Mark Scheme (Results)

Summer 2017

Pearson Edexcel GCE
In Further Pure Mathematics FP2 (6668/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
- M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations
   These are some of the traditional marking abbreviations that will appear in the mark schemes.
   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ∗ The answer is printed on the paper
   - ⎣ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

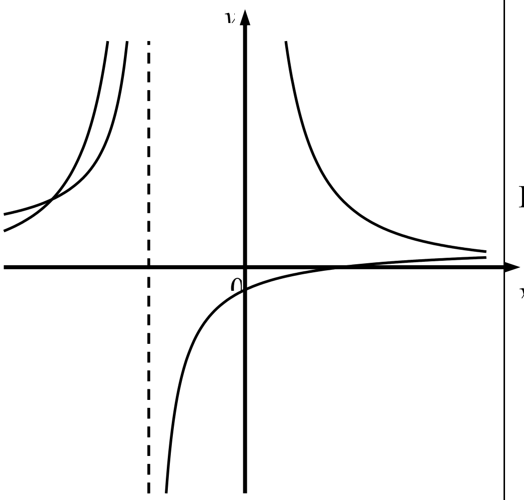
2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Question Number	Scheme	Notes	Marks
1(a)	$\frac{1}{r^2}-\frac{1}{(r+1)^2}=\frac{(r+1)^2-r^2}{r^2(r+1)^2}=\frac{2r+1}{r^2(r+1)^2}$	Correct proof (minimum as shown) ($(r+1)^2$ or r^2+2r+1 Can be worked in either direction.	B1
			(1)
(b)	$\sum_{r=1}^{n}\left(\frac{1}{r^2}-\frac{1}{(r+1)^2}\right) = 1-\frac{1}{4}+\frac{1}{4}-\frac{1}{9}......+\left(\frac{1}{n^2}\right)-\frac{1}{(n+1)^2}$ Terms of the series with $r=1$, $r=n$ and one of $r=2, r=n-1$ should be shown.		M1
	$1-\frac{1}{(n+1)^2}$	Extracts correct terms that do not cancel	A1
	$\frac{(n+1)^2-1}{(n+1)^2}=\frac{n(n+2)}{(n+1)^2}$ *	Correct completion with no errors	A1*cso
			(3)
(c)	$\sum_{r=n}^{3n}\frac{6r+3}{r^2(r+1)^2}=3\left(\frac{3n(3n+2)}{(3n+1)^2}-\frac{(n-1)(n+1)}{n^2}\right)$	Attempts to use $f(3n)-(f(n-1)\text{ or }f(n))$ 3 may be missing	M1
	$=3\left(\frac{3n^3(3n+2)-(3n+1)^2(n^2-1)}{n^2(3n+1)^2}\right)$	Attempt at common denominator, Denom to be $n^2(3n+1)^2$ or $(n+1)^2(3n+1)^2$ Numerator to be difference of 2 quartics. 3 may be missing	dM1
	$=\frac{24n^2+18n+3}{n^2(3n+1)^2}$	cao	A1cao
			(3)
			Total 7
	Alternative for part (c)		
	$\sum_{r=n}^{3n}\frac{6r+3}{r^2(r+1)^2}=3\left(\frac{1}{n^2}-\frac{1}{(3n+1)^2}\right)$ OR: $3\left(\frac{1}{(n+1)^2}-\frac{1}{(3n+1)^2}\right)$	Attempts the difference of 2 terms (either difference accepted) 3 may be missing	M1
	$=3\left(\frac{(3n+1)^2-n^2}{n^2(3n+1)^2}\right)$	Valid attempt at common denominator for their fractions 3 may be missing	dM1
	$=\frac{24n^2+18n+3}{n^2(3n+1)^2}$	cao	A1
	If (b) and/or (c) are worked with r instead of n do **NOT** award the final A mark for the parts affected. This applies even if r is changed to n at the end.		

