GradeMax

Pearson

Mark Scheme (Results)

Summer 2026

International Advanced Level in Further Pure Mathematics

WFM01/01

GradeMax

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and completing an attempt to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   1. ft – follow through
   2. cao – correct answer only
   3. cso - correct solution only. There must be no clear errors in this part of the question to obtain this mark
   4. isw – ignore subsequent working
   5. awrt – answers which round to
   6. SC: special case
   7. oe – or equivalent
   8. dM – dependent method mark
   9. dp decimal places
   10. sf significant figures
   11. ✱ The answer is given on the paper – apply cso
   12. d... Dependent mark

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1ft to indicate that previous wrong working is to be followed through.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out provided it is not cursory.
   - either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt

7. Ignore subsequent working ("ISW") - i.e., incorrect working or incorrect statements following a correct answer unless the mark scheme indicates otherwise.

8. Mark question parts separately unless the mark scheme indicates otherwise. If parts are not labelled or clearly mislabelled, mark in the order presented.

Method mark for solving a 3 term quadratic:

(Note: There may be other schemes where the below does not apply)

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q),\ \text{where}\left|pq\right|=\left|c\right|$, leading to x = ...

$(ax^2+bx+c)=(mx+p)(nx+q),\ \text{where}\left|pq\right|=\left|c\right|\ \text{and}\ \left|mn\right|=\left|a\right|$, leading to x = ...

2. Formula

Correct attempt to use the correct formula leading to

x = ... (unsimplified). There is no credit just for quoting the correct formula if it is incorrectly used or if only a simplified calculator answer is seen.

3. Completing the square (where *a* = 1; if *a* ≠ 1 must divide by *a* first but allow equivalent work e.g., if a is a perfect square)

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\ \ q\neq 0$, leading to x =...

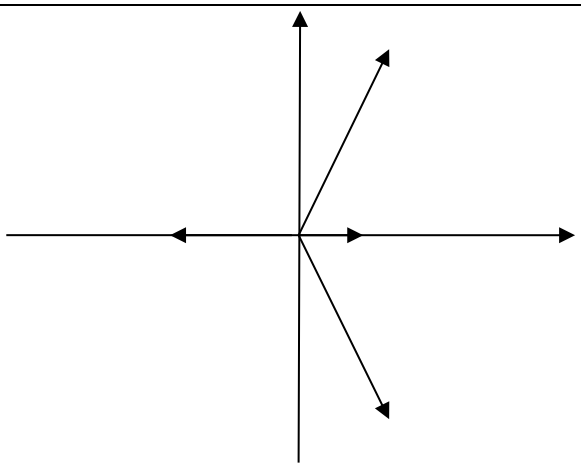
Question Number	Scheme	Notes	Marks
1	$\mathbf{C}=3\mathbf{A}+2\mathbf{B}=3\begin{pmatrix}2&4\\3&1\end{pmatrix}+2\begin{pmatrix}1&k\\5&3k\end{pmatrix}$ Condone any type of brackets (or even missing brackets) for matrices		
(a)	$\mathbf{C}=3\mathbf{A}+2\mathbf{B}=\begin{pmatrix}6&12\\9&3\end{pmatrix}+\begin{pmatrix}2&2k\\10&6k\end{pmatrix}$ $=\begin{pmatrix}8&12+2k\\19&3+6k\end{pmatrix}$	B1: Any 2 correct elements (terms collected but could be factorised) **d**B1: Fully correct **C** (terms collected but could be factorised)	**B1** **dB1**
			(2)
(b)	$\lvert\mathbf{C}\rvert=8(3+6k)-19(12+2k)$ $\{=24+48k-228-38k=10k-204\}$	**Correct** determinant explicitly seen (may be unsimplified but if only seen with the brackets multiplied out this must be done correctly) **for their C** of the correct form i.e., $\begin{pmatrix}a&b+ck\\d&e+fk\end{pmatrix}$ $a,b,c,d,e,f\in\mathbb{Z}^+$ Note that this mark cannot be scored in (c). If parts are unlabelled mark in the order presented. If there is any doubt look for using the determinant to find k	**M1**
	$\lvert\mathbf{C}\rvert=0\Rightarrow 8(3+6k)-19(12+2k)=0\Rightarrow k=...$ Finds a value for k following an attempt to apply $\lvert\mathbf{C}\rvert=0$ with their **C** which must have had at least 1 term in k. **Allow slips here** (e.g., errors multiplying out brackets which they may do immediately) provided there is a recognisable attempt at $ad-bc=0$ Note this mark is not dependent on the previous method mark.		**M1**
	$\{k=\}\frac{102}{5}$, $20\frac{2}{5}$, 20.4	Correct value. Any equivalent e.g., $\frac{204}{10}$	**A1**
			(3)
(c)	$\mathbf{C}^{-1}=\frac{1}{10k-204}\begin{pmatrix}3+6k&-12-2k\\-19&8\end{pmatrix}$ oe e.g., $\begin{pmatrix}\frac{3(1+2k)}{2(5k-102)}&\frac{-6-k}{5k-102}\\\frac{-19}{2(5k-102)}&\frac{8}{2(5k-102)}\end{pmatrix}$	M1: Award for a completed attempt at $\frac{1}{their\lvert\mathbf{C}\rvert}\times(\text{A }\textbf{changed}\ their\ \mathbf{C})$ $\lvert\mathbf{C}\rvert$ must be a 2 term linear function of k although there may have been slips when simplifying and they may reattempt the determinant **OR** sight of a correct ft Adj(their **C**) ignoring any determinant (it could be missing) provided **C** had at least one term in k A1: Fully correct inverse in simplest form (terms collected). Not ft. Elements/determinant may be factorised. Isw once a correct inverse is seen. Remember that ISW applies throughout this paper.	**M1** **A1**
			(2)
			(Total 7 marks)

