

June 2016 M1 UK Solutions Kprime2

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1. [In this question \mathbf{i} and \mathbf{j} are horizontal unit vectors due east and due north respectively and position vectors are given relative to a fixed origin O .]

Two cars P and Q are moving on straight horizontal roads with constant velocities. The velocity of P is $(15\mathbf{i} + 20\mathbf{j}) \text{ m s}^{-1}$ and the velocity of Q is $(20\mathbf{i} - 5\mathbf{j}) \text{ m s}^{-1}$

(a) Find the direction of motion of Q , giving your answer as a bearing to the nearest degree. (3)

At time $t = 0$, the position vector of P is $400\mathbf{i}$ metres and the position vector of Q is $800\mathbf{j}$ metres. At time t seconds, the position vectors of P and Q are \mathbf{p} metres and \mathbf{q} metres respectively.

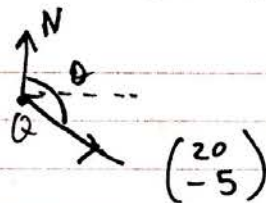
(b) Find an expression for

(i) \mathbf{p} in terms of t ,

(ii) \mathbf{q} in terms of t . (3)

(c) Find the position vector of Q when Q is due west of P . (4)

1.(a)



$$\theta = 90^\circ + \arctan\left(\frac{5}{20}\right) = 104.03\dots$$

$$\therefore \theta = 104^\circ \text{ (nearest degree)}$$

(b) Consider \mathbf{p} :

$$\underline{r}_0 = 400\mathbf{i} \quad \underline{v}_P = \begin{pmatrix} 15 \\ 20 \end{pmatrix}$$

$$\text{@ time } t, \quad \underline{r} = \begin{pmatrix} 400 \\ 0 \end{pmatrix} + t \begin{pmatrix} 15 \\ 20 \end{pmatrix}$$

$$\Rightarrow \underline{p} = \begin{pmatrix} 400 + 15t \\ 20t \end{pmatrix}$$



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Question 1 continued

$$(i) \underline{p} = (400 + 15t) \underline{i} + 20t \underline{j}$$

(ii) Consider θ :

$$\underline{r}_0 = \begin{pmatrix} 0 \\ 800 \end{pmatrix} \quad \underline{v}_a = \begin{pmatrix} 20 \\ -5 \end{pmatrix}$$

$$\Rightarrow \underline{r} = \begin{pmatrix} 0 \\ 800 \end{pmatrix} + t \begin{pmatrix} 20 \\ -5 \end{pmatrix}$$

$$\therefore \underline{q} = \begin{pmatrix} 20t \\ 800 - 5t \end{pmatrix} \Rightarrow \underline{q} = 20t \underline{i} + (800 - 5t) \underline{j}$$

$$(c) \begin{pmatrix} 20t \\ 800 - 5t \end{pmatrix} \cdot \begin{pmatrix} 400 + 15t \\ 20t \end{pmatrix}$$

$$\Rightarrow 800 - 5t = 20t$$

$$\therefore 25t = 800$$

$$\therefore t = 32$$

$$t = 32 \Rightarrow \underline{q} = 640 \underline{i} + 640 \underline{j}$$

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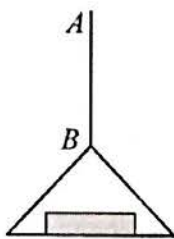


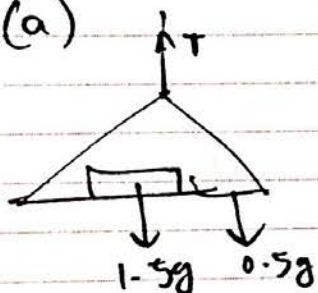
Figure 1

A vertical rope AB has its end B attached to the top of a scale pan. The scale pan has mass 0.5 kg and carries a brick of mass 1.5 kg , as shown in Figure 1. The scale pan is raised vertically upwards with constant acceleration 0.5 m s^{-2} using the rope AB . The rope is modelled as a light inextensible string.

