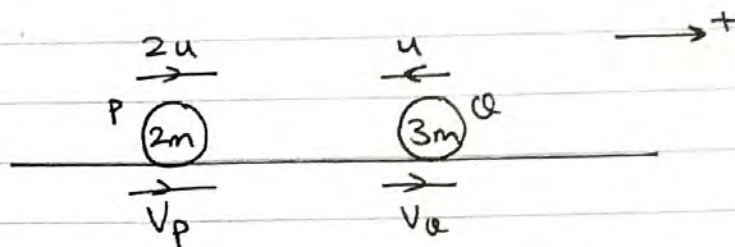


M1 October 2016 (IAL) (MA)

Q(a)



$$\begin{aligned} \text{C.L.M. : } 2m(2u) + 3m(-u) &= 2m(V_p) + 3m(V_q) \\ \Rightarrow u &= 2V_p + 3V_q \end{aligned}$$

Impulse on P acts towards the left.

$$\therefore -5mu = 2m(V - u) = -5mu = 2m(V_p - 2u)$$

$$\Rightarrow -5u = 2V_p - 4u$$

$$\Rightarrow 2V_p = -u \quad \therefore V_p = -\frac{u}{2}$$

$$\text{so speed of P} = \boxed{\frac{u}{2}}$$

- b) $V_p < 0$ so P is travelling in the opposite direction to that which we assumed in the diagram. So yes the dir. of P has been reversed.

c) from (a), $u = 2V_p + 3V_q$

$$\Rightarrow u = 2\left(-\frac{u}{2}\right) + 3V_q$$

$$\Rightarrow 3V_q = 2u \quad \therefore V_q = \boxed{\frac{2u}{3}} = \text{speed}_q.$$

$$(Q2a) \quad \underline{\Sigma F = ma} : \quad \overbrace{\begin{pmatrix} -10 \\ a \end{pmatrix} + \begin{pmatrix} b \\ -5 \end{pmatrix} + \begin{pmatrix} 2a \\ 7 \end{pmatrix}} = \underbrace{3}_{m} \overbrace{\begin{pmatrix} 3 \\ 4 \end{pmatrix}}^a$$

$$\Rightarrow \begin{pmatrix} 2a + b - 10 \\ 2 + a \end{pmatrix} = \begin{pmatrix} 9 \\ 12 \end{pmatrix}$$

equating \underline{i} : $2a + b - 10 = 9$
 $\Rightarrow 2a + b = 19 \quad \sim \textcircled{1}$

equating \underline{j} : $2 + a = 12$
 $\Rightarrow \boxed{a = 10} //$
 \downarrow
 $\textcircled{1} : 2(10) + b = 19$
 so $\boxed{b = -1} //$

b) $a = \frac{v-u}{t} : 3\underline{i} + 4\underline{j} = \frac{(20\underline{i} + 20\underline{j}) - (\underline{u})}{(4)}$

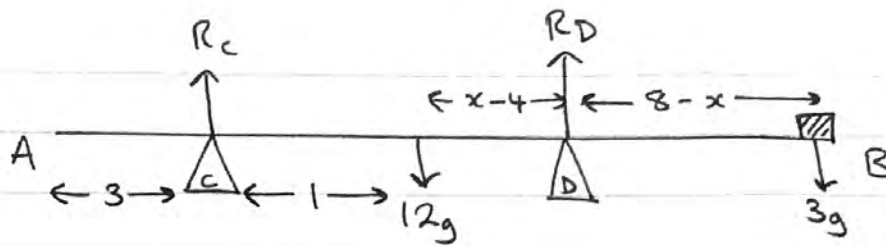
$$\stackrel{\times 4}{\Rightarrow} 12\underline{i} + 16\underline{j} = 20\underline{i} + 20\underline{j} - (\underline{u})$$

$$\Rightarrow \underline{u} = 20\underline{i} - 12\underline{i} + 20\underline{j} - 16\underline{j}$$

$$\Rightarrow \underline{u} = 8\underline{i} + 4\underline{j} //$$

hence $u = |\underline{u}| = \sqrt{8^2 + 4^2} = \boxed{4\sqrt{5}} \text{ ms}^{-1}$

Q3)



