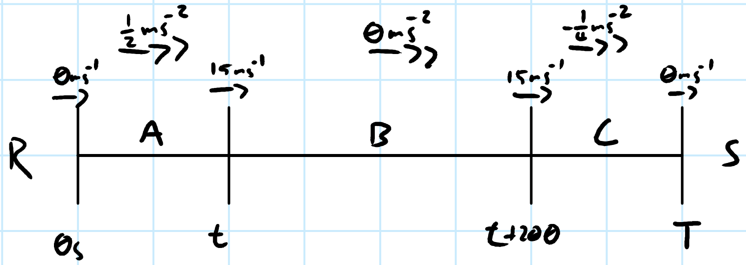
January 2017 (IAL) MA - M1

1)

a)



$15ms^{-1} = 0 + \frac{1}{2}t$

$t = 30s$

$0ms^{-1} = 15 - \frac{1}{4}t$

$t = 60s$

$T = 30 + 200 + 60$

$= 290s$

b)

(A)		(B)		(C)	
S	___	S	___	S	___
U	$0ms^{-1}$	U	$15ms^{-1}$	U	$15ms^{-1}$
V	$15ms^{-1}$	V	$15ms^{-1}$	V	$0ms^{-1}$
A	$\frac{1}{2}ms^{-2}$	A	$0ms^{-2}$	A	$-\frac{1}{4}ms^{-2}$
T	30s	T	200s	T	60s

(A)

$S = \left(\frac{u+v}{2}\right)t$

$S = \left(\frac{0+15}{2}\right)30$

$S = 225m$

(B)

$S = ut + \frac{1}{2}at^2$

$S = 15(200) + 0$

$S = 3000m$

(C)

$S = \left(\frac{u+v}{2}\right)t$

$S = \left(\frac{15+0}{2}\right)60$

$S = 450$

$|\vec{RS}| = 3,675m$

1)

c) $\overline{\text{speed}} = \frac{\text{distance}}{\text{time}}$

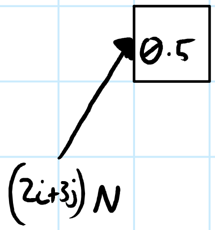
$\bar{s} = \frac{d}{t}$

$\bar{s} = \frac{3675}{290}$

$\bar{s} = 12.7\,ms^{-1}$ (3 s f)

2)

a)



$F = ma$

$2i + 3j = \frac{1}{2} a \quad \therefore a = (4i + 6j)\,ms^{-2}$

b) $V_t = 4i + 0j + t(4i + 6j$

$V_t = (4 + 4t)i + 6tj$

$|V_3| = |4 + 12i + 18j|$

$|V_3| = \sqrt{16^2 + 18^2}$

$|V_3| = \sqrt{580}$

$= 24.1\ ms^{-1}$ (3sf)

c) $V_T = k(2i + j)$

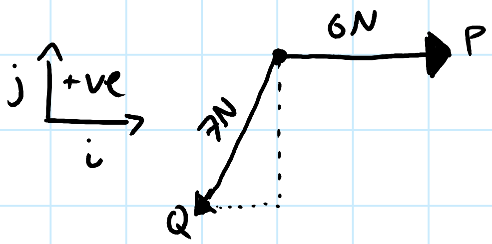
$2k = 4 + 4T$

$k = 6T$

$\therefore 12T = 4 + 4T$

$8T = 4 \qquad T = 0.5s$

3)