(a) Find the tension in the rope AB . (3)

(b) Find the magnitude of the force exerted on the scale pan by the brick. (3)

2(a)



$\uparrow 0.5 \text{ m s}^{-2}$

\therefore



$\uparrow 0.5 \text{ m s}^{-2}$

$\uparrow: F = ma$

$\Rightarrow T - 2g = 2 \times 0.5$

$\Rightarrow T = 1 + 2g \text{ N}$

$T = 20.6 \text{ N}$
(2sf)

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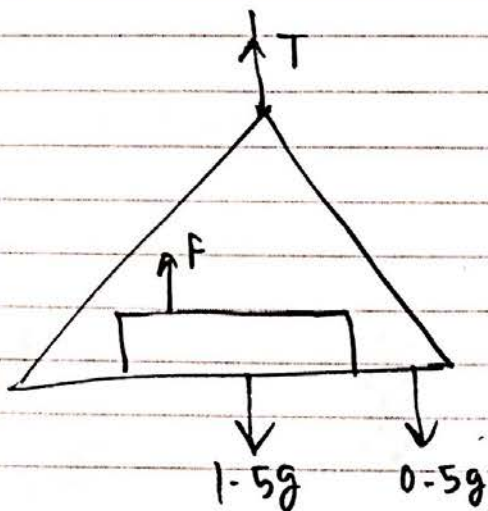
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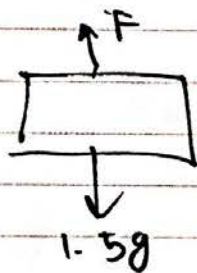
Question 2 continued

(b)



F is the contact force.

Consider just the block:



$a = 0.5 \text{ m s}^{-2}$

$\uparrow: F - 1.5g = 1.5 \times 0.5$

$\therefore F = \frac{3}{4} + 1.5g$

$\Rightarrow F = 15.5 \text{ N (3sf)}$

(Total 6 marks)

Q2

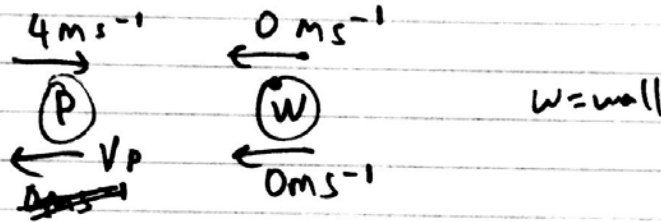
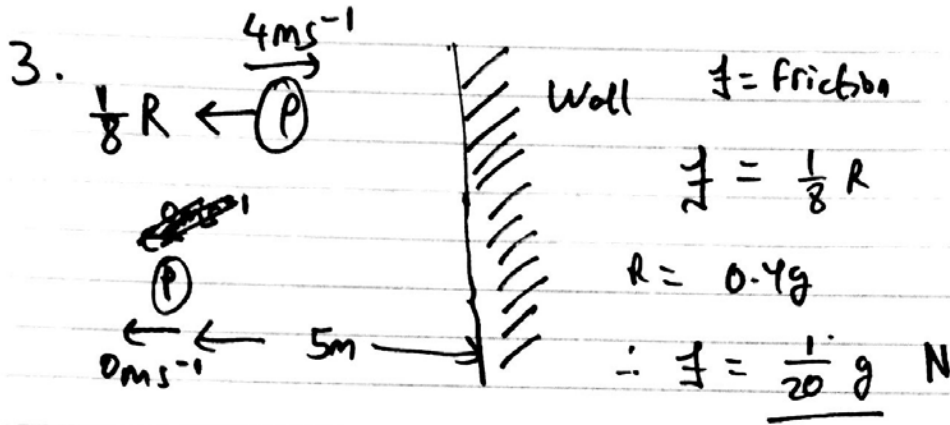


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3. A particle P of mass 0.4 kg is moving on rough horizontal ground when it hits a fixed vertical plane wall. Immediately before hitting the wall, P is moving with speed 4 m s^{-1} in a direction perpendicular to the wall. The particle rebounds from the wall and comes to rest at a distance of 5 m from the wall. The coefficient of friction between P and the ground is $\frac{1}{8}$.

Find the magnitude of the impulse exerted on P by the wall.

(7)



We need v_p to calculate Impulse.

