ At S, $0 = ma + c$ At D, $\dfrac{24a}{5} = -ma + c$ $\Rightarrow c = \dfrac{12a}{5},\ m = -\dfrac{12}{5}$	Uses $S(a, 0)$ and $D\left(\text{their "}-a\text{"}, \dfrac{24a}{5}\right)$ to find 2 simultaneous equations and solves to achieve $c = ...,\ m = ...$	M1
	$y = -\dfrac{12}{5}x + \dfrac{12a}{5} \Rightarrow 12x + 5y = 12a *$	Correct solution only leading to $12x + 5y = 12a$	A1*
			[2]
(b)	$m_{SP} = \dfrac{2ak}{ak^2 - a}\ \left\{= \dfrac{2k}{k^2 - 1}\right\}$	Attempts to find the gradient of SP	M1
	$m_l = -\left(\dfrac{ak^2 - a}{2ak}\right)$ or $m_{SP} = -\dfrac{1}{\left(-\frac{12}{5}\right)}\ \left\{= \dfrac{5}{12}\right\}$	Some evidence of applying $m_1 m_2 = -1$	M1
	So $\left\{\dfrac{2k}{k^2 - 1} = \dfrac{5}{12} \Rightarrow\right\}\ 24k = 5k^2 - 5$	Correct 3TQ in terms of k in any form.	A1
	$\left\{5k^2 - 24k - 5 = 0 \Rightarrow\right\}\ (k - 5)(5k + 1) = 0 \Rightarrow k = ...$	Attempt to solve their 3TQ for k	M1
	$\{\text{As } k > 0, \text{ so } k = 5\} \Rightarrow (25a, 10a)$	Uses their k to find P	M1
		$(25a, 10a)$	A1
			[6]

ALT 1 (b)	$SP: \ y - 0 = \dfrac{5}{12}(x-a)$		$y - 0 = m_{SP}(x-a)$	M1
			$m_{SP} = -\dfrac{1}{(-\frac{12}{5})}\left\{ = \dfrac{5}{12} \right\}$	M1
	$\left\{ y^2 = 4ax \Rightarrow \right\} \left(\dfrac{5}{12}(x-a) \right)^2 = 4ax$		Can sub for x and achieve $\dfrac{12}{5}y + a$	
	$25(x^2 - 2ax + a^2) = 576ax$			
	$25x^2 - 626ax + 25a^2 = 0$		Correct 3TQ in terms of a and x or $5y^2 - 48ay - 20a^2 = 0$	A1
	$(25x - a)(x - 25a) = 0 \Rightarrow x = ...$		Attempt to solve their 3TQ for x	M1
	$x = \dfrac{a}{25} \Rightarrow y = \dfrac{5}{12}\left(\dfrac{a}{25} - a \right) \left\{ = -\dfrac{2a}{5} \right\}$ $x = 25a \Rightarrow y = \dfrac{5}{12}(25a - a) \ \{ = 10a \}$		Uses their x to find y	M1
	$\{ \text{As } k > 0, \} \Rightarrow (25a, 10a)$		$(25a, 10a)$	A1
				[6]
ALT 2 (b)	$0 = m_{SP}a + c$		Subs S into $y = m_{SP}x + c$ to find c	M1
	$m_{SP} = -\dfrac{1}{(-\frac{12}{5})}\left\{ = \dfrac{5}{12} \right\}$		Some evidence of applying $m_1 m_2 = -1$	M1
	$y = \dfrac{5}{12}x - \dfrac{5}{12}a$			
	At P, $2ak = \dfrac{5}{12}ak^2 - \dfrac{5}{12}a$		Correct 3TQ in terms of k	A1
	then as part (b)			
				Total 8

Question Number	Scheme	Notes	Marks
8.	$\mathrm{f}(n) = 2^{n+2} + 3^{2n+1}$ divisible by 7		
	$\mathrm{f}(1) = 2^3 + 3^3 = 35$ {which is divisible by 7}.	Shows $\mathrm{f}(1) = 35$	B1
	$\{\therefore \mathrm{f}(n)$ is divisible by 7 when $n = 1$ }		
	{Assume that for $n = k$,		
	$\mathrm{f}(k) = 2^{k+2} + 3^{2k+1}$ is divisible by 7 for $k \in \mathbb{Z}^+$.}		
	$\mathrm{f}(k+1) - \mathrm{f}(k) = 2^{k+1+2} + 3^{2(k+1)+1} - (2^{k+2} + 3^{2k+1})$	Applies $\mathrm{f}(k+1)$ with at least 1 power correct	M1
	$\mathrm{f}(k+1) - \mathrm{f}(k) = 2(2^{k+2}) + 9(3^{2k+1}) - (2^{k+2} + 3^{2k+1})$		
	$\mathrm{f}(k+1) - \mathrm{f}(k) = 2^{k+2} + 8(3^{2k+1})$		
	$= (2^{k+2} + 3^{2k+1}) + 7(3^{2k+1})$	$(2^{k+2} + 3^{2k+1})$ or $\mathrm{f}(k)$; $7(3^{2k+1})$	A1; A1
	or $= 8(2^{k+2} + 3^{2k+1}) - 7(2^{k+2})$	or $8(2^{k+2} + 3^{2k+1})$ or $8\mathrm{f}(k)$; $-7(2^{k+2})$	
	$= \mathrm{f}(k) + 7(3^{2k+1})$ **or** $= 8\mathrm{f}(k) - 7(2^{k+2})$		
	$\therefore \mathrm{f}(k+1) = 2\mathrm{f}(k) + 7(3^{2k+1})$ **or** $\mathrm{f}(k+1) = 9\mathrm{f}(k) - 7(2^{k+2})$	**Dependent on at least one of the previous accuracy marks being awarded.** Makes $\mathrm{f}(k+1)$ the subject	dM1
	$\{\therefore \mathrm{f}(k+1) = 2\mathrm{f}(k) + 7(3^{2k+1})$ is divisible by 7 as both $2\mathrm{f}(k)$ and $7(3^{2k+1})$ are both divisible by 7}		
	If the result is **true for** $n = k$, then it is now **true for** $n = k+1$. As the result has shown to be **true for** $n = 1$, then the result is true **for all** n ($\in \mathbb{Z}^+$).	Correct conclusion seen at the end. Condone true for $n = 1$ stated earlier.	A1 cso
			[6]
ALT	$\mathrm{f}(k+1) - \alpha\mathrm{f}(k) = 2^{k+3} + 3^{2k+3} - \alpha(2^{k+2} + 3^{2k+1})$	Applies $\mathrm{f}(k+1)$ with at least 1 power correct	M1
	$\mathrm{f}(k+1) - \alpha\mathrm{f}(k) = (2-\alpha)2^{k+2} + (9-\alpha)3^{2k+1}$		
	$\mathrm{f}(k+1) - \alpha\mathrm{f}(k) = (2-\alpha)(2^{k+2} + 3^{2k+1}) + 7.3^{2k+1}$ or	$(2-\alpha)(2^{k+2} + 3^{2k+1})$ or $(2-\alpha)\mathrm{f}(k)$; 7.3^{2k+1} or	A1;A1
	$\mathrm{f}(k+1) - \alpha\mathrm{f}(k) = (9-\alpha)(2^{k+2} + 3^{2k+1}) - 7.2^{k+2}$	$(9-\alpha)(2^{k+2} + 3^{2k+1})$ or $(9-\alpha)\mathrm{f}(k)$; -7.2^{k+2}	
		NB: Choosing $\alpha = 0, \alpha = 2, \alpha = 9$ will make relevant terms disappear, but marks should be awarded accordingly.	
			Total 6

