M2 S14 uu

1. Three particles of mass $3m$, $2m$ and km are placed at the points whose coordinates are (1, 5), (6, 4) and (a, 1) respectively. The centre of mass of the three particles is at the point with coordinates (3, 3).

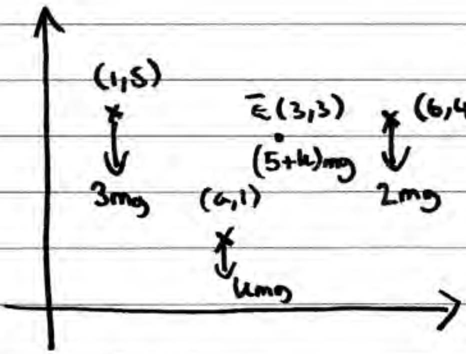
Find

(a) the value of k,

(3)

(b) the value of a.

(3)



a) $kmg \times 1 + 3mg \times 5 + 2mg \times 4 = (5+k)mg \times 3$

$\Rightarrow kmg + 23mg = 15mg + 3kmg$

$\Rightarrow 8mg = 2kmg \quad \therefore 2k = 8 \quad \therefore \underline{\underline{k = 4}}$

b) $3mg \times 1 + 4mg \times a + 2mg \times 6 = 9mg \times 3$

$4mg\,a + 15mg = 27mg$

$4mg\,a = 12mg$

$a = 3$

2. At time t seconds, where $t \geqslant 0$, a particle P is moving on a horizontal plane with acceleration $[(3t^2 - 4t)\mathbf{i} + (6t - 5)\mathbf{j}]$ m s^{-2}.

When $t = 3$ the velocity of P is $(11\mathbf{i} + 10\mathbf{j})$ m s^{-1}.

Find

(a) the velocity of P at time t seconds,

(5)

(b) the speed of P when it is moving parallel to the vector $\mathbf{i}$.

(4)

$$a = \begin{pmatrix} 3t^2 - 4t \\ 6t - 5 \end{pmatrix}$$

$$v = \int a\,dt = \begin{pmatrix} t^3 - 2t^2 + c_1 \\ 3t^2 - 5t + c_2 \end{pmatrix}$$

$$t = 3 \quad \begin{pmatrix} 9 + c_1 \\ 12 + c_2 \end{pmatrix} = \begin{pmatrix} 11 \\ 10 \end{pmatrix} \quad \therefore \; \begin{matrix} c_1 = 2 \\ c_2 = -2 \end{matrix}$$

$$\therefore \; v = \begin{pmatrix} t^3 - 2t^2 + 2 \\ 3t^2 - 5t - 2 \end{pmatrix}$$

b) moving parallel to i $\longrightarrow$ $\Rightarrow$ j component of V is 0

$$(3t^2 - 5t - 2) = 0 \qquad (3t + 1)(t - 2) = 0 \quad \therefore t = 2$$

$$v = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad \therefore \; \text{Speed} = 2$$

3.

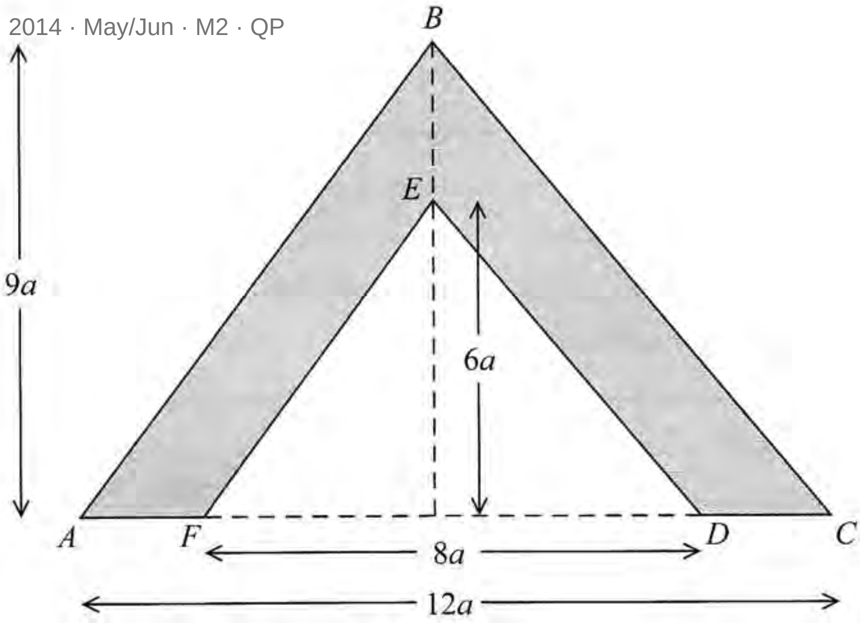


Figure 1

The uniform lamina *ABCDEF*, shown shaded in Figure 1, is symmetrical about the line through *B* and *E*. It is formed by removing the isosceles triangle *FED*, of height $6a$ and base $8a$, from the isosceles triangle *ABC* of height $9a$ and base $12a$.