We are told: $R_D = 2R_C$

$$R(\uparrow): R_C + R_D = 12g + 3g$$

$$R_C + 2R_C = 15g$$

$$\therefore R_C = \frac{15g}{3} = 5g \quad \rightarrow R_D = 10g$$

$$M(A): R_C(3) + R_D(x) = 12g(4) + 3g(8)$$

$$5g(3) + 10g(x) = 48g + 24g$$

$$10x + 15 = 72$$

$$10x = 57 \quad \therefore x = \frac{57}{10} = \boxed{5.7\text{m}}$$

Q4 a) $p = r_0 + vt$

$$p = \begin{pmatrix} -5 \\ 9 \end{pmatrix} + t \begin{pmatrix} 1 \\ -2 \end{pmatrix} \Rightarrow \boxed{p = (-5+t)\underline{i} + (9-2t)\underline{j}}$$

b) $\underline{j} = 2$ when P is due west of A.



$$\therefore 9 - 2t = 2$$

$$\Rightarrow 7 = 2t \quad \therefore t = \frac{7}{2}$$

$$\text{at } t = \frac{7}{2}, \quad p = \left(-5 + \frac{7}{2}\right)\underline{i} + (9 - 7)\underline{j}$$

$$\boxed{p = -\frac{3}{2}\underline{i} + 2\underline{j}}$$

c) if they move along parallel lines then their velocity vectors are parallel.

$$\Rightarrow \begin{pmatrix} 2b-1 \\ 5-2b \end{pmatrix} = c \begin{pmatrix} 1 \\ -2 \end{pmatrix} \quad \text{for some } c.$$

$$\therefore 2b-1 = c \quad \text{--- (1) (equating } i)$$

$$5-2b = -2c \quad \text{--- (2) (equating } j)$$

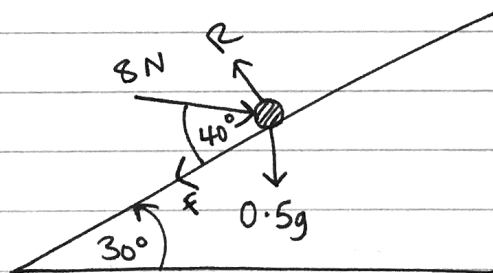
$$\underline{\text{(1) + (2)}}: \quad \cancel{2b} - 1 + 5 - \cancel{2b} = c - 2c$$

$$4 = -c \quad \therefore c = -4 //$$

$$\text{on d } 2b-1 = -4$$

$$\Rightarrow b = \frac{-4+1}{2} = \boxed{-\frac{3}{2}}$$

Q5)



$$R(\downarrow): \quad R = (8\sin 40 + 0.5g\cos 30)$$

$$R(\swarrow): \quad 8\cos 40 = F + 0.5g\sin 30$$

but $F = \mu R$ as P is on the point of sliding up the plane.

$$\therefore 8\cos 40 = \mu R + 0.5g\sin 30$$

$$8\cos 40 = \mu (8\sin 40 + 0.5g\cos 30) + 0.5g\sin 30$$

$$\text{So } \mu = \frac{8\cos 40 - 0.5g\sin 30}{8\sin 40 + 0.5g\cos 30}$$

$$= \boxed{0.392}$$

Q6)

For A :

$$S = S_A$$

$$u = 35$$

$$v = v_A$$

$$a = 0.4$$

$$t =$$

For B :

$$S = S_B$$

$$u = 44$$

$$v = v_B$$

$$a = 0.5$$

$$t =$$

When B overtakes A, $S_B - 200 = S_A$ // (since B starts 200m behind,

$$S_A = 35t + 0.2t^2$$

and $S_B = 44t + 0.25t^2$

but using $S_B - 200 = S_A$,

$$\Rightarrow 44t + 0.25t^2 - 200 = 35t + 0.2t^2$$

$$\Rightarrow 0.05t^2 + 9t - 200 = 0$$

$$\times 20 \Rightarrow t^2 + 180t - 4000 = 0$$

By Quadratic Formula :