Question Number	Scheme		Marks
9.(i) (a)	$\dfrac{3w+7}{5} = \dfrac{(p-4\mathrm{i})}{(3-\mathrm{i})} \times \dfrac{(3+\mathrm{i})}{(3+\mathrm{i})}$	Multiplies by $\dfrac{(3+\mathrm{i})}{(3+\mathrm{i})}$ or divide by $(9-3\mathrm{i})$ then multiply by $\dfrac{(9+3\mathrm{i})}{(9+3\mathrm{i})}$	M1
	$= \left(\dfrac{3p+4}{10}\right) + \left(\dfrac{p-12}{10}\right)\mathrm{i}$	Evidence of $(3-\mathrm{i})(3+\mathrm{i}) = 10$ or $3^2 + 1^2$ or $9^2 + 3^2$	B1
		Rearranges to $w = ...$	dM1
	So, $w = \left(\dfrac{3p-10}{6}\right) + \left(\dfrac{p-12}{6}\right)\mathrm{i}$	At least one of either the real or imaginary part of w is correct in any equivalent form.	A1
		Correct w in the form $a + b\mathrm{i}$. Accept $a + \mathrm{i}b$.	A1
			[5]
ALT (i) (a)	$(3-\mathrm{i})(3w+7) = 5(p-4\mathrm{i})$		
	$9w + 21 - 3\mathrm{i}w - 7\mathrm{i} = 5p - 20\mathrm{i}$		
	$w(9 - 3\mathrm{i}) = 5p - 21 - 13\mathrm{i}$		
	Let $w = a + b\mathrm{i}$, so $(a + b\mathrm{i})(9 - 3\mathrm{i}) = 5p - 21 - 13\mathrm{i}$		
	$9a + 3b - 3a\mathrm{i} + 9b\mathrm{i} = 5p - 21 - 13\mathrm{i}$		
	Real: $9a + 3b = 5p - 21$ Imaginary: $-3a + 9b = -13$	Sets $w = a + b\mathrm{i}$ and equates at least either the real or imaginary part.	M1
		$9a + 3b = 5p - 21$	B1
	$b = \dfrac{p-12}{6}$, $a = \dfrac{3p-10}{6}$	Solves to finds $a = ...$ and $b = ...$	dM1
		At least one of a or b is correct in any equivalent form.	A1
	$w = \left(\dfrac{3p-10}{6}\right) + \left(\dfrac{p-12}{6}\right)\mathrm{i}$	Correct w in the form $a + b\mathrm{i}$. Accept $a + \mathrm{i}b$.	A1
			[5]
(b)	$\left\{\arg w = -\dfrac{\pi}{2} \Rightarrow \left(\dfrac{3p-10}{6}\right) = 0\right\} \Rightarrow p = \dfrac{10}{3}$	$p = \dfrac{10}{3}$ Follow through provided $p < 12$	B1ft
			[1]

(ii)	$(x+iy+1-2i)^* = 4i(x+iy)$	Replaces z with $x+iy$ on both sides of the equation	M1
	$x-iy+1+2i = 4i(x+iy)$ or $x+iy+1-2i = -4i(x-iy)$	Fully correct method for applying the conjugate	M1
	$x-iy+1+2i = 4ix-4y$		
	Real: $\qquad x+1 = -4y$ Imaginary: $\quad -y+2 = 4x$	$x+1 = -4y$ and $-y+2 = 4x$	A1
	$4x+16y = -4$ $4x+y = 2$ $\Rightarrow 15y = -6 \Rightarrow y = ...$	Solves two equations in x and y to obtain at least one of x or y	ddM1
	So, $x = \dfrac{3}{5},\ y = -\dfrac{2}{5}\quad \left\{z = \dfrac{3}{5} - \dfrac{2}{5}i\right\}$	At least one of either x or y are correct	A1
		Both x and y are correct	A1
			[6]
			Total 12