Let i be a horizontal vector, parallel to the line of action of P
Let j be a vertical vector, perpendicular to the line of action of P

$$P = 6i$$

$$Q = 7\cos(-120)i + 7\sin(-120)j$$

(note: angles for unit vectors are done anti-clockwise, hence the negative sign seen here)

$$R = \left(6 + 7\left(-\tfrac{1}{2}\right)\right)i + 7\,\frac{-\sqrt{3}}{2}\,j$$

$$R = \frac{5}{2}i - \frac{7\sqrt{3}}{2}j$$

i) $$|R| = \sqrt{\left(\frac{5}{2}\right)^2 + \left(\frac{-7\sqrt{3}}{2}\right)^2}$$

$$|R| = \sqrt{43}$$

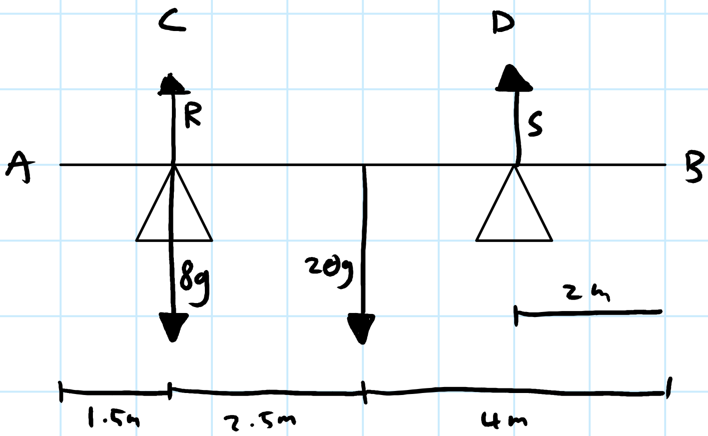
$$|R| = 6.56\,N \text{ (3sf)}$$

ii) $$\arg(R) = \tan^{-1}\left(\frac{\frac{-7\sqrt{3}}{2}}{\frac{5}{2}}\right)$$

$$= -67.6^\circ \text{ anticlockwise}$$

4)

a)



$R + S = 8g + 20g$

moments around C $(\circlearrowleft) = (\circlearrowright)$

$0 = 2.5(-20g) + 4.5(S)$

$490 = 4.5S$

$S = \frac{980}{9}$

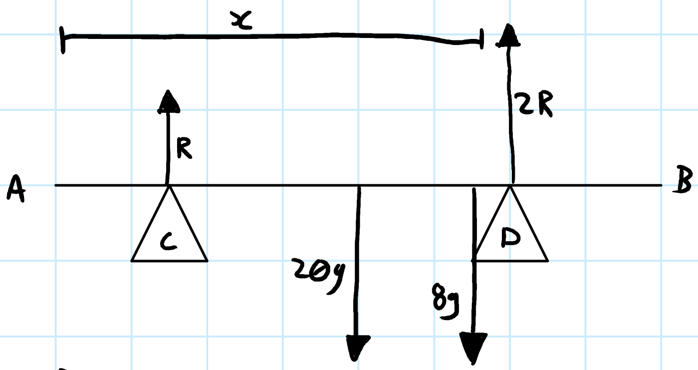
i) $S = 109$ (3sf)

$R + 109 = 274.4$

ii) $R = 166\text{ N}$ (3sf)

4)

b)



$R + 2R = 20g + 8g$

moments around A (clockwise) : (anticlockwise)

$0 = 1.5R + 4(-20g) + x(-8g) + 6(2R)$

$80g + 8xg = 13.5R$

$240g + 24xg = 13.5(3R)$

$240g + 24xg = 13.5(28g)$ (g cancelled)

$240 + 24x = 378$

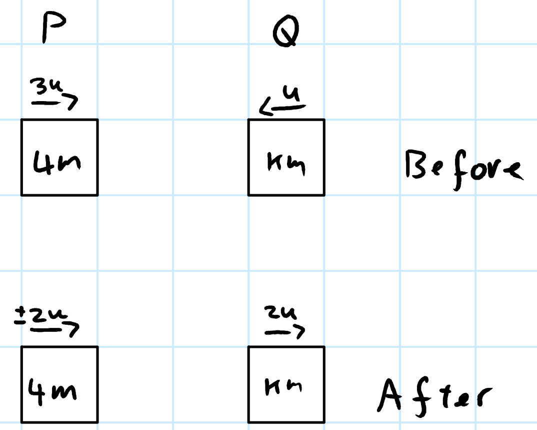
$24x = 138$

$x = 5.75\text{m}$

c) The weight of the package acts directly on the point stated (C or E)