(a) Find, in terms of a, the distance of the centre of mass of the lamina from *AC*.

(5)

The lamina is freely suspended from *A* and hangs in equilibrium.

(b) Find, to the nearest degree, the size of the angle between *AB* and the downward vertical.

(4)

mass per unit area $a^2 = k$

$g_1 (0, 2a)$ $M = 24a^2 k$

$g_2 (0, \bar{y})$ $M = 30a^2 k$

$24\cancel{a^2}\cancel{kg} \times 2a + 30\cancel{a^2}\cancel{kg} \times \bar{y}$

$= 54\cancel{a^2}\cancel{kg} \times 3a$

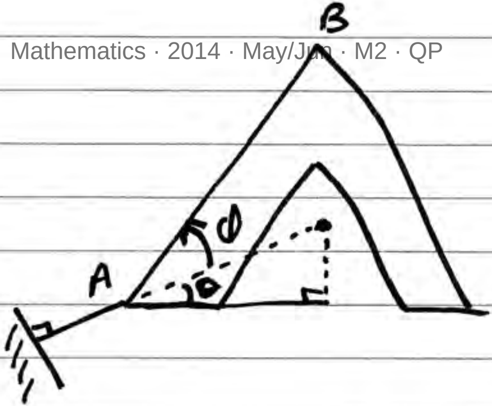
$\therefore 24 \times 2a + 30\bar{y} = 162a$

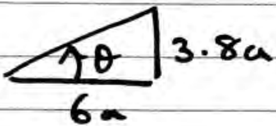
$G (0, 3a)$

$M = 54a^2 k$

$30\bar{y} = 114a$

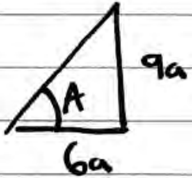
$\bar{y} = 3.8a$





$\therefore \theta = \tan^{-1}\left(\frac{3.8}{6}\right) = 32.3^\circ$

$\approx 32^\circ$



$A = \tan^{-1}\left(\frac{9a}{6a}\right) = 56.3$

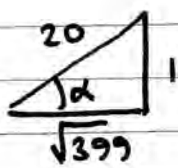
$\therefore \phi = 56.3 - 32.3 \approx 24^\circ$

4. A truck of mass 1800 kg is towing a trailer of mass 800 kg up a straight road which is inclined to the horizontal at an angle α, where $\sin \alpha = \frac{1}{20}$. The truck is connected to the trailer by a light inextensible rope which is parallel to the direction of motion of the truck. The resistances to motion of the truck and the trailer from non-gravitational forces are modelled as constant forces of magnitudes 300 N and 200 N respectively. The truck is moving at constant speed v m s^{-1} and the engine of the truck is working at a rate of 40 kW.

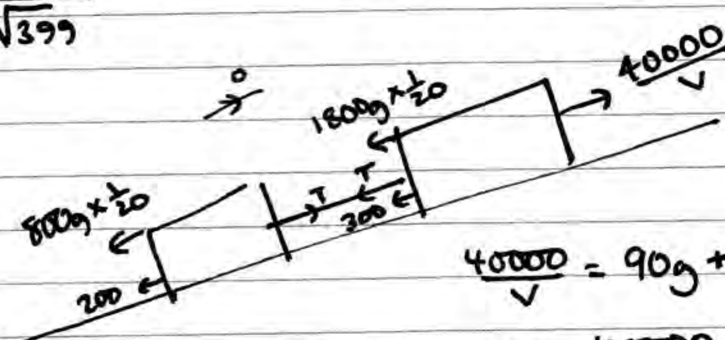
(a) Find the value of v.
(5)

As the truck is moving up the road the rope breaks.

(b) Find the acceleration of the truck immediately after the rope breaks.
(4)

a) 

$\tan\alpha = \frac{1}{\sqrt{399}}$ $\cos\alpha = \frac{\sqrt{399}}{20}$



$\frac{40000}{v} = 90g + 40g + 300 + 200$

$v = \frac{40000}{1774} = 22.5$ (3sf)

b) $RF = ma$ $\frac{40000}{1774} - 90g - 300 = 1800a$

$592 = 1800a \therefore a = 0.329$

5. A particle of mass m kg lies on a smooth horizontal surface. Initially the particle is at rest at a point O midway between a pair of fixed parallel vertical walls. The walls are 2 m apart. At time $t = 0$ the particle is projected from O with speed u m s^{-1} in a direction perpendicular to the walls. The coefficient of restitution between the particle and each wall is $\frac{2}{3}$. The magnitude of the impulse on the particle due to the first impact with a wall is λmu N s.

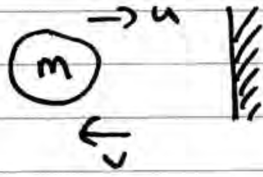
(a) Find the value of λ.

(3)

The particle returns to O, having bounced off each wall once, at time $t = 3$ seconds.