	Alternative for (b) - by induction. **NB:** No marks available if result in (a) is not used.		
	Assume true for $n=k$		
	$\sum_{r=1}^{k+1}\frac{2r+1}{r^2(r+1)^2}=\frac{k(k+2)}{(k+1)^2}+\frac{1}{(k+1)^2}-\frac{1}{(k+2)^2}$	Uses $\sum_{r=1}^{k}$ together with the $(k+1)$th term as 2 fractions (see (a))	M1
	$.=\frac{k^2+2k+1}{(k+1)^2}-\frac{1}{(k+2)^2}.$		
	$1-\frac{1}{(k+2)^2}=\frac{k^2+4k+3}{(k+2)^2}=\frac{(k+1)(k+3)}{(k+2)^2}$	Combines the 3 fractions to obtain a single fraction. Must be correct but numerator need not be factorised.	A1
	Show true for $n=1$	This must be seen somewhere	
	Hence proved by induction	Complete proof with no errors and a concluding statement.	A1

Question Number	Scheme	Notes	Marks
2.	$\frac{x-2}{2(x+2)} \le \frac{12}{x(x+2)}$		
NB	Question states "Use algebra..." so purely graphical solutions score max 1/9 (the B1). A sketch and some algebra to find CVs or intersection points can score according to the method used.		
	Can use $\le, <$ or $=$ for the first 6 marks **in all methods**		
	$\frac{x-2}{2(x+2)} - \frac{12}{x(x+2)} (\le 0)$	Collects expressions to one side.	M1
	$\frac{x^2-2x-24}{2x(x+2)} (\le 0)$	M1: Attempt common denominator A1: Correct single fraction	M1A1
	$x = 0, -2$	Correct critical values	B1
	$x^2-2x-24 \Rightarrow (x+4)(x-6)(=0) \Rightarrow x = ...$	Attempt to solve their quadratic as far as $x = ...$	M1
	$x = -4, 6$	Correct critical values. May be seen on a sketch.	A1
	$-4 \le x < -2,\ \ 0 < x \le 6$ with $\le$ or $<$ throughout	M1: Attempt two inequalities using their 4 critical values in ascending order. (dependent on at least one previous M mark) A1: All 4 CVs in the inequalities correct	dM1A1
	$-4 \le x < -2,\ \ 0 < x \le 6$ $[-4,-2) \cup (0,6]$	A1:Inequality signs correct Set notation may be used. $\cup$ or "or" but not "and"	A1cao **(9)**
			Total 9
	Alternative 1: Multiplies both sides by $x^2(x+2)^2$		
	$x^2(x-2)(x+2) \le 24x(x+2)$ $x^3(x+2) - 2x^2(x+2) \le 24x(x+2)$	Both sides $\times x^2(x+2)^2$ May multiply by more terms but must be a positive multiplier containing $x^2(x+2)^2$	M1
	$x^3(x+2) - 2x^2(x+2) - 24x(x+2)(\le 0)$	M1: Collects expressions to one side A1: Correct inequality	M1A1
	$x = 0, -2$	Correct critical values	B1
	$x^4 - 28x^2 - 48x(=0)$ $x(x+2)(x-6)(x+4)(=0) \Rightarrow x = ...$	Attempt to solve their quartic as far as x = ... to obtain the **other** critical values Can cancel x and solve a cubic or x and $(x+2)$ and solve a quadratic.	M1
	$x = -4, 6$	Correct critical values	A1
	$-4 \le x < -2,\ \ 0 < x \le 6$ with $\le$ or $<$ throughout	M1: Attempt two inequalities using their 4 critical values in ascending order. (dependent on at least one previous M mark) A1: All 4 CVs in the inequalities correct	dM1A1
	$-4 \le x < -2,\ \ 0 < x \le 6$ $[-4,-2) \cup (0,6]$	A1:Inequality signs correct Set notation may be used. $\cup$ or "or" but not "and"	A1cao **(9)**
			Total 9

