C1 January 2014 (IAL) (MA)

Q1a) $(2\sqrt{x})^2 = \boxed{4x}$

b) $\dfrac{5+\sqrt{7}}{2+\sqrt{7}} = \dfrac{5+\sqrt{7}}{2+\sqrt{7}} \times \dfrac{2-\sqrt{7}}{2-\sqrt{7}}$

$= \dfrac{(5+\sqrt{7})(2-\sqrt{7})}{(2+\sqrt{7})(2-\sqrt{7})}$

$= \dfrac{10 - 5\sqrt{7} + 2\sqrt{7} - 7}{4 - 2\sqrt{7} + 2\sqrt{7} - 7}$

$= \dfrac{3 - 3\sqrt{7}}{-3}$

$= -1 + \sqrt{7}$

$= \boxed{\sqrt{7} - 1}$

Q2) $y = 2x^2 - \dfrac{4}{\sqrt{x}} + 1, \quad x > 0$

$y = 2x^2 - 4x^{-\frac{1}{2}} + 1$

a) $\dfrac{dy}{dx} = 4x + 2x^{-\frac{3}{2}}$

$= 4x + \dfrac{2}{x^{3/2}}$

$= \boxed{4x + \dfrac{2}{x\sqrt{x}}}$

b) $\frac{d^2y}{dx^2} = 4 - 3x^{-\frac{5}{2}}$

$= 4 - \frac{3}{x^{5/2}}$

$= \boxed{4 - \frac{3}{x^2\sqrt{x}}}$

Q3) $x - 2y - 1 = 0 \Rightarrow x = 2y + 1$ ①

$x^2 + 4y^2 - 10x + 9 = 0 \Rightarrow x^2 + 4y^2 - 10x + 9 = 0$ ②

Substitute ① into ②:

$(2y+1)^2 + 4y^2 - 10(2y+1) + 9 = 0$

$(2y+1)(2y+1) + 4y^2 - 10(2y+1) + 9 = 0$

$4y^2 + 4y + 1 + 4y^2 - 20y - 10 + 9 = 0$

$8y^2 - 16y = 0$

$y^2 - 2y = 0$

$y(y-2) = 0$

Either $y = 0$ or $y = 2$

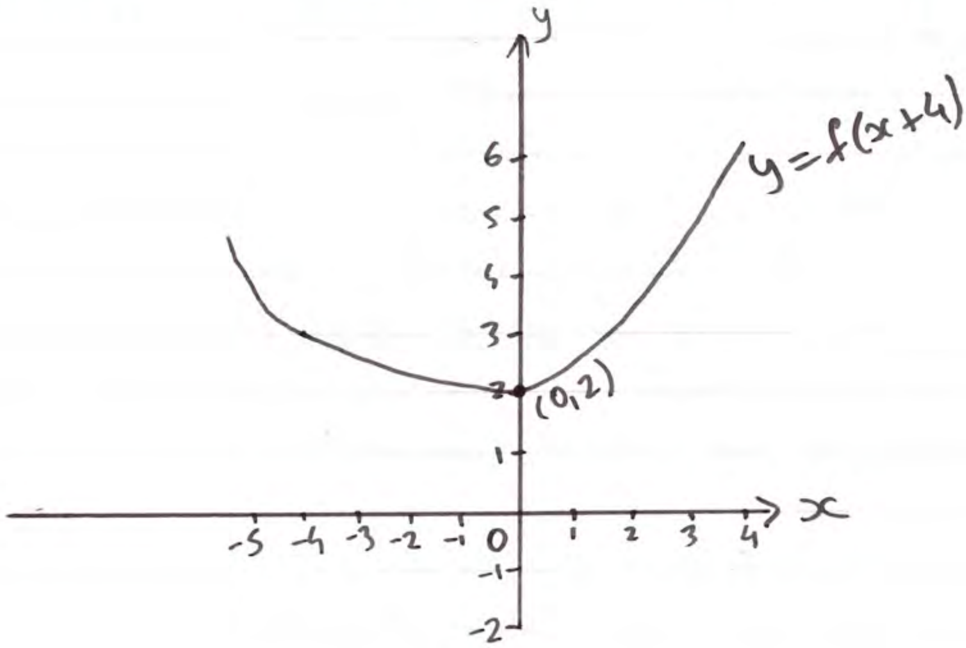
Substitute into ① for x:

When $y = 0$, $x = 2(0) + 1$ $\underline{= 1}$

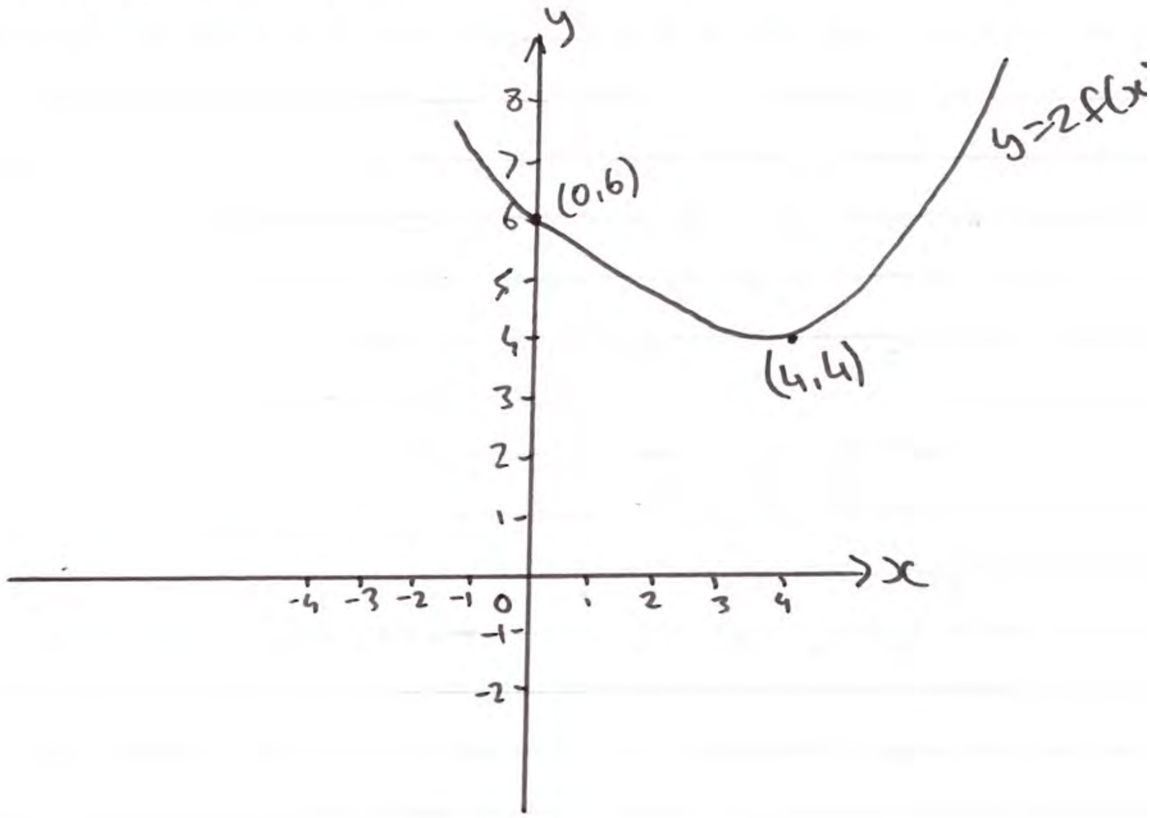
When $y = 2$, $x = 2(2) + 1$ $\underline{= 5}$

Solution set: $\boxed{x = 1, y = 0 \text{ and } x = 5, y = 2}$

Q4a) $y = f(x+4)$ - transformation horizontally of



b) $y = 2f(x)$ - transformation vertically of $\times 2$:



Q5) $\sum_{r=1}^{n} a_r = 12 + 4n^2$

a) $\sum_{r=1}^{5} a_r = 12 + 4(5)^2$

$= 12 + 100$

$\boxed{= 112}$

b) $\sum_{r=1}^{6} a_r = 12 + 4(6)^2$

$= 12 + 144$

$= 156$

$a_6 = \sum_{r=1}^{6} a_r - \sum_{r=1}^{5} a_r$

$a_6 = 156 - 112$

$\boxed{a_6 = 44}$

Q6a) l_1 has equation $2y = 3x + 7$

i) $y = \frac{3}{2}x + \frac{7}{2}$

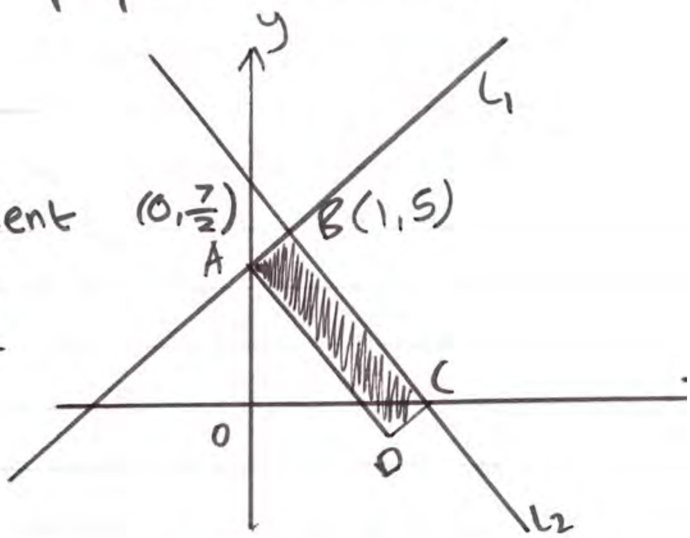
$\boxed{\text{Gradient of } l_1 = \frac{3}{2}}$

ii) when l_1 cuts the y-axis, $x = 0$

So, $y = \frac{7}{2} \Rightarrow$ $\boxed{A \text{ is at } \left(0, \frac{7}{2}\right)}$

b) l_2 intersects l_1 at $B(1,5)$ and crosses the x-axis at

From diagram, l_1 is perpendicular to l_2, since $A\hat{B}C = 90°$.



If the gradient of $l_1 = \frac{3}{2}$, then the gradient of $l_2 = \frac{-2}{3}$

Equation of l_2:

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -\frac{2}{3}(x-1)$$

Using $B(1,5)$ and $m = -\frac{2}{3}$

$$3(y-5) = -2(x-1)$$

$$3y - 15 = -2x + 2$$

$$3y + 2x - 17 = 0$$

$$\boxed{2x + 3y - 17 = 0}$$

c) When l_2 cuts the x-axis, $y = 0$

So, $2x + 3(0) - 17 = 0$

$$2x = 17$$

$$x = \frac{17}{2} \Rightarrow C \text{ is at } \left(\frac{17}{2}, 0\right)$$

$$\text{Length } AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(1-0)^2+(5-\tfrac{7}{2})^2}$$

$$= \sqrt{1^2+(\tfrac{3}{2})^2}$$

$$= \sqrt{1+\tfrac{9}{4}}$$

$$= \sqrt{\tfrac{13}{4}} = \frac{\sqrt{13}}{\sqrt{4}} = \frac{\sqrt{13}}{2}$$

$$= \frac{\sqrt{13}}{2} \text{ units}$$

$$\text{Length } BC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(\tfrac{17}{2}-1)^2+(0-5)^2}$$

$$= \sqrt{(\tfrac{15}{2})^2+(-5)^2}$$

$$= \sqrt{\tfrac{225}{4}+25}$$

$$= \sqrt{\tfrac{325}{4}} = \sqrt{\tfrac{325}{4}} = \frac{\sqrt{325}}{\sqrt{4}} = \frac{\sqrt{25}\sqrt{13}}{2}$$

$$= \frac{5\sqrt{13}}{2} \text{ units}$$

$$\text{Area } ABCD = bh = (AB)(BC)$$

$$= \left(\frac{\sqrt{13}}{2}\right)\left(\frac{5\sqrt{13}}{2}\right)$$

$$\boxed{= \frac{65}{4} \text{ units}^2}$$

Q7) First term, $a = £14\,000$
Common difference, $d = £1500$

a) $U_n = a + (n-1)d$

$U_9 = 14000 + (9-1)1500$

$= 14000 + 12000$

$= \boxed{£26\,000}$

b) $S_n = \frac{n}{2}(2a + (n-1)d$

$S_9 = \frac{9}{2}(2(14000) + (9-1)\ 1500)$

$= \frac{9}{2}(28000 + 12000)$

$= \frac{9}{2} \times 40000$

$= \boxed{£180\,000}$

c) For Anna, first term, $a = £A$
common difference, $d = £1000$

Max. salary of £26000 reached in year 10

$\therefore U_{10} = 26000$

$U_n = a + (n-1)d$

$U_{10} = A + (10-1)1000$

$$26\,000 = A + 9000$$

$$\therefore \underline{A = £17\,000}$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{10} = \frac{10}{2}(2(17\,000) + (10-1)1000)$$

$$= 5(34\,000 + 9000)$$

$$= 5(43\,000)$$

$$\underline{= £215\,000}$$

For Shelim, $S_9 = £180,000$

Since his salary does not go above £26 000 a year, he will receive £26 000 in the 10th year, making his total:

$$S_{10} = S_9 + 26\,000$$

$$S_{10} = 180\,000 + 26\,000$$

$$\underline{= £206\,000}$$

Difference between Shelim and Anna:

$$= 215\,000 - 206\,000$$

$$\boxed{= £9\,000}$$

∴ Anna earns £9 000 more in the first 10 yea

Q 8) $2x^2 + 2kx + (k+2) = 0$ has two distinct real roots.