$s = 5$
 $u = v_p$
 $v = 0$
 $a = -\frac{g}{8}$
 $t =$

$\leftarrow F = ma$
 $\Rightarrow -f = 0.4a$
 $\Rightarrow -\frac{g}{20} = 0.4a$



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Question 3 continued

$$\therefore a = -\frac{1}{8}g \text{ ms}^{-2}$$

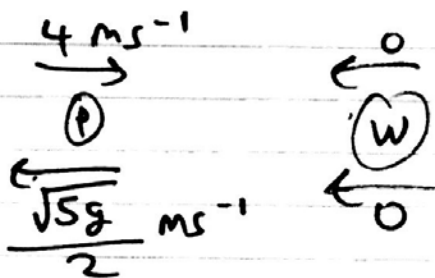
$$\therefore v^2 = u^2 + 2as$$

$$\Rightarrow 0 = (v_p)^2 - 2\left(\frac{g}{8}\right)(5)$$

$$\therefore 0 = (v_p)^2 - \frac{5}{4}g$$

$$(v_p)^2 = \frac{5}{4}g$$

$$\therefore v_p = \frac{\sqrt{5g}}{2} \text{ ms}^{-1}$$



$$+ \leftarrow I = m(v - u)$$

$$\therefore I = 0.4 \left(\frac{\sqrt{5g}}{2} + 4 \right)$$

$$\Rightarrow \underline{\underline{I = 3 \text{ N s}}}$$

(Total 7 marks)

Q3



4. Two trains M and N are moving in the same direction along parallel straight horizontal tracks. At time $t = 0$, M overtakes N whilst they are travelling with speeds 40 m s^{-1} and 30 m s^{-1} respectively. Train M overtakes train N as they pass a point X at the side of the tracks.

After overtaking N , train M maintains its speed of 40 m s^{-1} for T seconds and then decelerates uniformly, coming to rest next to a point Y at the side of the tracks.

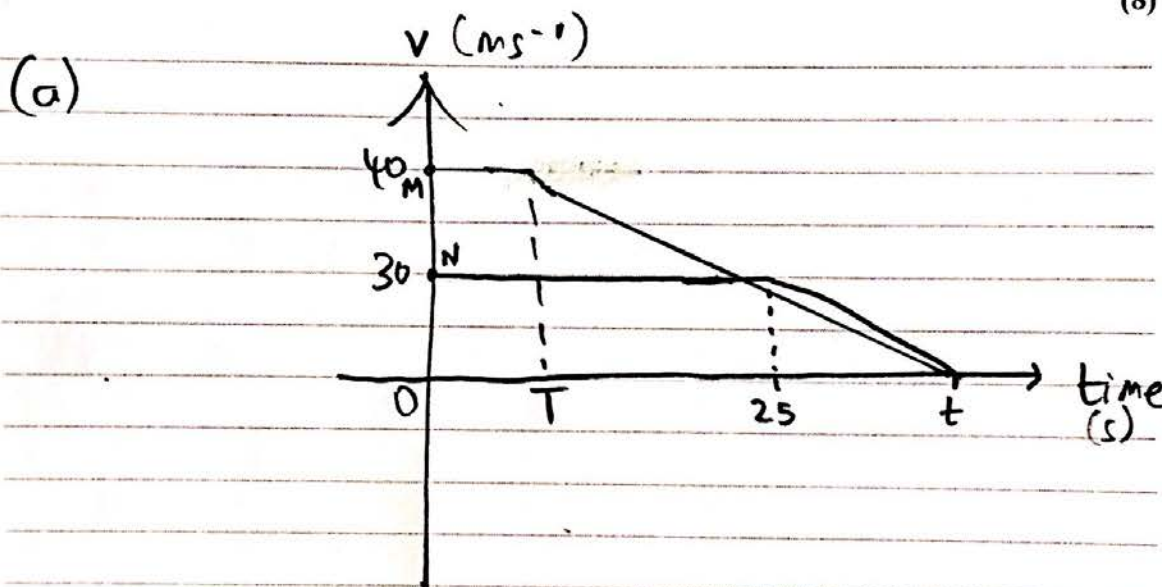
After being overtaken, train N maintains its speed of 30 m s^{-1} for 25 s and then decelerates uniformly, also coming to rest next to the point Y .

The times taken by the trains to travel between X and Y are the same.