	Alternative 2: using a sketch graph (probably from calculator)		
		Draw graphs of $y=\frac{x-2}{2(x+2)}$ and $y=\frac{12}{x(x+2)}$	
	CVs $x=0,-2$	(Vertical asymptotes of graphs.)	B1
	$\frac{x-2}{2(x+2)}=\frac{12}{x(x+2)}$	Eliminate y	M1
	$x(x-2)=24$	M1: Obtains a quadratic equation A1: Correct equation	M1A1
	$x^2-2x-24\Rightarrow(x+4)(x-6)=0\Rightarrow x=..$	Attempt to solve their quadratic as far as $x=...$	M1
	CVs $x=-4,6$	Correct critical values	A1
	$-4\le x<-2,\ \ 0<x\le 6$ with $\le$ or $<$ throughout	M1: Attempt two inequalities using their 4 critical values in ascending order. (dependent on at least one previous M mark)	dM1
	$-4\le x<-2,\ \ 0<x\le 6$	A1: All 4 CVs in the inequalities correct	A1
		A1: All inequality signs correct	A1cao **(9)**
NB	As above, but with no sketch graph shown: CVs $x=0,-2$ **must** be stated somewhere.		B1
	Otherwise no marks available.		

	Scheme	Notes	Marks
3.	$z^3 + 32 + 32\mathrm{i}\sqrt{3} = 0$		
	$\arg(z^3) = \frac{4\pi}{3}$ or $-\frac{2\pi}{3}$	M1: Uses tan to find arg z^3 $\arctan\sqrt{3}$, $\arctan\frac{1}{\sqrt{3}}$, $\frac{\pi}{3}$ or $\frac{\pi}{6}$ seen. Allow equivalent angles A1: Either of values shown	M1A1
	$\lvert z\rvert = r = 4$	Correct r seen anywhere (eg only in answers)	B1
	$3\theta = \frac{4\pi}{3},\ -\frac{2\pi}{3},\ -\frac{8\pi}{3}$		
	$\theta = \frac{4\pi}{9},\ -\frac{2\pi}{9},\ -\frac{8\pi}{9}$	Divides by 3 to obtain at least 2 values of θ which differ by $\frac{2\pi}{3}$ or $\frac{4\pi}{3}$.	M1
	$\theta = \frac{4\pi}{9},\ -\frac{2\pi}{9}$ or $\frac{16\pi}{9}$, $-\frac{8\pi}{9}$ or $\frac{10\pi}{9}$	At least 2 correct (and distinct) values from list shown	A1
	$z = 4\mathrm{e}^{\frac{4\pi}{9}\mathrm{i}},\ 4\mathrm{e}^{-\frac{2\pi}{9}\mathrm{i}},\ 4\mathrm{e}^{-\frac{8\pi}{9}\mathrm{i}}$ or $4\mathrm{e}^{\mathrm{i}\theta}$ where $\theta = ...$	A1: All correct and in either of the forms shown Ignore extra answers outside the range	A1 **(6)**
			Total 6

Question Number	Scheme	Notes	Marks
4.	$y=\ln\left(\frac{1}{1-2x}\right)$		
(a)	$y=\ln(1-2x)^{-1}=(\ln 1)-\ln(1-2x)$ $\frac{dy}{dx}=-\frac{1}{1-2x}\times -2\left(=\frac{2}{1-2x}\right)$	M1: $\frac{dy}{dx}=\frac{-1}{(1-2x)}\times\frac{d(1-2x)}{dx}$ Must use chain rule ie $\frac{k}{1-2x}$ with $k\neq\pm1$ needed. Minus sign may be missing. A1: Correct derivative	M1A1
OR	$\frac{dy}{dx}=(1-2x)\times-(1-2x)^{-2}\times-2$ $\left(=\frac{2}{1-2x}\right)$	M1: $\frac{dy}{dx}=\frac{1}{(1-2x)^{-1}}\times\frac{d(1-2x)^{-1}}{dx}$ Must use chain rule. Minus sign may be missing. A1: Correct derivative	M1A1
	$\frac{d^2y}{dx^2}=-2\times(1-2x)^{-2}\times-2$ $\left(=\frac{4}{(1-2x)^2}\right)$	Correct second derivative obtained from a correct first derivative.	A1
	$\frac{d^3y}{dx^3}=-8\times(1-2x)^{-3}\times-2$ $\left(=\frac{16}{(1-2x)^3}\right)$	Correct third derivative obtained from correct first and second derivatives	A1
			(4)
	Alternative by use of exponentials and implicit differentiation		
(a)	$y=\ln\left(\frac{1}{1-2x}\right)\Rightarrow e^y=\frac{1}{1-2x}=(1-2x)^{-1}$		
	$e^y\frac{dy}{dx}=2(1-2x)^{-2}$	Differentiates using implicit differentiation and chain rule.	M1
	$\frac{dy}{dx}=2e^{-y}(1-2x)^{-2}$ or $\frac{2}{(1-2x)}$	Correct derivative in either form. Equivalents accepted.	A1
	If $\frac{dy}{dx}=\frac{2}{(1-2x)}$ has been used from here, see main scheme for second and third derivatives		

