S15 M1

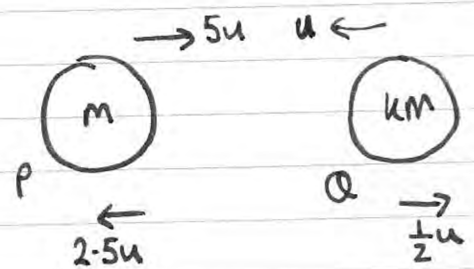
1. Particle P of mass m and particle Q of mass km are moving in opposite directions on a smooth horizontal plane when they collide directly. Immediately before the collision the speed of P is $5u$ and the speed of Q is u. Immediately after the collision the speed of each particle is halved and the direction of motion of each particle is reversed.

Find

(a) the value of k, **(3)**

(b) the magnitude of the impulse exerted on P by Q in the collision. **(3)**

a)



Total Mom before

$5mu - kmu$

Total mom after

$-2.5mu + \frac{1}{2}kmu$

CLM ⇒ $5mu - kmu = -2.5mu + \frac{1}{2}kmu$

$7.5\cancel{mu} = 1.5k\cancel{mu}$ $\quad \therefore k = \frac{7.5}{1.5} = 5$

b) Mom p before $= 5mu$ $\quad \therefore$ Impulse = change in Mom

Mom P after $= -2.5mu$

$= 7.5mu$

2. A small stone is projected vertically upwards from a point O with a speed of 19.

Modelling the stone as a particle moving freely under gravity,

(a) find the greatest height above O reached by the stone,

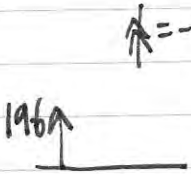
(2)

(b) find the length of time for which the stone is more than 14.7 m above O.

(5)

a)

$0\uparrow$ • $v=0$ at gh.



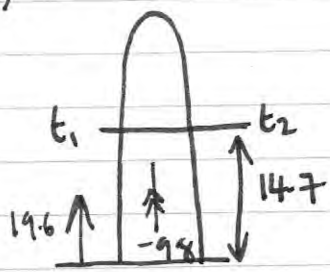
s
$u = 19.6$
$v = 0$
$a = -9.8$
t

$v^2 = u^2 + 2as$

$0 = 19.6^2 - 19.6s$

$s = \frac{19.6^2}{19.6} = 19.6\text{m}$

b)



total time above 14.7 $= t_2 - t_1$

$s = 14.7$
$u = 19.6$
v
$a = -9.8$
t

$s = ut + \frac{1}{2}at^2$

$14.7 = 19.6t - 4.9t^2$

$4.9t^2 - 19.6t + 14.7 = 0$

$\div 4.9$ $\quad t^2 - 4t + 3 = 0$

$(t-3)(t-1) = 0$

$t_1 = 1 \quad t_2 = 3$

$\therefore$ total time above

= 2 seconds

3.

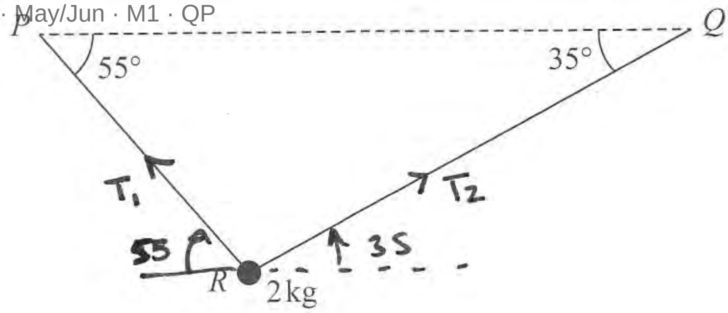


Figure 1

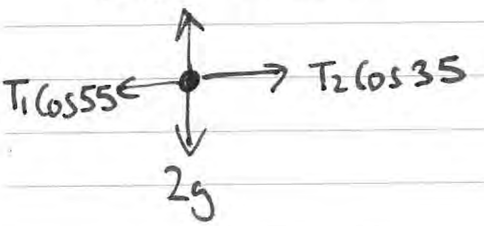
A particle of mass 2 kg is suspended from a horizontal ceiling by two light inextensible strings, PR and QR. The particle hangs at R in equilibrium, with the strings in a vertical plane. The string PR is inclined at 55° to the horizontal and the string QR is inclined at 35° to the horizontal, as shown in Figure 1.