Question Number	Scheme	Notes	Marks
2	$y^2 = 10x$ $\quad y^2 = 4ax,\ x = at^2,\ y = 2at,$ focus : $(a, 0)$, directrix : $x = -a$		
(a) (i) & (ii)	Focus is $\left(\frac{5}{2}, 0\right)$ **or** directrix is $x = -\frac{5}{2}$ oe e.g., $2x + 5 = 0$	Correct focus (may be given as $x = \frac{5}{2},\ y = 0$) **OR** any correct **equation** for directrix	**B1**
	Focus is $\left(\frac{5}{2}, 0\right)$ **and** directrix is $x = -\frac{5}{2}$ oe	Correct focus (may be given as $x = \frac{5}{2},\ y = 0$) **and** directrix. Allow $\frac{10}{4}$ or $2\frac{1}{2}$ or 2.5 for both marks	**dB1**
	SC: Allow B1dB0 for $(k, 0)$ & $x = -k \quad k > 0,\ k \neq 4$ Just $a = \frac{5}{2}$ is 00 unless $(a, 0),\ x = -a$ are seen		
			(2)
(b)	20	Correct distance	**B1**
			(1)
(c)	$x_P = 20 - "\frac{5}{2}" = ... \left\{= \frac{35}{2}\right\}$	Correct ft value for the x coordinate of P with 20 and their a where $0 < a < 20$. If a = ... is not seen or implied and their focus and directrix are not consistent then look for $20 - a = ...$ from a directrix of $x = -a,\ a > 0$. Ignore any extra x-coordinates	**B1ft**
	$y_P = (\pm)\sqrt{20^2 - (17.5 - 2.5)^2}$ or $y = (\pm)\sqrt{10 \times \frac{35}{2}}$	A correct numerical expression or value for one of the y coordinates of P. Allow awrt $(\pm)13.2$ **Not an ft mark**	**M1**
	$\left(\frac{35}{2}, \pm 5\sqrt{7}\right)$ or e.g., $\left(17.5, \sqrt{175}\right), \left(17.5, -\sqrt{175}\right)$	Fully correct exact coordinates of P May be given as $x = ...\ y = ...$ Answer only is acceptable. No extra coordinates unless rejected	**A1**
	Using a parameter (scheme above still applies): $P\left(at^2, 2at\right) = P\left(\frac{5}{2}t^2, 5t\right) \Rightarrow \frac{5}{2}t^2 = 20 - \frac{5}{2} \Rightarrow t = \pm\sqrt{\frac{2}{5} \times \frac{35}{2}} = \sqrt{7} \Rightarrow P\left(\frac{35}{2}, \pm 5\sqrt{7}\right)$ Allow B1ft for a consistent $x = ...$ from $"\frac{5}{2}"t^2 + "\frac{5}{2}" = 20$ provided $0 < "\frac{5}{2}" < 20$ May also see e.g., $20^2 = \left(\frac{5}{2}t^2 - \frac{5}{2}\right)^2 + (5t)^2 \Rightarrow t^4 + 2t^2 - 63 = \left(t^2 - 7\right)\left(t^2 + 9\right) = 0 \Rightarrow t = \pm\sqrt{7} \Rightarrow P\left(\frac{35}{2}, \pm 5\sqrt{7}\right)$ Calculators may be used to solve equations in solutions such as: $\left(x - \frac{5}{2}\right)^2 + y^2 = 20^2 \Rightarrow x^2 + 5x - \frac{1575}{4} = 0 \Rightarrow 4x^2 + 20x - 1575 = (2x - 35)(2x + 45) = 0 \Rightarrow x = \frac{35}{2}$		
			(3)
			(Total 6 marks)