(b)	$(y_0=0), y'_0=2, y''_0=4, y'''_0=16$	Attempt values at $x=0$ using their derivatives from (a) $y_0=0$ need not be seen but other 3 values must be attempted.	M1
	$(y=)(0)+2x+\frac{4x^2}{2!}+\frac{16x^3}{3!}$	Uses their values in the correct Maclaurin series. Must see x^3 term Can be implied by a final series which is correct for their values. 2!,3! or 2 and 6	M1
	$y=2x+2x^2+\frac{8}{3}x^3$	Correct expression. Must start $y=$ ... or $\ln\left(\frac{1}{1-2x}\right)=$... f($x$) = ... allowed **only** if f(x) is defined to be one of these.	A1cao
			(3)
	Alternative (b)		
	$y=\ln\left(\frac{1}{1-2x}\right)=-\ln(1-2x)$	Log power law applied correctly	M1
	$=-\left((-2x)-\frac{(-2x)^2}{2}+\frac{(-2x)^3}{3}\right)$	Replaces x with -2x in the expansion for $\ln(1+x)$ (in formula book)	M1
	$y=2x+2x^2+\frac{8}{3}x^3$	Correct expression	A1cao
(c)	$\frac{1}{1-2x}=\frac{3}{2}\Rightarrow x=\frac{1}{6}$	Correct value for x, seen explicitly or substituted in their expansion	B1
	$\ln\left(\frac{3}{2}\right)\approx 2\left(\frac{1}{6}\right)+2\left(\frac{1}{6}\right)^2+\frac{8}{3}\left(\frac{1}{6}\right)^3$	Substitute their value of x into their expansion. May need to check this is correct for their expansion and their x. (Calculator value for $\ln\left(\frac{3}{2}\right)$ is 0.405)	M1
	$=0.401$	Must come from correct work	A1cso
NB:	$\ln 3-\ln 2$ or $\ln 3+\ln\left(\frac{1}{2}\right)$ scores 0/3 as $\lvert x\rvert$ must be $<\frac{1}{2}$		
	Answer with no working scores 0/3		**(3)**
			Total 10