Find

(i) the tension in the string PR,

(ii) the tension in the string QR.

(7)



$\overrightarrow{Rf} = 0$

$\therefore \; T_2 \cos 35 = T_1 \cos 55$

$T_2 = \dfrac{\cos 55}{\cos 35} T_1$

$R f \uparrow = 0 \quad \therefore \; T_1 \sin 55 + T_2 \sin 35 = 19.6$

$T_1 \sin 55 + T_1 \dfrac{\cos 55}{\cos 35} \times \sin 35 = 19.6$

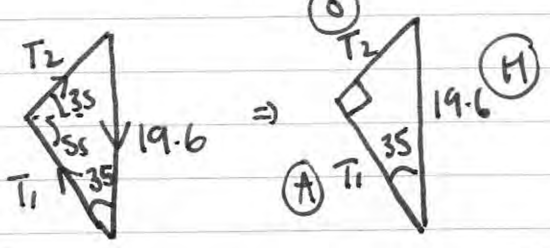
$T_1 (\sin 55 + \cos 55 \tan 35) = 19.6$

$\div$

$T_1 = 16.055\ldots$ $\qquad T_1 = 16.1\,\text{N}$

b) $T_2 = \dfrac{\cos 55}{\cos 35} \times T_1 \quad \therefore \; T_2 = 11.24\ldots \qquad T_2 = 11.2\,\text{N}$

alt



$\therefore \; T_1 = 19.6 \cos 35 = 16.1$

$T_2 = 19.6 \sin 35 = 11.2$

4.

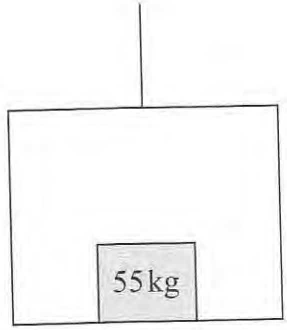


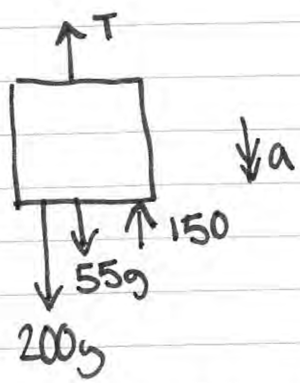
Figure 2

A lift of mass 200 kg is being lowered into a mineshaft by a vertical cable attached to the top of the lift. A crate of mass 55 kg is on the floor inside the lift, as shown in Figure 2. The lift descends vertically with constant acceleration. There is a constant upwards resistance of magnitude 150 N on the lift. The crate experiences a constant normal reaction of magnitude 473 N from the floor of the lift.

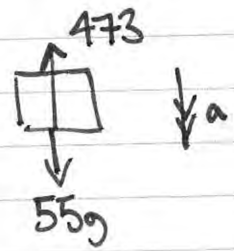
(a) Find the acceleration of the lift. **(3)**

(b) Find the magnitude of the force exerted on the lift by the cable. **(4)**

a) Lift



Crate



from the crate $55g - 473 = 55a \quad \therefore \; a = 1.2 \text{ ms}^{-2}$

b) from the lift $200g + 55g - 150 - T = 255a$

$2349 - T = 306 \quad \therefore \; T = 2043\text{N}$

5.