Question Number	Scheme	Notes	Marks
3(a)	$\sum_{r=1}^{n}(2r-3)(3r+2)=\sum_{r=1}^{n}6r^2-5r-6=6\times\frac{1}{6}n(n+1)(2n+1)-\frac{5}{2}n(n+1)-6n$ M1: Expands to a quadratic and uses at least one of the standard formulae correctly for that term. If quadratic has two terms in r must correctly use the formula twice if scoring for using the sum of r A1: Correct expression - no need to simplify it		**M1** **A1**
	$=n\left[(n+1)(2n+1)-\frac{5}{2}(n+1)-6\right]$ or $=\frac{n}{2}\left[2(n+1)(2n+1)-5(n+1)-12\right]$	Completes an attempt to factorise out n or $\frac{1}{2}n$. May expand completely first. Sign/coefficient errors only. Requires an expression of the correct form i.e., 3TC with no contant ± 2TQ with no constant ± kn	**dM1**
	$=\frac{1}{2}n\left[2\left(2n^2+3n+1\right)-5n-5-12\right]$ $=\frac{1}{2}n\left(4n^2+n-15\right)$	Correct expression in this form. Not just values for the constants unless $\frac{1}{2}n\left(an^2+bn+c\right)$ is seen in the work	**A1**
	Alternative for the last 2 marks - expands expression and given form and compares coefficients: $6\times\frac{1}{6}n(n+1)(2n+1)-\frac{5}{2}n(n+1)-6n=2n^3+3n^2+n-\frac{5}{2}n^2-\frac{5}{2}n-6n=2n^3+\frac{1}{2}n^2-\frac{15}{2}n$ $\frac{1}{2}n\left(an^2+bn+c\right)=\frac{1}{2}an^3+\frac{1}{2}bn^2+\frac{1}{2}cn$ $\Rightarrow a=4,\ b=1,\ c=-15$ **d**M1: Expands both sides. LHS must have been of the form 3TC with no contant ± 2TQ with no constant ± kn and must achieve 2 3TCs with no constants A1: Correct values. $\frac{1}{2}n\left(an^2+bn+c\right)$ must be explicitly seen in the work.		
			(4)
(b)	$21\times38+23\times41+\ldots+97\times152$ $=\frac{1}{2}\times50\left(4\times50^2+50-15\right)-\frac{1}{2}\times11\left(4\times11^2+11-15\right)$ Attempts $\sum_{r=1}^{50}(2r-3)(3r+2)-\sum_{r=1}^{11}(2r-3)(3r+2)$ **explicitly** using their result from part (a) and obtains a numerical expression. Allow slips but it must clearly be an attempt at f(50) - f(11). There must be evidence of use of the formula for both summations so accept e.g., $\frac{1}{2}\times501750-\frac{1}{2}\times5280$ as a minimum but **not** $250875-2640$		**M1**
	= 248 235	Correct answer from correct work	**A1**
			(2)
			(Total 6 marks)

Question Number	Scheme	Notes	Marks
4	$f(x)=\frac{1}{10}x-\frac{(x-4)^2}{\sqrt{x}}=\frac{1}{10}x-\frac{x^2-8x+16}{\sqrt{x}}=\frac{1}{10}x-x^{\frac{3}{2}}+8x^{\frac{1}{2}}-16x^{-\frac{1}{2}}$ $x>0$		
(a)	$f(5)=0.05278...$ $f(5.1)=-0.02579...$ Obtains values for both f (5) and f (5.1) with one correct (1 sf as shown below) if just values are given. Accept $f(5)=\frac{5-2\sqrt{5}}{10}$ for the M mark but the A mark requires a convincing argument that this is > 0		**M1**
	Sign change (or > 0, < 0, or positive, negative or product negative) and f(x) is **continuous** therefore a **root** α is between x = 5 and x = 5.1	f (5) = awrt 0.05 and f (5.1) = awrt –0.03 (or truncated: -0.02), sign change, any reference to continuity (which could be poor e.g., "x/it/the graph is continual") and a minimal conclusion.	**A1**
			(2)
(b)	$f(5.05)=0.01439...$ $(\Rightarrow 5.05<\alpha<5.1)$	For f (5.05) = awrt 0.01	**B1**
	$f(5.075)=...$ $\{-0.00547...\}$	Obtains a value for f (5.075) following a positive value for f (5.05)	**M1**
	$[5.05,\ 5.075]$ or e.g., $5.05<\alpha<5.075$ Correct interval following no clearly incorrect values for f(5.05) and f(5.075). Allow any equivalent notation but must be a **mathematical** interval. May use x or a etc for α. Note real root is 5.068164... Ignore further bisections and additional work/comment		**A1**
			(3)
(c)	$f'(x)=\frac{1}{10}-\frac{3}{2}x^{\frac{1}{2}}+4x^{-\frac{1}{2}}+8x^{-\frac{3}{2}}$ M1: For $...x^n\rightarrow...x^{n-1}$ once, e.g., $\frac{1}{10}x\rightarrow\frac{1}{10}$ & allow unprocessed indices e.g., $\frac{1}{10}x^{1-1}$ A1: An expression of the correct form (sign and coefficient slips only). No unprocessed indices nor x^0 A1: All correct. Accept $...+\frac{8}{2}x^{-\frac{1}{2}}+\frac{16}{2}x^{-\frac{1}{2}}$		**M1** **A1** **A1**
	If quotient/product rule is used on the fraction you may see: $\frac{1}{10}-\frac{2x^{\frac{1}{2}}(x-4)-\frac{1}{2}x^{-\frac{1}{2}}(x-4)^2}{x}$ or $\frac{1}{10}-\frac{x^{\frac{1}{2}}(2x-8)-\frac{1}{2}x^{-\frac{1}{2}}(x^2-8x+16)}{x}$ or $\frac{1}{10}-\left(2x^{-\frac{1}{2}}(x-4)-\frac{1}{2}x^{-\frac{3}{2}}(x-4)^2\right)$ or $\frac{1}{10}-\left(x^{-\frac{1}{2}}(2x-8)-\frac{1}{2}x^{-\frac{3}{2}}(x^2-8x+16)\right)$ and remember to ISW once a correct answer is seen		
			(3)
(d)	$x_1=3.3-\frac{f(3.3)}{f'(3.3)}=3.3-\frac{0.06026387754...}{0.9115437451...}=...$ $\{3.23388811...\}$ Attempts to apply a correct Newton-Raphson formula with 3.3, obtaining a value. Implied by awrt 3.234 (i.e., correct to 3 dp) and could just give this value which scores both marks. Accept as minimal evidence e.g., "$\{x_1=\}\,x_0-\frac{f(x_0)}{f'(x_0)}=...$" and no need to check value. Note: real root is 3.2368768...		**M1**
	= awrt 3.234 Ignore how the value is labelled and it can follow an incorrect f '(x)		**A1**
			(2)
			(Total 10 marks)