(a) Sketch, on the same diagram, the speed-time graphs for the motions of the two trains between X and Y . (4)

Given that $XY = 975 \text{ m}$,

(b) find the value of T . (8)



(b) $XY = 975 \text{ m}$ let $t = \Sigma \text{time}$

Consider Area train N:

$$\frac{30}{2} (t + 25) = 15(t + 25)$$

$$\therefore 15(t + 25) = 975 \Rightarrow t = 40$$

~~$t = 40$~~

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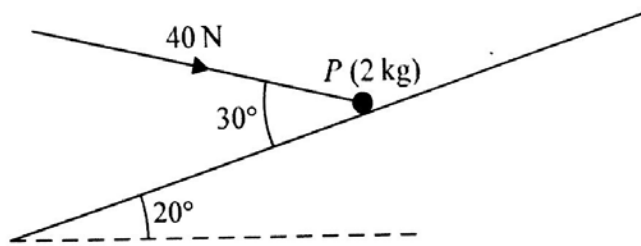
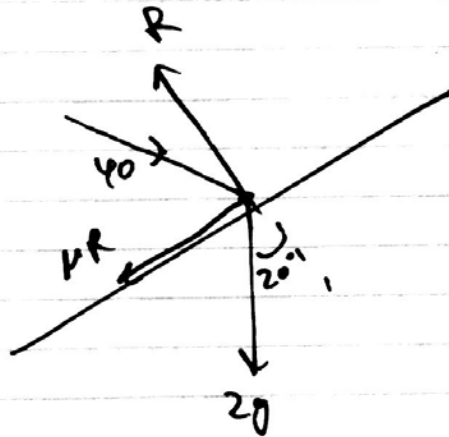


Figure 2

A particle P of mass 2 kg is held at rest in equilibrium on a rough plane by a constant force of magnitude 40 N. The direction of the force is inclined to the plane at an angle of 30° . The plane is inclined to the horizontal at an angle of 20° , as shown in Figure 2. The line of action of the force lies in the vertical plane containing P and a line of greatest slope of the plane. The coefficient of friction between P and the plane is μ .

Given that P is on the point of sliding up the plane, find the value of μ .

(10)



$$\rightarrow: 40 \cos 30 = \mu R + 2g \sin 20$$

$$\uparrow: R = 2g \cos 20 + 40 \sin 30 = 38.417 \dots$$

$$\mu = \frac{40 \cos 30 - 2g \sin 20}{38.417 \dots} = 0.7271 \dots$$

$$\Rightarrow \mu = \underline{\underline{0.727}} \text{ (3sf)}$$



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6. A non-uniform plank AB has length 6 m and mass 30 kg. The plank rests in equilibrium in a horizontal position on supports at the points S and T of the plank where $AS = 0.5$ m and $TB = 2$ m.

When a block of mass M kg is placed on the plank at A , the plank remains horizontal and in equilibrium and the plank is on the point of tilting about S .

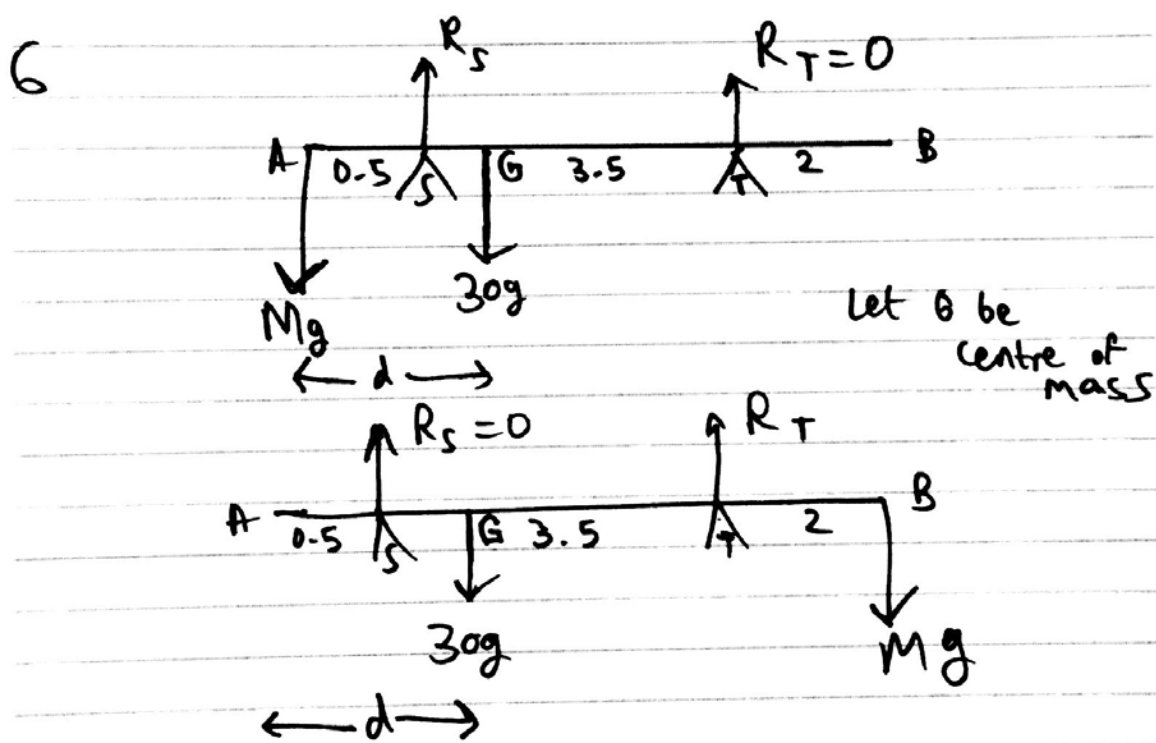
When the block is moved to B , the plank remains horizontal and in equilibrium and the plank is on the point of tilting about T .