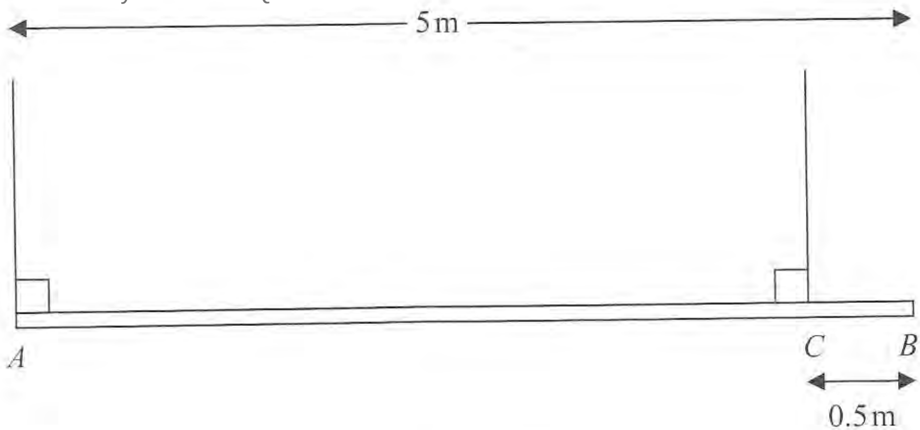


Figure 3

A beam AB has length 5 m and mass 25 kg. The beam is suspended in equilibrium in a horizontal position by two vertical ropes. One rope is attached to the beam at A and the other rope is attached to the point C on the beam where $CB = 0.5$ m, as shown in Figure 3. A particle P of mass 60 kg is attached to the beam at B and the beam remains in equilibrium in a horizontal position. The beam is modelled as a uniform rod and the ropes are modelled as light strings.

(a) Find

(i) the tension in the rope attached to the beam at A,

(ii) the tension in the rope attached to the beam at C.

(6)

Particle P is removed and replaced by a particle Q of mass M kg at B. Given that the beam remains in equilibrium in a horizontal position,

(b) find

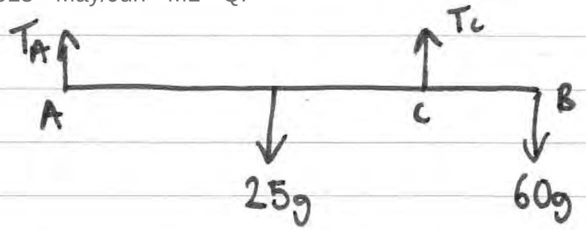
(i) the greatest possible value of M,

(ii) the greatest possible tension in the rope attached to the beam at C.

(6)

GradeMax

a)



$A\circlearrowright$ $25g \times 2.5 + 60g \times 5 = T_C \times 4.5$

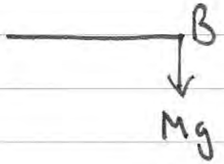
$\frac{725}{\cancel{2}}g = \frac{9}{\cancel{2}}T_C$ $\quad \therefore T_C = 80.\dot{5}g$

$R(\uparrow) = 0$ $\quad T_A + T_C = 25g + 60g$ $\quad \therefore T_A = 85g - 80.\dot{5}g$

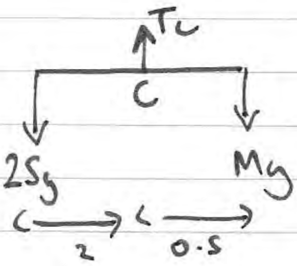
$= 4.4g$

$\therefore T_A = 43.6N$ $\qquad T_C = 789.4N$

b)



greatest value of M would result in $T_A = 0$



$C\circlearrowright$ $25g \times 2 = Mg \times \frac{1}{2}$

$100\cancel{g} = M\cancel{g}$

$\therefore$ Max $M = 100$ kg

$R(\uparrow) = 0 \Rightarrow T_C = 125g = 1225N$

6. A particle P is moving with constant velocity. The position vector of P at time t ($t \geqslant 0$) is $\mathbf{r}$ metres, relative to a fixed origin O, and is given by

$$\mathbf{r} = (2t - 3)\mathbf{i} + (4 - 5t)\mathbf{j}$$

(a) Find the initial position vector of P.