a) If a quadratic has two distinct real roots, then the discriminant is greater than 0.

$$\therefore b^2 - 4ac > 0$$

$$(2k)^2 - (4)(2)(k+2) > 0$$

$$4k^2 - 8(k+2) > 0$$

$$4k^2 - 8k - 16 > 0$$

$$\therefore \boxed{k^2 - 2k - 4 > 0}$$

b) For $k^2 - 2k - 4 = 0$

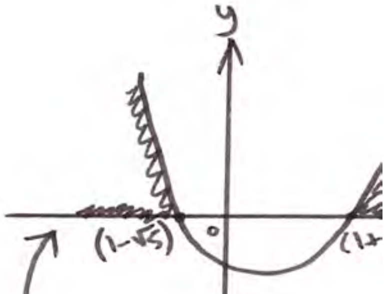
$$(k-1)^2 - 1^2 - 4 = 0$$

$$(k-1)^2 - 1 - 4 = 0$$

$$(k-1)^2 = 5$$

$$k - 1 = \pm\sqrt{5}$$

$$k = 1 \pm \sqrt{5}$$



Choosing the val
'above' the x-
Since $k^2-2k-4 >$

Set of possible values of k:

$$\boxed{k < 1-\sqrt{5} \quad \text{or} \quad k > 1+\sqrt{5}}$$

Q9a) $f'(x) = (x-2)(3x+4)$

$f'(x) = 3x^2 + 4x - 6x - 8$

$f'(x) = 3x^2 - 2x - 8$

$f(x) = \int (3x^2 - 2x - 8)\,dx$

$f(x) = \frac{3x^3}{3} - \frac{2x^2}{2} - 8x + C$

$\underline{f(x) = x^3 - x^2 - 8x + C}$

Since $f(x)$ passes through $(3, 6)$, substitute in $x = 3$ and $f(x) = 6$:

$6 = 3^3 - 3^2 - 8(3) + C$

$6 = 27 - 9 - 24 + C$

$6 = -6 + C$

$\therefore \underline{C = 12}$

$\boxed{f(x) = x^3 - x^2 - 8x + 12}$

b) $f(x) = (x-2)^2(x+p)$

$f(x) = (x-2)(x-2)(x+p)$

$f(x) = (x^2 - 4x + 4)(x+p)$

$$x^3 - x^2 - 8x + 12 = (x^2 - 4x + 4)(x+3)$$

$$\therefore \boxed{p = 3}$$

$$x^3 - x^2 - 8x + 12 = \underline{(x-2)^2(x+3)}$$

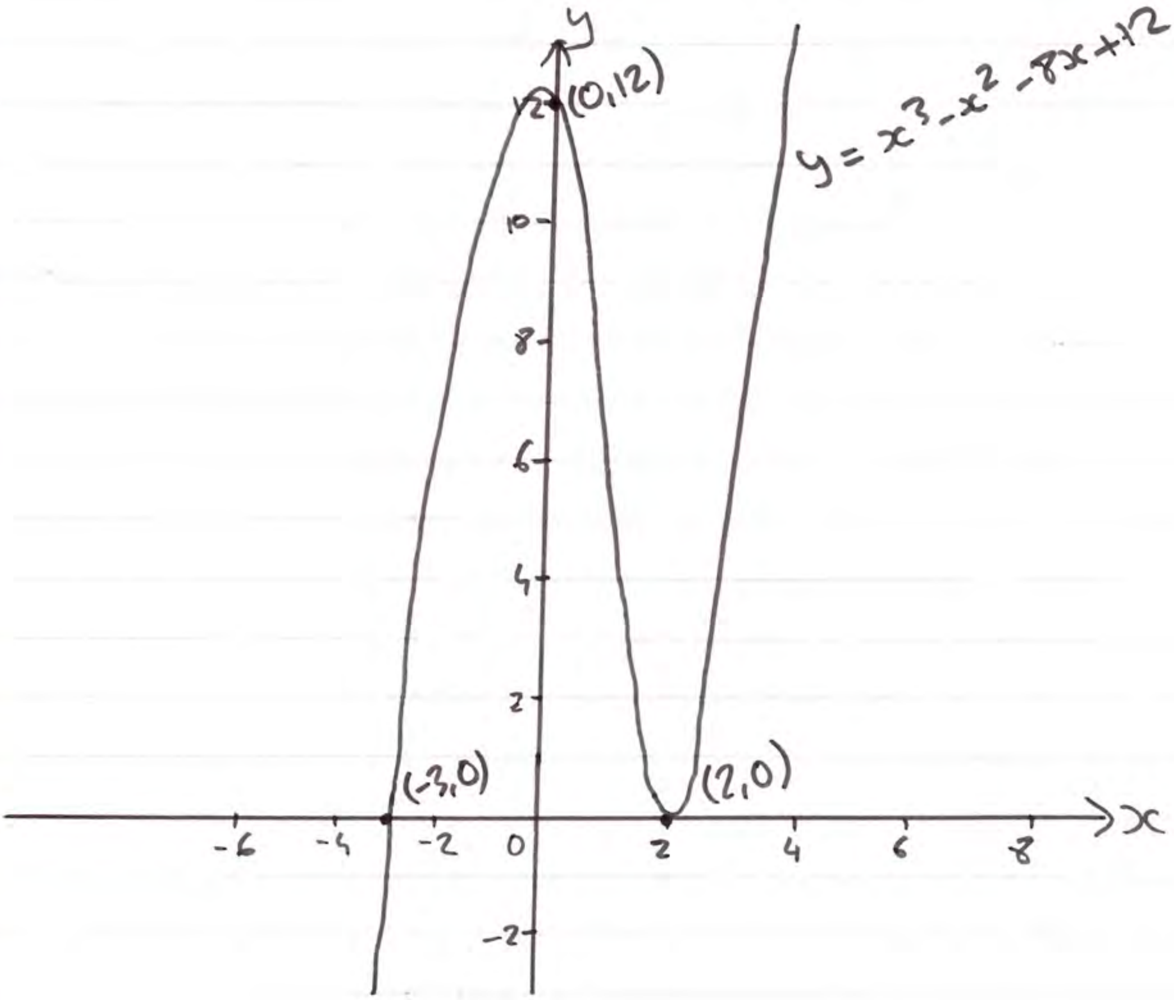
c) $y = f(x) = x^3 - x^2 - 8x + 12$

When $\underline{x = 0,\ y = 12}$

When $y = 0$, $x^3 - x^2 - 8x + 12 = 0$

$(x-2)^2(x+3) = 0$

$\underline{\text{Either } x = 2 \text{ or } x = 2 \text{ or } x = -3}$



Q10a) $y = x^3 - 2x^2 - x + 3$

$$\frac{dy}{dx} = 3x^2 - 4x - 1$$

At $x = 2$, $\frac{dy}{dx} = 3(2)^2 - 4(2) - 1$

$$= 12 - 8 - 1$$

$$= 3$$

$\therefore$ the gradient of the tangent at P is 3.

Equation of tangent: $y - y_1 = m(x - x_1)$

Using $P(2,1)$ and $m = 3$ $\longrightarrow$ $y - 1 = 3(x - 2)$

$$y - 1 = 3x - 6$$

$$\boxed{y = 3x - 5}$$

b) If the tangent at Q is parallel to the tangent at P, then the gradients of both ar

$$\therefore \frac{dy}{dx} = 3 = 3x^2 - 4x - 1$$

$$3x^2 - 4x - 1 = 3$$

$$3x^2 - 4x - 4 = 0$$

$$(3x + 2)(x - 2) = 0$$

Either $x = -\frac{2}{3}$ or $x = 2$

We already know that when $x=2$, $y=1$ and this is point P.

So, when $x = -\frac{2}{3}$, this is point Q.

$x = -\frac{2}{3}$, $y = x^3 - 2x^2 - x + 3$

$$y = \left(-\frac{2}{3}\right)^3 - 2\left(-\frac{2}{3}\right)^2 - \left(-\frac{2}{3}\right) + 3$$

$$y = \left(-\frac{8}{27}\right) - 2\left(\frac{4}{9}\right) + \frac{2}{3} + 3$$

$$y = \frac{-8}{27} - \frac{8}{9} + \frac{2}{3} + 3$$

$$y = -\frac{8}{27} - \frac{24}{27} + \frac{18}{27} + \frac{81}{27}$$

$$\underline{y = \frac{67}{27}}$$

$\therefore$ Q has coordinates $\boxed{\left(-\frac{2}{3}, \frac{67}{27}\right)}$