Question Number	Scheme	Notes	Marks
5	$2x^2 - x + 3 = 0$ Note that solving the equation and clearly using the values $\alpha = \frac{1+\sqrt{23}\mathrm{i}}{4}$ and $\beta = \frac{1-\sqrt{23}\mathrm{i}}{4}$ throughout i.e. for $\alpha^3 + \beta^3$, $\alpha^4\beta - 2 + \alpha\beta^4 - 2$ & $\left(\alpha^4\beta - 2\right)\left(\alpha\beta^4 - 2\right)$ is likely to only be able to access the final M mark in (c). However, if identities are used in (b) and (c) allow all the marks to be accessible.		
(a)	$\alpha + \beta = \frac{1}{2}, \quad \alpha\beta = \frac{3}{2}$	Both values correct. B0 if these have clearly come from solving the equation.	**B1**
			(1)
(b)	$\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = \left(\frac{1}{2}\right)^3 - 3\times\frac{3}{2}\times\frac{1}{2}$ Obtains a numerical expression for $\alpha^3 + \beta^3$ with a **correct** identity. Condone substitution slips e.g., mixing up "$\frac{1}{2}$" and "$\frac{3}{2}$" and allow M1 if a correct identity is seen or implied. Sight of a correct identity followed by a value is sufficient but if only a value is seen it must be $-\frac{17}{8}$		**M1**
	$= -\frac{17}{8}$ oe, -2.125	Correct value. Must not be fortuitous but allow all part (c) marks.	**A1**
			(2)
(c)	Work for the new sum/product may be embedded within a quadratic		
	new sum $= \left\{\alpha^4\beta - 2 + \alpha\beta^4 - 2 =\right\}\ \alpha\beta\left(\alpha^3 + \beta^3\right) - 4 = \frac{3}{2}\left(-\frac{17}{8}\right) - 4 \quad \left\{= -\frac{115}{16}\right\}$ Obtains a numerical expression for the new sum with a **correct** identity. Condone substitution slips e.g., mixing up "$\frac{1}{2}$" and "$\frac{3}{2}$" and allow M1 if a correct identity is seen or implied. Also ft their $\alpha^3 + \beta^3$ which may be recalculated wrongly and may be 0. Sight of a correct identity followed by a value is sufficient but if only a value is given it must be $-\frac{115}{16}$		**1st M1 (sum)**
	new product $= \left\{\left(\alpha^4\beta - 2\right)\left(\alpha\beta^4 - 2\right) =\right\}\ (\alpha\beta)^5 - 2\alpha\beta\left(\alpha^3 + \beta^3\right) + 4 = \left(\frac{3}{2}\right)^5 - 2\times\left(\frac{3}{2}\right)\left(-\frac{17}{8}\right) + 4 \quad \left\{= \frac{575}{32}\right\}$ Obtains a numerical expression for the new product with a **correct** identity. Condone substitution slips e.g., mixing up "$\frac{1}{2}$" and "$\frac{3}{2}$" and allow M1 if a correct identity is seen or implied. Also ft their $\alpha^3 + \beta^3$ which may be recalculated wrongly and may be 0. Sight of a correct identity followed by a value is sufficient but if only a value is given it must be $\frac{575}{32}$		**2nd M1 (prod.)**
	new sum $= -\frac{115}{16}$ **or** new product $= \frac{575}{32}$	One correct value. Allow -7.1875, 17.96875 **Requires one previous M mark.**	**A1 (M1 on ePen)**
	$x^2 - \left(-\frac{115}{16}\right)x + \frac{575}{32}(=0)$ Correctly applies x^2 – (their new sum)x + their new product. If e.g., the "x" is missing it must be recovered. Allow a different variable e.g., z. May use rounded decimals. Accept values for p, q, r		**M1**
	$32x^2 + 230x + 575 = 0$	Correct **equation in x**. Three terms in any order on same side. Allow any integer multiple. If just values are given $px^2 + qx + r = 0$ must be seen in the work	**A1**
			(5)
			(Total 8 marks)