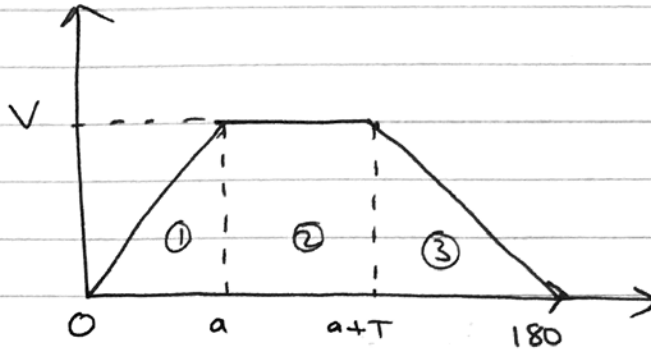
$$t = 20 \quad \text{or} \quad t = -200$$

// \underbrace{\hspace{10em}}_{t > 0}

So at $t = 20$ B overtakes A.

$$\text{using } v = u + at \text{ with B} = v_B = 44 + 0.5(20) \\ = \boxed{54}$$

(Q7a)



$$b) \quad a = \frac{v-u}{t} \text{ for (1)} : \quad 1 = \frac{V-0}{a} \\ \therefore V = a.$$

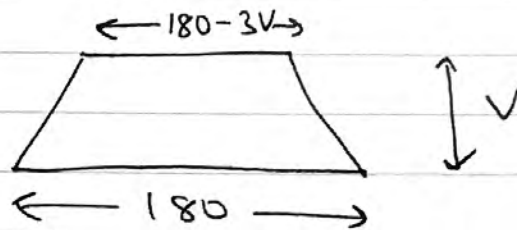
$$a = \frac{v-u}{t} \text{ for (3)} : \quad -0.5 = \frac{0-V}{180-(v+T)}$$

$$-90 + 0.5(T+V) = -V \\ 90 - \frac{1}{2}(T+V) = V$$

$$180 - T - V = 2V \\ \therefore T = 180 - 3V$$

c) Total area under graph = 4800 //

$$\text{Area of trapezium} = \frac{(a+b)h}{2} (= 4800)$$



$$\therefore 4800 = \frac{1}{2} (180 + 180 - 3V)V$$

$$9600 = 360V - 3V^2$$

$$\textcircled{\div 3} : 3200 = 120V - V^2$$

$$V^2 - 120V + 3200 = 0$$

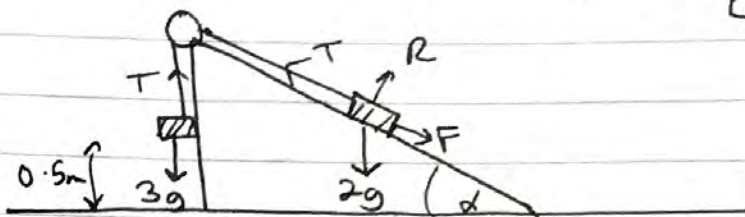
$$(V-40)(V-80) = 0$$

$$\text{so } \boxed{V=40} \text{ or } V=80$$

can't be $V=80$ as then area of $\textcircled{1}$ would be greater than 4800m which isn't possible.

Q8a

$$\begin{aligned} \cos \alpha &= \frac{3}{5} \\ \sin \alpha &= \frac{4}{5} \\ \tan \alpha &= \frac{3}{4} \end{aligned}$$



N2L(Q): $3g - T = 3a \quad \sim \textcircled{1}$

N2L(P): $T - 2g \sin \alpha - F = 2a \quad \sim \textcircled{2}$

(Q) $\left. \begin{array}{l} s = 0.5 \\ u = 0 \\ v = 1.4 \\ a = a \\ t = \end{array} \right\} \begin{array}{l} v^2 = u^2 + 2as \\ 1.4^2 = 0^2 + 2a(0.5) \\ 1.4^2 = a \end{array}$

from $\textcircled{1}$: $T = 3g - 3a = 3g - 3(1.4^2)$
 $= \boxed{23.5 \text{ N}}$

b) $\textcircled{2}$: $T - 2g \sin \alpha - \mu R = 2a$

$$23.5 - 2g \left(\frac{3}{5}\right) - \mu (2g \cos \alpha) = 2(1.4^2)$$

$$23.5 - \frac{6g}{5} - \mu \left(2g \times \frac{4}{5}\right) = 2(1.4^2)$$

$$\therefore \mu = \frac{23.5 - \frac{6g}{5} - 2(1.4^2)}{2g \times \frac{4}{5}} = \boxed{0.5}$$