Question Number	Scheme	Notes	Marks
5.	$\frac{d^2y}{dx^2}-2\frac{dy}{dx}=26\sin 3x$		
(a)	$m^2-2m=0\Rightarrow m=0,\ 2$	Solves AE	M1
	$(\text{CF or } y=)\ A+Be^{2x}$ or Ae^0+Be^{2x} oe	Correct CF (CF or $y=$ not needed)	A1
	$(\text{PI or } y=)\ a\cos 3x+b\sin 3x$	Correct form for PI (PI or $y=$ not needed)	B1
	$\frac{dy}{dx}=-3a\sin 3x+3b\cos 3x,\ \frac{d^2y}{dx^2}=-9a\cos 3x-9b\sin 3x$		M1A1
	M1: Differentiates twice; change of trig functions needed, ± 1 or ± 3 for coeffs for first derivative, ± 1, ± 3 or ± 9 for second derivative (1/3 etc indicates integration) A1: Correct derivatives		
	$-9a\cos 3x-9b\sin 3x+6a\sin 3x-6b\cos 3x=26\sin 3x$		
	$\therefore -9a-6b=0,\ -9b+6a=26\Rightarrow a=...,b=...$	Substitutes and forms simultaneous equations (by equating coeffs) and attempts to solve for a and b Depends on the second M mark	dM1
	$a=\frac{4}{3},\ b=-2$	Correct a and b	A1
	$y=A+Be^{2x}+\frac{4}{3}\cos 3x-2\sin 3x$	Forms the GS (ft their CF and PI) Must start $y=$	A1ft **(8)**
(b)	$0=A+B+\frac{4}{3}$	Substitutes $x=0$ and $y=0$ into their GS	M1
	$\left(\frac{dy}{dx}\right)=2Be^{2x}-4\sin 3x-6\cos 3x\Rightarrow 0=2B-6$ Differentiates and substitutes $x=0$ and $y'=0$ (change of trig functions needed, ± 1 or ± 3 for coeffs)		M1
	$0=A+B+\frac{4}{3},\ 0=2B-6\Rightarrow A=..,B=..$	Solves simultaneously to obtain values for A and B Depends on the second M mark	dM1
	$A=\frac{-13}{3},\quad B=3$	Correct values	A1
	$y=3e^{2x}-\frac{13}{3}+\frac{4}{3}\cos 3x-2\sin 3x$	Follow through their GS and A and B Must start $y=$ ...	A1ft **(5)**
			Total 13
ALT for (a)	$\frac{d^2y}{dx^2}-2\frac{dy}{dx}=26\sin 3x\Rightarrow\frac{dy}{dx}-2y=-\frac{26}{3}\cos 3x+c$	M1: Integrates both sides wrt x A1: Correct expression	M1A1
	$I=e^{\int -2dx}=e^{-2x}$	Correct integrating factor	B1
	$ye^{-2x}=\int e^{-2x}\left(-\frac{26}{3}\cos 3x+c\right)dx$	M1: Uses $yI=\int I\left(-\frac{26}{3}\cos 3x+c\right)dx$ A1: Correct expression	M1A1
	$=\frac{4}{3}e^{-2x}\cos 3x-2e^{-2x}\sin 3x-\frac{1}{2}ce^{-2x}+B$	M1: Integration by parts twice A1: Correct expression	M1A1
	$y=-\frac{1}{2}c+Be^{2x}+\frac{4}{3}\cos 3x-2\sin 3x$	Must start $y=$ ...	

Question Number	Scheme	Notes	Marks
6.	$r=6+a\sin\theta$		
	$A=\frac{1}{2}\int(6+a\sin\theta)^2\,d\theta$	Use of $\frac{1}{2}\int r^2(d\theta)$ Limits not needed. Can be gained if $\frac{1}{2}$ appears later	B1
	$(6+a\sin\theta)^2=36+12a\sin\theta+a^2\sin^2\theta$		
	$(6+a\sin\theta)^2=36+12a\sin\theta+a^2\left(\frac{1-\cos2\theta}{2}\right)$	M1: Squares ($36+k\sin^2\theta$, where $k=a^2$ or a as min) and attempts to change : $\sin^2\theta$ to an expression in $\cos2\theta$ A1: Correct expression	M1A1
	$\left(\frac{1}{2}\right)\left[36\theta-12a\cos\theta+\frac{a^2}{2}\theta-\frac{a^2}{4}\sin2\theta\right]$	dM1: Attempt to integrate $\cos2\theta\to\pm\frac{1}{2}\sin2\theta$ Limits not needed A1: Correct integration limits not needed	dM1A1
	$=36\pi+\frac{\pi a^2}{2}$	Correct area obtained from correct integration and correct limits. No need to simplify but trig functions must be evaluated.	A1
	$36\pi+\frac{\pi a^2}{2}=\frac{97\pi}{2}\Rightarrow a=...$	Set their area = $\frac{97\pi}{2}$ and attempt to solve for a (depends on both M marks above) If $\frac{1}{2}$ omitted from the initial formula and area set = 97π, give the B1 by implication as well as this mark.	ddM1
	$a=5$	cao and cso $a=\pm5$ or $a=-5$ scores A0	A1cso
			Total 8
	Alternatives: Splitting the area and so using 2 integrals with different limits.		
	Marks the same as the main scheme.		
1	Limits 0 to π (area above initial line) and limits π to 2π (area below initial line) and add the two results.		
2	Limits 0 to $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ to 2π Twice the sum of the results needed.		