The distance of the centre of mass of the plank from A is d metres. The block is modelled as a particle and the plank is modelled as a non-uniform rod. Find

- (i) the value of d ,

- (ii) the value of M .

(7)



Consider when block is placed at A:

$$M(A): 30g(d) = 0.5 R_S \quad (1)$$



Question 6 continued

Consider when block is placed at B:

$$M(B): 30g(6-d) = 2R_T$$

$$\Rightarrow 180g - 30gd = 2R_T \quad (2)$$

Sub m ① in ②:

$$180g - \frac{1}{2}R_S = 2R_T$$

$$\text{Also: } (M+30)g = R_S = R_T$$

$$\therefore 180g - \frac{1}{2}(M+30)g = 2(M+30)g$$

$$\therefore 180g - \frac{g}{2}M - 15g = 2gM + 60g$$

$$\therefore \cancel{120g} = 105g = \left(2g + \frac{g}{2}\right)M$$

$$\therefore 105 = \left(2 + \frac{1}{2}\right)M = \frac{5}{2}M$$

$$\Rightarrow M = 42 \text{ kg}$$

$$M=42 \Rightarrow R_S = 72g$$



Sub in ①

$$30g(d) = 0.5 \times 72g$$

$$30g d = 36g$$

$$30d = 36 \Rightarrow d = 1.2$$

$$\therefore (i) \quad d = \underline{\underline{1.2 \text{ m}}}$$

$$(ii) \quad M = \underline{\underline{42 \text{ kg}}}$$



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7. Two forces F_1 and F_2 act on a particle P .

The force F_1 is given by $F_1 = (-i + 2j)$ N and F_2 acts in the direction of the vector $(i + j)$.

Given that the resultant of F_1 and F_2 acts in the direction of the vector $(i + 3j)$,

(a) find F_2 (7)

The acceleration of P is $(3i + 9j) \text{ m s}^{-2}$. At time $t = 0$, the velocity of P is $(3i - 22j) \text{ m s}^{-1}$

(b) Find the speed of P when $t = 3$ seconds. (4)

$$7. (a) \quad \vec{F}_1 = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad \vec{F}_2 = k \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\text{Let } \vec{R} = \text{resultant}, \quad \vec{R} = \lambda \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$\vec{R} = \vec{F}_1 + \vec{F}_2 \Rightarrow \vec{R} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} + k \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\Rightarrow \vec{F}_2 = \vec{R} - \vec{F}_1$$

$$\therefore \vec{R} = \begin{pmatrix} k-1 \\ 2+k \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$\Rightarrow \begin{aligned} k-1 &= \lambda & \Rightarrow k &= \lambda+1 \\ \& \quad k+2 &= 3\lambda \end{aligned}$$