GradeMax

5)



a)

$$-u(km) + I = 2u(km)$$

$$I = 3ukm$$

b)

Momentum is conserved
Total momentum before = total momentum after

$$3u(4m) + -u(km) = 2u(4m) + 2u(km)$$

$$12um - kum = 8um + 2kum$$

$$4\cancel{um} = 3k\cancel{um}$$

$$k = \frac{4}{3}$$

OR

$$3u(4m) + -u(km) = -2u(4m) + 2u(km)$$

$$12um - kum = -8um + 2kum$$

$$20\cancel{um} = 3k\cancel{um}$$

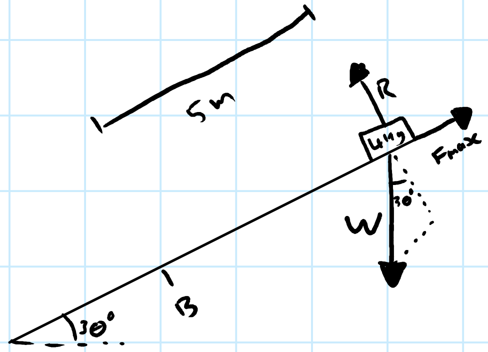
$$k = \frac{20}{3}$$

The question states the ***speed*** of both particles is 2u, but only mentions the direction of this speed for Q, leaving P's velocity ambiguous. The 2 values of K are for when P is moving towards Q (k = 4/3) and away form Q (20/3)

6)

a)

$\mu = 0.3$



$R = 4g\cos(30°)$

$= \frac{98\sqrt{3}}{5}$

$F_{max} = \mu R$

$= 0.3 \cdot \frac{98\sqrt{3}}{5}$

$= 10.2N$ (3sf)

$W(\swarrow) = 4g\sin(30°)$

$= 19.6N$

$19.6 - 10.2 = 9.42N$ (3sf)

$F = ma$

$9.42 = 4a \quad \therefore a = 2.35\,ms^{-2}$

S 5m

U $0\,ms^{-1}$

V —

A $2.35\,ms^{-2}$

T X

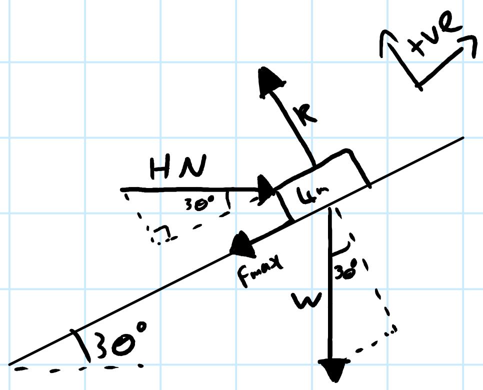
$v^2 = u^2 + 2as$

$v^2 = 0 + 23.5$

$|v| = 4.85\,ms^{-1}$ (3sf)

6)

b)



$$R = 4g\cos(30°) + H\sin(30°)$$

$$F_{max} = 0.3\left[33.9 + \tfrac{1}{2}H\right]$$

$$= 10.2 + \tfrac{3}{20}H$$

Forces parallel to R

$$H\cos(30) - 4g\sin(30°) - F_{max} = 0$$

Forces perpendicular to R

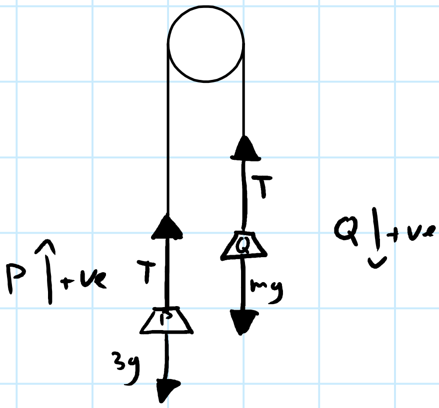
$$\frac{\sqrt{3}}{2}H - 2g - 10.2 - \frac{3}{20}H = 0$$

$$10\sqrt{3}H - 3H = 595.7$$

$$H(10\sqrt{3} - 3) = 595.7$$

$$H = 41.6\,N$$

7)



a) P: $T - 3g = 3a$

$33.6 - 29.4 = 3a$

$4.2 = 3a$

$a = 1.4 ms^{-2}$

b)