Question Number	Scheme	Notes	Marks
7.	$\cos x\frac{dy}{dx}+y\sin x=2\cos^3 x\sin x+1$		
(a)	$\frac{dy}{dx}+y\tan x=2\cos^2 x\sin x+\frac{1}{\cos x}$	Divides by cos x LHS both terms divided RHS min 1 term divided	M1
	$I=e^{\int\tan x\,dx}=e^{\ln\sec x}=\sec x$	M1: Attempt integrating factor $e^{\int\tan x\,dx}$ needed A1: Correct integrating factor, $\sec x$ or $\frac{1}{\cos x}$	dM1A1
	$y\sec x=\int\left(2\sin x\cos x+\sec^2 x\right)dx$	Multiply through by their IF and integrate LHS (integration may be done later) $yI=\int\left(\text{their RHS}\right)I\,dx$	M1
	$y\sec x=-\frac{1}{2}\cos 2x+\tan x(+c)$	M1: Attempt integration of at least one term on RHS (provided both sides have been multiplied by their IF.) OR $\sec^2 x\to K\tan x$ A1: $-\frac{1}{2}\cos 2x$ or equivalent integration of $2\sin x\cos x$ ($\sin^2 x$ or $-\cos^2 x$) A1: $\tan x$ constant not needed.	M1A1A1
	$y=\left(-\frac{1}{2}\cos 2x+\tan x+c\right)\cos x$ $y=\left(-\cos^2 x+\tan x+c\right)\cos x$ $y=\left(\sin^2 x+\tan x+c\right)\cos x$	Include the constant and deal with it correctly. Must start $y=$ ... Or equivalent eg $y=-\frac{1}{2}\cos 2x\cos x+\sin x+c\cos x$ Follow through from the line above	A1ft
			(8)
(b)	$x=\frac{\pi}{4}\Rightarrow 5\sqrt{2}=.....\Rightarrow c=.....$	Substitutes for x and y and solves for c (If substitution not shown award for at least one term evaluated correctly.)	M1
	$x=\frac{\pi}{6}\Rightarrow y=.........$	Substitutes $x=\frac{\pi}{6}$ to find a value for y	M1
	$y=\frac{1}{2}+\frac{35}{8}\sqrt{3}$ or $y=0.5+4.375\sqrt{3}$	Must be in given form. Equivalent fractions allowed. ...	A1cao
			(3)
NB	(b) There may be no working shown due to use of calculator. In such cases: Final answer correct (and in required form with no decimals instead of $\sqrt{3}$ seen), score 3/3. Final answer incorrect (or decimals instead of $\sqrt{3}$ seen), score 0/3. This applies whether (a) is correct or not.		
			Total 11

Question Number	Scheme	Notes	Marks
8.	$w=\dfrac{z+3\mathrm{i}}{1+\mathrm{i}z}$		
(a)	$z=\dfrac{w-3\mathrm{i}}{1-\mathrm{i}w}$ oe	M1: Attempt to make z the subject A1: Correct equation	M1A1
	$\lvert z\rvert=1\Rightarrow\left\lvert\dfrac{w-3\mathrm{i}}{1-\mathrm{i}w}\right\rvert=1\Rightarrow\lvert w-3\mathrm{i}\rvert=\lvert 1-w\mathrm{i}\rvert$ $\therefore\ \lvert u+\mathrm{i}v-3\mathrm{i}\rvert=\lvert(u+\mathrm{i}v)\mathrm{i}-1\rvert$	Uses $\lvert z\rvert=1$ and introduce "$u+\mathrm{i}v$" (or $x+\mathrm{i}y$) for w	M1
	$u^2+(v-3)^2=u^2+(v+1)^2$	Correct use of Pythagoras on either side.	M1
	$v=1$ oe	$v=1$ or $y=1$	A1
			(5)
	Alternative 1 for (a)		
	eg $w(1)=\dfrac{1+3\mathrm{i}}{1+\mathrm{i}}=2+\mathrm{i}$	M1: Maps one point on the circle using the given transformation A1:Correct mapping	M1A1
	eg $w(-\mathrm{i})=\dfrac{2\mathrm{i}}{2}=\mathrm{i}$	Maps a second point on the circle	M1
	$v=1$ oe	M1: Forms Cartesian equation using their 2 points A1: $v=1$ or $y=1$	M1A1
	Alternative 2 for (a)		
	$z=\dfrac{w-3\mathrm{i}}{1-\mathrm{i}w}$ oe	M1: Attempt to make z the subject A1: Correct equation	M1A1
	$\lvert z\rvert=1\Rightarrow\left\lvert\dfrac{w-3\mathrm{i}}{1-\mathrm{i}w}\right\rvert=1\Rightarrow\lvert w-3\mathrm{i}\rvert=\lvert 1-w\mathrm{i}\rvert$ $\lvert w-3\mathrm{i}\rvert=\lvert w+\mathrm{i}\rvert=\lvert w-(-\mathrm{i})\rvert$	Uses $\lvert z\rvert=1$ and changes to form $\lvert w-...\rvert=\lvert w-...\rvert$ or draws a diagram	M1
	Perpendicular bisector of points $(0,3)$ and $(0,-1)$	Uses a correct geometrical approach	M1
	$v=1$ oe	$v=1$ or $y=1$	A1