Question Number	Scheme	Notes	Marks
6	$f(z) = 12z^4 + Az^3 + Bz^2 + Cz - 306$ Allow work in x		
(a)	$(z =) 1 - 4i$	Correct conjugate seen anywhere	**B1**
			(1)
(b)	$z = 1 \pm 4i \Rightarrow (z - (1+4i))(z - (1-4i)) = ... \quad \{z^2 - 2z + 17\}$ or e.g., sum of roots $= ... \{2\}$, product of roots $= (1+4i)(1-4i) = ... \{17\} \Rightarrow ... \{z^2 - 2z + 17\}$ M1: Applies an appropriate strategy and finds a 3 term quadratic factor with real coefficients. Condone poor algebra provided there is a completed attempt using $1+4i$ and $1-4i$ A1: A correct 3 term quadratic factor. Could be just written down. Ignore if "= 0" or f(z) =...		**M1 A1**
			(2)
	Mark parts (c) and (d) together		
(c)	The other factor is $(4z \pm k)$	Sight of $(4z \pm k)$ as the other linear factor. See note below. No requirement to find k but do not award for just $4z$	**B1**
	e.g., $k \times -2 \times "17" = -306 \Rightarrow k = ...(9)$ $(Dz + E)("3z^3 - 8z^2 + 55z - 34") = f(z) \Rightarrow D = 4,\ E = ...$ Any appropriate method using $(4z \pm k)$ to obtain k seen. See note below		**M1**
	Note: If $(z \pm \frac{j}{4})$ is used then a multiplier of 4 must be recovered later to score marks		
	$\{f(z) =\}(z^2 - 2z + 17)(3z - 2)(4z + 9)$ oe e.g., $(z - (1+4i))(z - (1-4i))(3z - 2)(4z + 9)$ $12(z^2 - 2z + 17)(z - \frac{2}{3})(z + \frac{9}{4})$	All correct on one line. Factors in any order. Ignore irrelevant errors. Answer could just be written down.	**A1**
			(3)
(d)	$f(z) = (z^2 - 2z + 17)(3z - 2)(4z + 9) = ...$	Expands and collects terms or obtains values for all 3 constants via any credible method. If method unclear look for one of the correct values below	**M1**
	$f(z) = (12z^4) - 5z^3 + 148z^2 + 359z(-306)$ or $A = -5,\ B = 148,\ C = 359$	A1: 2 correct (with 3 found) A1: All 3 correct	**A1 A1**
			(3)
(e)		B1: Conjugate pair (broadly symmetrical but anywhere in Q1 and Q4) **or** real pair (positive root closer to origin). **d**B1: All plotted such that the distance to the origin for the real positive root < the shortest distance to the y-axis for the complex roots **and** the positive real root positioned closer to the origin than the real negative root. Points or vectors/lines. Ignore root/axes labelling. These are not ft marks so do not award if their work clearly indicates they are using any roots which are not $1 \pm 4i$, $\frac{2}{3}$, $-\frac{9}{4}$ but ignore the labelling on the diagram	**B1 dB1**
			(2)
			(Total 11 marks)

Question Number	Scheme	Notes	Marks
7	$\mathbf{M}=\begin{pmatrix}1 & 0\\ 0 & 3\end{pmatrix}$ $\mathbf{Q}=\begin{pmatrix}5 & -1\\ 5 & 3\end{pmatrix}$	Allow any (or no) brackets for matrices	
(i)(a)	A stretch, (scale) factor 3, parallel to the/in the/on the/up the/down the/along the/across the *y*-axis/ *y* direction/of the y-coordinates or vertically/ perpendicular to *x*-axis M1: Stretch. Must be a single transformation and no alternatives also offered. Ignore additional "and parallel to the *x*-axis scale of factor 1" & similar May be mis-spelt but must be an attempt at "stretch" - not e.g., "expansion". A1: Fully correct description. Just "in *y*" is insufficient. Accept surrogates for "factor" e.g., "scale" but not just "stretch of/by 3". Ignore reference to "about O"		**M1** **A1**
			(2)
(b)	$\{\mathbf{N}=\}\ \begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix}$	Correct matrix. No elements as trig expressions	**B1**
			(1)
(c)	$\{\mathbf{P}=\mathbf{NM}=\}\ "\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix}"\begin{pmatrix}1 & 0\\ 0 & 3\end{pmatrix}$ Indicates intention to multiply the right way round. If this is set up as a grid multiplication then they must get a consistent answer. M0 if written the wrong way around even if the correct answer results. The **NM** must be seen if **N** is incorrect and their **N** must have at least two non-zero elements and cannot be **I**		**M1**
	$\begin{pmatrix}0 & -3\\ -1 & 0\end{pmatrix}$	Correct matrix. No incorrect work. Accept answer only.	**A1**
			(2)
(ii)	$\left\{\det\mathbf{Q}=\begin{vmatrix}5 & -1\\ 5 & 3\end{vmatrix}=\right\}\ 5\times3-5\times(-1)\ \{=20\}$	Correct value or numerical expression for det **Q** Could be seen in an attempt at $\mathbf{Q}^{-1}$	**B1**
	e.g., Area of H is $\frac{200}{"20"}\ \{=10\}$ $A_H\times"20"=200$ M1: A correct ft equation or statement linking the area of H and 200 ignoring any incorrect expression for A_H A1: Correct statement/equation but ignore any incorrect expression for A_H		**M1** **A1**
	Area of $H=6\times\frac{1}{2}\times x\times x\sin60^\circ$ or e.g., $\frac{3\sqrt{3}}{2}x^2$ or e.g., $2\left(\frac{1}{2}x^2\sin120°\right)+\sqrt{3}\,x.x$	Any correct expression for the area of H in terms of x. Trig expressions may be unevaluated. Not dependent.	**M1**
	$\{x=\}$ awrt 1.96	Not $\pm$. Must be decimal so A0 for e.g., $\sqrt{\frac{20\sqrt{3}}{9}}$	**A1**
	Alternative via x' : B1 and 2nd M1 as before $6\times\frac{1}{2}\times x'\times x'\sin60^\circ=200\Rightarrow\frac{3\sqrt{3}}{2}(x')^2=200\Rightarrow(x')^2=\frac{400}{3\sqrt{3}}$ or $\frac{400\sqrt{3}}{9}\Rightarrow x'=\sqrt{\frac{400}{3\sqrt{3}}}=\frac{20\sqrt{3}}{3}=8.7738...$ $x'=\frac{"8.7738..."}{\sqrt{20}}(1\text{st M1 A1})=1.961887...\approx1.96\,(\text{A1})$		
			(5)
			(Total 10 marks)