Q: $mg - T = ma$

$4.8m - 33.6 = 1.4m$

$8.4m = 33.6$

$m = 4 kg$

c) Q:

S $10.5 m$

U $0 ms^{-1}$

V x

A $1.4 ms^{-2}$

T —

$s = ut + \frac{1}{2}at^2$

$10.5 = 0 + 0.7t^2$

$t^2 = 15$

$t = \pm\sqrt{15}$

$t > 0 \therefore t = 3.87$ (3 sf)

7)

Speed of particles when Q hits the ground

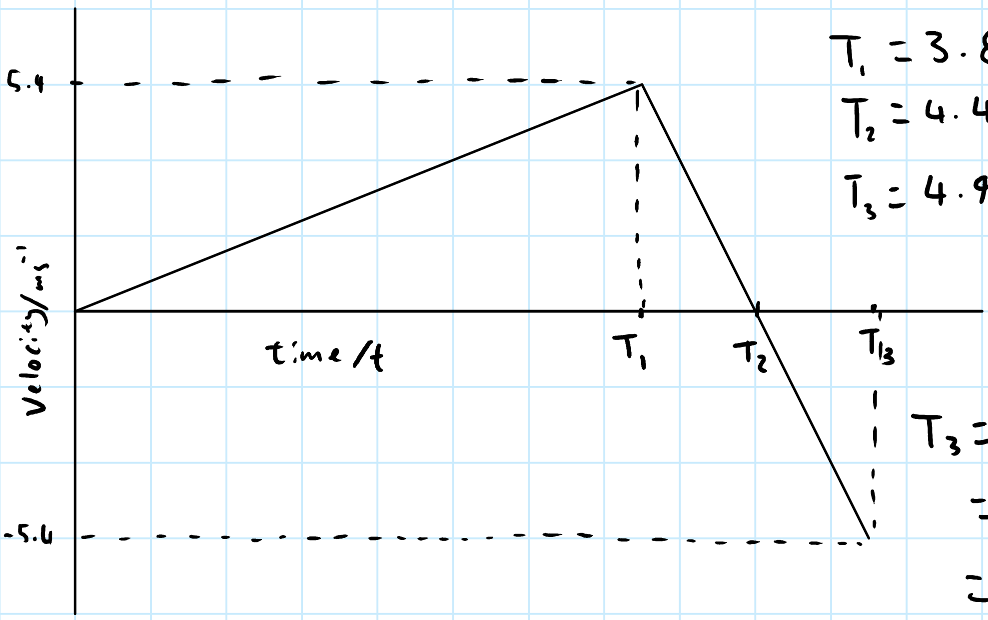
d)

S	10.5 m	$v^2 = u^2 + 2as$
U	0	$v^2 = 0^2 + 29.4$
V	—	$v = 5.42\,ms^{-1}$
A	$1.4\,ms^{-2}$	
T	x	

S	x	$v = u + at$
U	$5.42\,ms^{-1}$	$0 = 5.42 - 9.8t$
V	$0\,ms^{-1}$	$9.8t = 5.42$
A	$-9.8\,ms^{-2}$	$t = 0.553\,s$
T	—	

$T_2 = 3.87 + 0.553 = 4.43\,s$

e)



$T_1 = 3.87\,s$

$T_2 = 4.42\,s$

$T_3 = 4.98\,s$

$T_3 = T_2 + (T_2 - T_1)$

$= \frac{8\sqrt{15}}{7} + \frac{8\sqrt{15}}{7} - \sqrt{15}$

$= \frac{9\sqrt{15}}{7}$