(1)

The particle P passes through the point with position vector $(3.4\mathbf{i} - 12\mathbf{j})$ m at time T seconds.

(b) Find the value of T.

(3)

(c) Find the speed of P.

(4)

a) $r = \begin{pmatrix} -3+2t \\ 4-5t \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix} + t\begin{pmatrix} 2 \\ -5 \end{pmatrix}$ $\therefore$ original pos vector $= \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

b) $\begin{pmatrix} -3+2t \\ 4-5t \end{pmatrix} = \begin{pmatrix} 3.4 \\ -12 \end{pmatrix}$ $\therefore$ $2t = 6.4$ $\therefore$ $T = 3.2\text{sec}$

2

c) $\text{Vel} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$ $\therefore$ speed $= \sqrt{2^2+5^2} = \sqrt{29}$

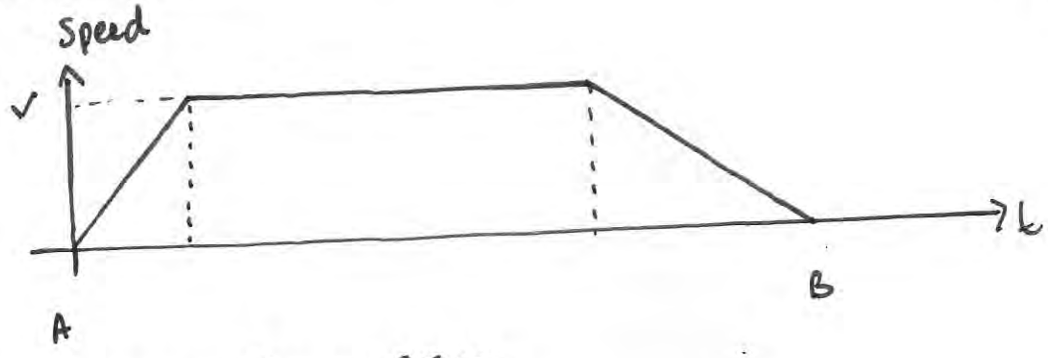
$= 5.39 \text{ ms}^{-1}$

2

7. A train travels along a straight horizontal track between two stations, A and B. The train starts from rest at A and moves with constant acceleration 0.5 m s^{-2} until it reaches a speed of $V \text{ m s}^{-1}$, ($V < 50$). The train then travels at this constant speed before it moves with constant deceleration 0.25 m s^{-2} until it comes to rest at B.

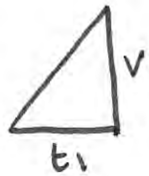
(a) Sketch in the space below a speed-time graph for the motion of the train between the two stations A and B. **(2)**

a)



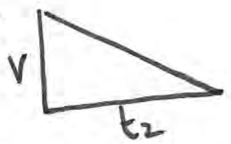
b) total time = 5min = 300sec

i)



acc = gradient = $\frac{1}{2}$ $\quad \frac{V}{t_1} = \frac{1}{2}$ $\quad \therefore t_1 = 2V$

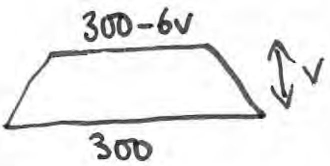
ii)



$\frac{V}{t_2} = \frac{1}{4}$ $\quad \therefore t_2 = 4V$

iii) $300 - 2V - 4V = 300 - 6V$

c)



$\frac{(300-6V+300)}{2} \times V = 6300$

The total time for the journey from A to B is 5 minutes.

(b) Find, in terms of V, the length of time, in seconds, for which the train is

(i) accelerating,

(ii) decelerating,

(iii) moving with constant speed. **(5)**

Given that the distance between the two stations A and B is 6.3 km,

(c) find the value of V. **(6)**

$$\therefore \left(\frac{600-6v}{2}\right)v = 6300$$

$$300v - 3v^2 = 6300 \quad \Rightarrow \quad 3v^2 - 300v + 6300 = 0$$

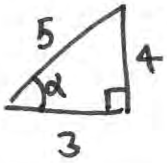
$$\div 3 \quad v^2 - 100v + 210 = 0$$

$$(v-30)(v-70) = 0$$

$$\therefore \underline{\underline{v = 30}}$$

8.