Question Number	Scheme	Notes	Marks
8 **Way 1 f(*k*+1)=...** **Way 2 overleaf**	$f(n)=3^{2n+4}-2^{2n}$ May see subtle variations e.g., replacing f(*k*) with "5α" Accept a proof that is done in terms of another variable including *n*		
	$f(1)=3^6-2^2=725$	Correct value (or 5 x 145) for $n=1$	**B1**
	Assume true for $n=k$ so that $3^{2k+4}-2^{2k}$ is divisible by 5		
	$\{f(k+1)=\}3^{2(k+1)+4}-2^{2(k+1)}$	Attempts f (k + 1).	**M1**
	$\{f(k+1)=\}3^{2k+6}-2^{2k+2}=9\times3^{2k+4}-9\times2^{2k}+5\times2^{2k}$ or $4\times3^{2k+4}-4\times2^{2k}+5\times3^{2k+4}$	Obtains a 3 term expression which includes terms of the form $...\times3^{2k+4}$ & $...\times2^{2k}$ which might be implied by introduction of f(*k*). 3rd term must include a function of *k* as a power	**dM1**
	$f(k+1)=9f(k)+5\times2^{2k}$ or $f(k+1)=4f(k)+5\times3^{2k+4}$ Accept e.g., $f(k+1)=4f(k)+405\times3^{2k}$ or $f(k+1)=5(9\alpha+2^{2k})$ Correctly expresses f (k + 1) in terms of f (k) with f (k) isolated. Allow $f(k+1)=9(3^{2k+4}-2^{2k})+5\times2^{2k}$ or $f(k+1)=4(3^{2k+4}-2^{2k})+5\times3^{2k+4}$ f(k + 1) must be explicitly seen at some point. Allow recovery from poor bracketing.		**A1**
	If the result is **true for *n* = *k* then** it is **true for *n* = *k* + 1**. As the result has been shown to be **true for *n* = 1**, then the result is **true for** (all) ***n*** Correct conclusion or narrative. Minimum in bold. Accept surrogates for "true". Accept e.g.,"$n\in\mathbb{Z}$" for "$n\in\mathbb{Z}^+$" but not "$n\in\mathbb{R}$". If "assume true for $n=k$" is seen the "if...then" is not required. There is no requirement to say or demonstrate that e.g., 725, 405, 45 etc are divisible by 5. Requires all previous marks.		**A1**
			(Total 5 marks)

Question Number	Scheme	Notes	Marks
8 **Way 2** **f(*k*+1) -*m*f(*k*)=...** **Note: It is not essential to choose a value for *m***	$f(n)=3^{2n+4}-2^{2n}$ May see subtle variations e.g., replacing f(k) with "5α" Accept a proof that is done in terms of another variable including n		
	$f(1)=3^6-2^2=725$	Correct value (or 5 x 145) for n = 1	**B1**
	$f(k+1)-mf(k)=3^{2(k+1)+4}-2^{2(k+1)}-m\left(3^{2k+4}-2^{2k}\right)$ If $m=1$: $f(k+1)-f(k)=3^{2(k+1)+4}-2^{2(k+1)}-\left(3^{2k+4}-2^{2k}\right)$ If $m=-1$: $f(k+1)+f(k)=3^{2(k+1)+4}-2^{2(k+1)}+3^{2k+4}-2^{2k}$	Attempts f (k + 1), can be scored as Way 1 but is likely to be within an attempt at f (k + 1) – m f (k)	**M1**
	$f(k+1)-mf(k)=(9-m)\times\left(3^{2k+4}-2^{2k}\right)+5\times2^{2k}$ or $f(k+1)-mf(k)=(4-m)\left(3^{2k+4}-2^{2k}\right)+5\times3^{2k+4}$ If $m=1$: $f(k+1)-f(k)=8\times\left(3^{2k+4}-2^{2k}\right)+5\times2^{2k}$ or $3\left(3^{2k+4}-2^{2k}\right)+5\times3^{2k+4}$ If $m=-1$: $f(k+1)+f(k)=10\times\left(3^{2k+4}-2^{2k}\right)+5\times2^{2k}$ or $5\left(3^{2k+4}-2^{2k}\right)+5\times3^{2k+4}$ If $m=4$: $f(k+1)-4f(k)=5\times3^{2k+4}$ or $5\times\left(3^{2k+4}-2^{2k}\right)+5\times2^{2k}$ If $m=9$: $f(k+1)-9f(k)=5\times2^{2k}$ or $-5\times\left(3^{2k+4}-2^{2k}\right)+5\times3^{2k+4}$ Obtains a 3 term expression (**but see note below**) which includes terms of the form $...\times3^{2k+4}$ or $...\times2^{2k}$ but allow only seeing 1 of these if 4 or 9 is used for m and allow e.g., $5\times3^{2k+4}$ as 405×3^{2k}. 3rd term must include a function of k as a power. **Note** that it is possible that the expression on the RHS could be written as a multiple of 5 without involving f(k) e.g., $f(k+1)+f(k)=3^{2k+6}-2^{2k+2}+3^{2k+4}-2^{2k}=9\times3^{2k+4}-4\times2^{2k}+3^{2k+4}-2^{2k}=10\times3^{2k+4}-5\times2^{2k}$ In such cases the work must be correct.		**dM1**
	$f(k+1)=9f(k)+5\times2^{2k}$ or $f(k+1)=4f(k)+5\times3^{2k+4}$ Accept e.g., $f(k+1)=4f(k)+45\times3^{2k+2}$ Correctly expresses f (k + 1) in terms of f (k) with f (k) isolated. Must make f (k + 1) the subject unless there is mention of e.g., "RHS is divisible by 5 so $\Rightarrow$ f (k + 1) – m f (k) is divisible by 5" oe. Allow $f(k+1)=9\left(3^{2k+4}-2^{2k}\right)+5\times2^{2k}$ or $f(k+1)=4\left(3^{2k+4}-2^{2k}\right)+5\times3^{2k+4}$ $f(k+1)$ must be explicitly seen at some point. Allow recovery from poor bracketing.		**A1**
	If the result is **true for *n* = *k* then** it is **true for *n* = *k* + 1**. As the result has been shown to be **true for *n* = 1**, then the result is **true for** (all) ***n*** Correct conclusion or narrative. Minimum in bold. Accept surrogates for "true". Accept e.g.,"$n\in\mathbb{Z}$" for "$n\in\mathbb{Z}^+$" but not "$n\in\mathbb{R}$". If "assume true for $n=k$" is seen the "if...then" is not required. There is no requirement to say or demonstrate that e.g., 725, 405, 45 etc are divisible by 5. Requires all previous marks.		**A1**
			(Total 5 marks)