$$\Rightarrow \lambda+1+2=3\lambda$$

$$\therefore 2\lambda=3 \Rightarrow \lambda = \frac{3}{2}$$

$$\therefore k = \frac{3}{2} + 1 = \frac{5}{2}$$

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Question 7 continued

$$\therefore \underline{\underline{F_2}} = \frac{5}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\therefore \underline{\underline{F_2}} = \frac{5}{2} \underline{\underline{i}} + \frac{5}{2} \underline{\underline{j}}$$

$$(b) \quad \underline{\underline{a}} = \begin{pmatrix} 3 \\ 9 \end{pmatrix} \quad \underline{\underline{u}} = \begin{pmatrix} 3 \\ -22 \end{pmatrix} \quad t = 3$$

$$\underline{\underline{v}} = \underline{\underline{u}} + t\underline{\underline{a}}$$

$$\therefore \underline{\underline{v}} = \begin{pmatrix} 3 \\ -22 \end{pmatrix} + 3 \begin{pmatrix} 3 \\ 9 \end{pmatrix}$$

$$\therefore \underline{\underline{v}} = \begin{pmatrix} 12 \\ 5 \end{pmatrix}$$

$$\therefore |\underline{\underline{v}}| = \sqrt{12^2 + 5^2} = 13 \text{ m s}^{-1}$$

$$\underline{\underline{\text{Speed}}} = 13 \text{ m s}^{-1}$$



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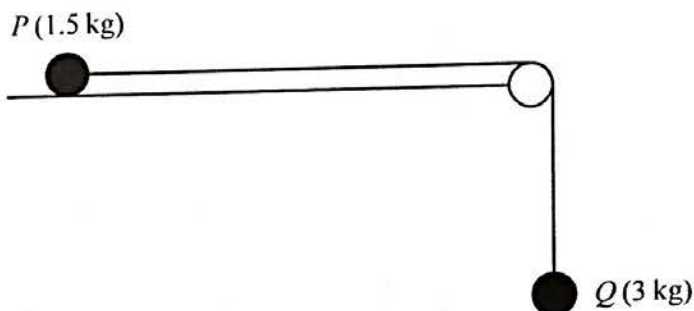


Figure 3

Two particles P and Q have masses 1.5 kg and 3 kg respectively. The particles are attached to the ends of a light inextensible string. Particle P is held at rest on a fixed rough horizontal table. The coefficient of friction between P and the table is $\frac{1}{5}$. The string is parallel to the table and passes over a small smooth light pulley which is fixed at the edge of the table. Particle Q hangs freely at rest vertically below the pulley, as shown in Figure 3. Particle P is released from rest with the string taut and slides along the table.

Assuming that P has not reached the pulley, find

- (a) the tension in the string during the motion, (8)
- (b) the magnitude and direction of the resultant force exerted on the pulley by the string. (4)

8 (a) Consider P:

$\rightarrow \quad R = 1.5g \quad \quad \quad \rightarrow \quad F = ma$

$\frac{1}{5}R = \frac{3}{10}g \quad \leftarrow \quad \text{P} \quad \rightarrow \quad T$

$\downarrow 1.5g$

$\therefore T - \frac{3}{10}g = 1.5a$

Consider Q:

$\uparrow T \quad \quad \quad \downarrow F = ma$

$\text{Q} \quad \downarrow$

$\downarrow 3g$

$3g - T = 3a$

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Question 8 continued

$$T - \frac{3}{10}g = \frac{3}{2}a \Rightarrow 2T - \frac{3}{5}g = 3a$$

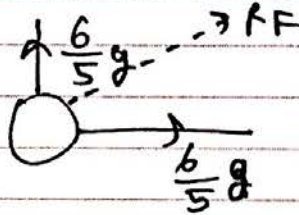
$$\& 3g - T = 3a$$

$$\therefore 3g - T = 2T - \frac{3}{5}g$$

$$\therefore 3T = 3g + \frac{3}{5}g = \frac{18}{5}g$$

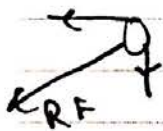
$$\Rightarrow T = \frac{6}{5}g \text{ N}$$

(b)



$$RF = \sqrt{\left(\frac{6}{5}g\right)^2 + \left(\frac{6}{5}g\right)^2} = 16.63\dots$$

$$\therefore RF = \underline{\underline{16.6 \text{ N} \text{ (3sf)}}}$$



Direction: 45° below pulley (south west)

Q8

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

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