	Alternative 3 for (a)		
	Let $z = x + \mathrm{i}y$, $\lvert z\rvert = 1 \Rightarrow x^2 + y^2 = 1$		
	$w = \dfrac{z+3\mathrm{i}}{1+\mathrm{i}z} = \dfrac{x+\mathrm{i}y+3\mathrm{i}}{1+\mathrm{i}(x+\mathrm{i}y)} = \dfrac{x+\mathrm{i}(y+3)}{(1-y)+\mathrm{i}x}$		
	$w = \dfrac{x+\mathrm{i}(y+3)}{(1-y)+\mathrm{i}x} \times \dfrac{(1-y)-\mathrm{i}x}{(1-y)-\mathrm{i}x}$	Substitute $z = x + \mathrm{i}y$ and multiply numerator and denominator by complex conjugate of their denominator	M1
	$w = \dfrac{x(1-y)-\mathrm{i}x^2+\mathrm{i}(y+3)(1-y)-\mathrm{i}^2x(y+3)}{(1-y)^2-\mathrm{i}x(1-y)+\mathrm{i}x(1-y)-\mathrm{i}^2x^2}$		
	$w = \dfrac{[x(1-y)+x(y+3)]+\mathrm{i}[-x^2+(y+3)(1-y)]}{(1-y)^2+x^2}$	M1: Multiply out and collect real and imaginary parts in numerator. Denominator must be real. A1: all correct	M1 A1
	$w = \dfrac{[x-xy+xy+3x]+\mathrm{i}[-x^2+y-y^2+3-3y]}{1-2y+y^2+x^2}$		
	$w = \dfrac{[4x]+\mathrm{i}[-1+3-2y]}{2-2y}$	Applies $x^2 + y^2 = 1$	M1
	$w = \dfrac{4x+\mathrm{i}[2-2y]}{2-2y} = \dfrac{4x}{2-2y} + \mathrm{i}$		
	$y = 1$	$y = 1$ or $v = 1$	A1
(b)	$\lvert w\rvert = 5 \Rightarrow \left\lvert \dfrac{z+3\mathrm{i}}{1+\mathrm{i}z} \right\rvert = 5 \Rightarrow \lvert z+3\mathrm{i}\rvert = 5\lvert 1+\mathrm{i}z\rvert$ $\therefore\ \lvert x+\mathrm{i}y+3\mathrm{i}\rvert = 5\lvert (x+\mathrm{i}y)\mathrm{i}+1\rvert$	Uses $\lvert w\rvert = 5$ and introduce "$x + \mathrm{i}y$"	M1
	$x^2 + (y+3)^2 = 25\left(x^2 + (1-y)^2\right)$	M1: Correct use of Pythagoras Allow 25 or 5 A1: Correct equation	M1A1
	$x^2 + y^2 - \dfrac{7}{3}y + \dfrac{2}{3} = 0$		
	$x^2 + \left(y - \dfrac{7}{6}\right)^2 = \dfrac{25}{36}$	Attempt circle form or attempt r^2 from the line above.	M1
	$a = 0,\ b = \dfrac{7}{6},\ c = \dfrac{5}{6}$	A1: 2 correct A1: All correct	A1, A1
			(6)
			Total 11
	Or, for the last 3 marks:		
	$\left\lvert z - 0 - \dfrac{7}{6}\mathrm{i} \right\rvert = \dfrac{5}{6}$		M1A1A1
	If 0 not shown score M1A1A0		
	No need to list a, b, c separately if answer in this form.		

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