Question Number	Scheme	Notes	Marks
9(a)	$xy=c^2 \Rightarrow y=c^2x^{-1} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-c^2x^{-2}$ or $xy=c^2 \Rightarrow x\frac{\mathrm{d}y}{\mathrm{d}x}+y=0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{y}{x}$ or $x=cp, y=\frac{c}{p} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-cp^{-2}}{c}$ Any correct expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$		**B1**
	$m_T=-\frac{c^2}{c^2p^2} \Rightarrow m_N=p^2$ or $m_T=-\frac{c/p}{cp} \Rightarrow m_N=p^2$ or $m_T=-p^{-2} \Rightarrow m_N=p^2$ Correct use of the perpendicular gradient rule and the point P to obtain a normal gradient in terms of p (and possibly c)		**M1**
	$y-\frac{c}{p}=p^2(x-cp)$ or $y=p^2x+k,\ \frac{c}{p}=p^2\times cp+k \Rightarrow k=...\ \left\{\frac{c}{p}-cp^3\right\}$ Forms the equation of the normal correctly for their changed gradient in terms of p (and possibly c). Only reaching k = ... requires requires sight of y = "p^2"$x+k$		**M1**
	$py-c=p^3x-cp^4$ or $y=p^2x+\frac{c}{p}-cp^3$ $\Rightarrow p^3x-py=c\left(p^4-1\right)$*	Obtains the given answer with an intermediate step and no errors. Last line: p^3x and $-py$ on one side, $c\left(p^4-1\right)$ on other	**A1***
			(4)
(b)	Treat the M and dM marks like a B mark and a dB mark - so correct work is required. The first mark is for a significant correct first step and the second is for reaching an equation from which the given result can clearly be easily obtained. **Use this guidance for any potentially creditworthy alternative approaches but use Review if you are in doubt.**		
Way 1 **grad PQ**	gradient $PQ=\frac{\frac{c}{p}-\frac{c}{q}}{cp-cq}$	Correct expression for the gradient of the chord PQ	**M1**
	$\frac{\frac{c}{p}-\frac{c}{q}}{cp-cq}=p^2 \Rightarrow \frac{\frac{q-p}{pq}}{p-q}=p^2$ or $\frac{\frac{c(q-p)}{pq}}{c(p-q)}=p^2$	Equates the 2 gradients (may repeat the work in (a)) and correctly makes the numerator a single fraction or otherwise manipulates correctly to reach an equation from which the given result can clearly be easily obtained - all correct work	**dM1**
	$\Rightarrow p^3q=-1$* Allow e.g., $-1=qp^3$	Fully correct proof. Can proceed directly to the given answer from the expressions shown for the **d**M1	**A1*** **(3)**
Way 2 **Sub. Q into normal**	Substituting Q into the normal: $Q\left(cq, \frac{c}{q}\right)$ in $p^3x-py=c\left(p^4-1\right) \Rightarrow cp^3q-\frac{cp}{q}=c\left(p^4-1\right)$ (M1: correct equation) e.g., $p^3q+1-\frac{p}{q}-p^4=p^3q+1-\frac{p}{q}\left(1+p^3q\right)=\left(p^3q+1\right)\left(1-\frac{p}{q}\right)=0$ (**d**M1) $p^3q=-1$(A1*)		
Way 3 **Sub. $xy=c^2$ into normal**	$y=\frac{c^2}{x}$ in $p^3x-py=c\left(p^4-1\right) \Rightarrow p^3x-\frac{pc^2}{x}=c\left(p^4-1\right)$ (M1: correct equation) $p^3x^2-c\left(p^4-1\right)x-pc^2=\left(p^3x+c\right)(x-pc)=0 \Rightarrow x=-\frac{c}{p^3}=cq$ (dM1) $p^3q=-1$* (A1*) OR $x=\frac{c^2}{y}$ in $p^3x-py=c\left(p^4-1\right) \Rightarrow \frac{p^3c^2}{y}-py=c\left(p^4-1\right)$(M1: correct equation) $py^2+cy\left(p^4-1\right)-p^3c^2=\left(py+cp^4\right)\left(y-\frac{c}{p}\right)=0 \Rightarrow y=-cp^3=\frac{c}{q}$ (dM1) $p^3q=-1$* (A1*)		