$\tan\alpha = \frac{4}{3}$ $\alpha = \tan^{-1}\left(\frac{4}{3}\right) \approx 53.1$



$\sin\alpha = 0.8$
$\cos\alpha = 0.6$

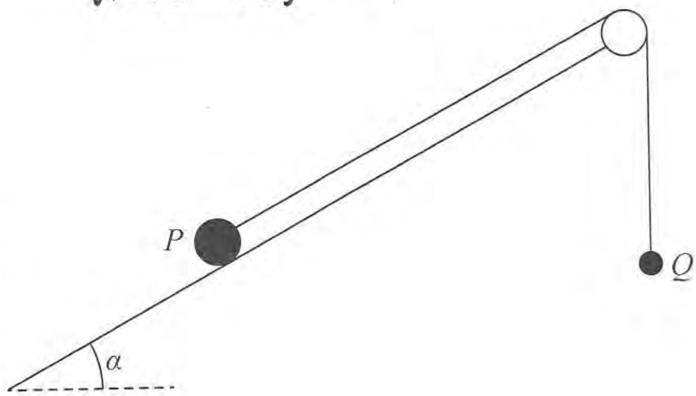


Figure 4

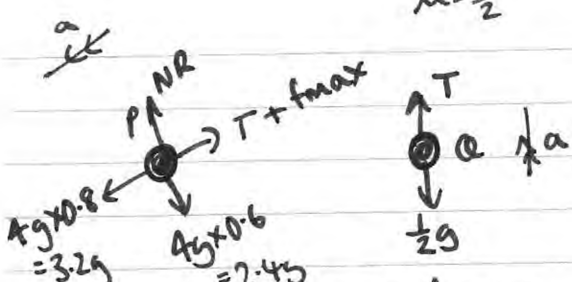
Two particles P and Q have mass 4 kg and 0.5 kg respectively. The particles are attached to the ends of a light inextensible string. Particle P is held at rest on a fixed rough plane, which is inclined to the horizontal at an angle α where $\tan\alpha = \frac{4}{3}$. The coefficient of friction between P and the plane is 0.5. The string lies along the plane and passes over a small smooth light pulley which is fixed at the top of the plane. Particle Q hangs freely at rest vertically below the pulley. The string lies in the vertical plane which contains the pulley and a line of greatest slope of the inclined plane, as shown in Figure 4. Particle P is released from rest with the string taut and slides down the plane.

Given that Q has not hit the pulley, find

(a) the tension in the string during the motion, **(11)**

(b) the magnitude of the resultant force exerted by the string on the pulley. **(4)**

$\mu = \frac{1}{2}$



$R(\uparrow) = ma$

$T - \frac{1}{2}g = \frac{1}{2}a$

$f_{max} = \mu NR = \frac{1}{2} \times 2.4g$
$= 1.2g$

$R(\swarrow) = ma$ $\quad 3.2g - T - f_{max} = 4a$

$3.2g - T - 1.2g = 4a$

$2g - T = 4a$
$T - \frac{1}{2}g = \frac{1}{2}a$ $+$

$\frac{3}{2}g = \frac{9}{2}a \quad \therefore a = \frac{1}{3}g$

$\therefore T = \frac{1}{2}a + \frac{1}{2}g = \frac{1}{6}g + \frac{1}{2}g$

$\therefore T = \frac{4}{6}g \quad \therefore T = \frac{2}{3}g = 6.53\,\text{N}$

b) $2T\cos\left(\frac{90 - \alpha}{2}\right)$ $\quad$ 101

$= 2\left(\frac{2}{3}g\right)\cos\left(\frac{90 - 53.1}{2}\right)$

$= 12.4\,\text{N}$