Question Number	Scheme	Notes	Marks
9(c)	$q^3x-qy=c\left(q^4-1\right)$ and $xy=c^2\Rightarrow$ $q^3x-q\dfrac{c^2}{x}=c\left(q^4-1\right)$ or $q^3\dfrac{c^2}{y}-qy=c\left(q^4-1\right)$ $\Rightarrow q^3x^2-c\left(q^4-1\right)x-c^2q=0$ or $qy^2+c\left(q^4-1\right)y-q^3c^2=0$ Or with $q=-\dfrac{1}{p^3}\Rightarrow-\dfrac{1}{p^9}x+\dfrac{1}{p^3}.\dfrac{c^2}{x}=c\left(\dfrac{1}{p^{12}}-1\right)\Rightarrow\dfrac{1}{p^9}x^2+cx\left(\dfrac{1}{p^{12}}-1\right)-\dfrac{c^2}{p^3}=0$ or $-\dfrac{c^2}{p^9y}+\dfrac{y}{p^3}=c\left(\dfrac{1}{p^{12}}-1\right)\Rightarrow\dfrac{y^2}{p^3}-cy\left(\dfrac{1}{p^{12}}-1\right)-\dfrac{c^2}{p^9}=0$ Substitutes the equation of the hyperbola into the equation of the normal (which they may obtain from scratch) to obtain a 3 term quadratic equation in x or y Could be uncollected terms in x (or y) and allow slips if the intention is clear		**M1**
	$\Rightarrow\left(x-cq\right)\left(q^3x+c\right)=0\Rightarrow x=-\dfrac{c}{q^3}$ or $\Rightarrow\left(qy-c\right)\left(y+cq^3\right)=0\Rightarrow y=-cq^3$ or $p^3x^2+cx\left(1-p^{12}\right)-c^2p^9=\left(p^3x+c\right)\left(x-cp^9\right)=0\Rightarrow x=cp^9$ $p^9y^2-cy\left(1-p^{12}\right)-p^3c^2=\left(p^9y-c\right)\left(y+cp^3\right)=0\Rightarrow y=\frac{c}{p^9}$ Completes a credible attempt to find the x or y coordinate of R (not Q) Note that this may result from work in (b) but this must be restated or used in (c)		**dM1**
	R is $\left(-\dfrac{c}{q^3},-cq^3\right)$ or $\left(cp^9,\dfrac{c}{p^9}\right)$	Correct coordinates of R May be seen as $x=...,\ y=...$	**A1**
	Gradient PR is $\dfrac{\frac{c}{p}+cq^3}{cp+\frac{c}{q^3}}$ or $\dfrac{\frac{c}{p}-\frac{c}{p^9}}{cp-cp^9}$	Forms a **correct** gradient of PR	**ddM1**
	gradient $PR=\dfrac{\frac{c}{p}+cq^3}{cp+\frac{c}{q^3}}=\dfrac{\frac{1+pq^3}{p}}{\frac{pq^3+1}{q^3}}=\dfrac{q^3}{p}$ and $q=-\dfrac{1}{p^3}\Rightarrow-\dfrac{1}{p^9}\times\dfrac{1}{p}=-\dfrac{1}{p^{10}}$ or gradient $PR=\dfrac{\frac{c}{p}-\frac{c}{p^9}}{cp-cp^9}=\dfrac{\frac{c\left(p^8-1\right)}{p^9}}{cp\left(1-p^8\right)}=-\dfrac{1}{p^{10}}$ Correct answer. Accept $k=10$ from appropriate work		**A1**
			(5)
			(Total 12 marks)
Alt	Let R have coordinates $\left(cr,\dfrac{c}{r}\right)$ and $q^3r=-1$	Introduces a 3rd parameter for R e.g. r and states/uses $q^3"r"=-1$ somewhere	**M1**
	Gradient PR is $\dfrac{\frac{c}{p}-\frac{c}{r}}{cp-cr}=-\dfrac{1}{pr}$	**d**M1: Correctly forms the gradient of PR A1: Correct simplified gradient in terms of p and "r"	**dM1** **A1**
	$r=-\frac{1}{q^3}\Rightarrow$ gradient $=\frac{q^3}{p}$	Eliminates "r"	**ddM1**

	and $q = -\frac{1}{p^3} \Rightarrow$ gradient $PR = -\frac{1}{p^9} \times \frac{1}{p} = -\frac{1}{p^{10}}$	Correct answer	**A1**