(b) Find the value of u.

(6)



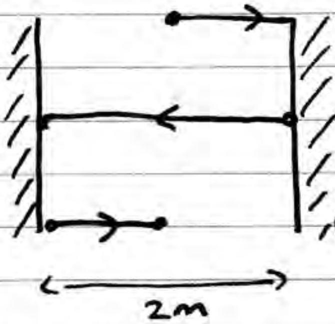
$e = \frac{\text{sep}}{\text{app}} = \frac{v}{u} = \frac{2}{3} \quad \therefore v = \frac{2}{3}u$

Mom before $= mu$

Mom after $= m(-\frac{2}{3}u) = -\frac{2}{3}mu \quad \therefore \text{Impulse} = \frac{5}{3}mu$

$\frac{5}{3}mu = \lambda mu \quad \therefore \lambda = \frac{5}{3}$.

b)



Dist = speed × time

① Speed $= u$ $\quad 1 = u \times t_1$

② Speed $= \frac{2}{3}u$ $\quad 2 = \frac{2}{3}u \times t_2$

③ speed $= \frac{4}{9}u$ $\quad 1 = \frac{4}{9}u \times t_3$

$t_1 = \frac{1}{u} \quad t_2 = \frac{3}{u} \quad t_3 = \frac{9}{4u} \qquad t_1 + t_2 + t_3 = 3$

$\frac{1}{u} + \frac{3}{u} + \frac{9}{4u} = 3 \qquad \frac{4^{\times 4}}{u_{\times 4}} + \frac{9}{4u} = \frac{25}{4u} = 3$

$\therefore 25 = 12u \quad \therefore u = \frac{25}{12}$

2

6.

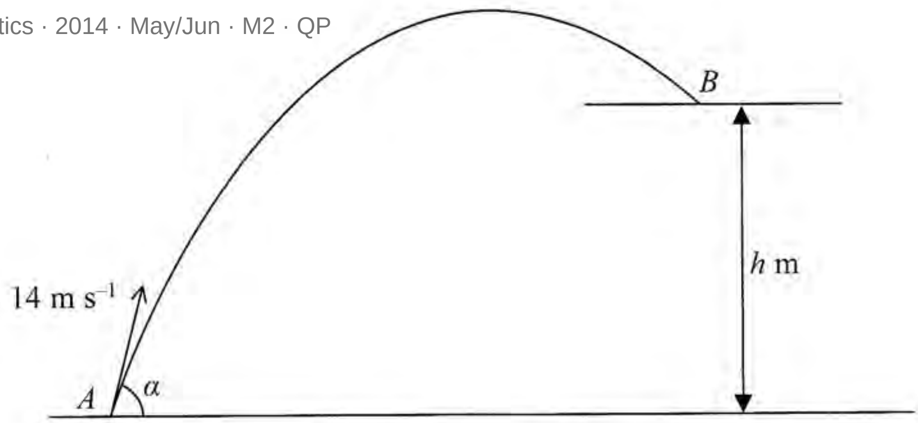


Figure 2

A small ball is projected with speed 14 m s^{-1} from a point A on horizontal ground. The angle of projection is α above the horizontal. A horizontal platform is at height h metres above the ground. The ball moves freely under gravity until it hits the platform at the point B, as shown in Figure 2. The speed of the ball immediately before it hits the platform at B is 10 m s^{-1}.

(a) Find the value of h.

(4)

Given that $\sin \alpha = 0.85$,

(b) find the horizontal distance from A to B.

(8)

$\uparrow$ $s = h$ $\qquad$ $\vec{u}$ Dist $= x$

$u = 14\sin\alpha$ $\qquad$ speed $= 14\cos\alpha$

v

$a = -9.8$

t

$KE_A = \frac{1}{2}m(14)^2$ $\qquad$ $KE_B = \frac{1}{2}m(10)^2$

$PE_A = 0$ $\qquad$ $PE_B = mgh$

Total A $= 98m$ $\qquad$ Total $= 9.8mh + 50m$

CE $\Rightarrow$ $98\cancel{m} = 9.8\cancel{m}h + 50\cancel{m}$ $\Rightarrow 9.8h = 48 \Rightarrow h = \frac{240}{49}$

$\approx 4.9m$

b) $s = \frac{240}{49}$

$u = 14\sin\alpha = 11.9$

v

$a = -9.8$

$t =$

$s = ut + \frac{1}{2}at^2$

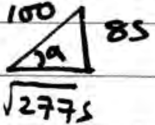
$\frac{240}{49} = 11.9t - 4.9t^2$

$4.9t^2 - 11.9t + \frac{240}{49} = 0$

$t_1 = 0.525$

$t_2 = 1.903$

$\overrightarrow{H}$ $x = 14\cos\alpha \times 1.903\ldots$



$x = 14\,\dfrac{\sqrt{2775}}{100} \times 1.903\ldots$

$\cos\alpha = \dfrac{\sqrt{2775}}{100}$

$x \simeq 14\,m$

7.

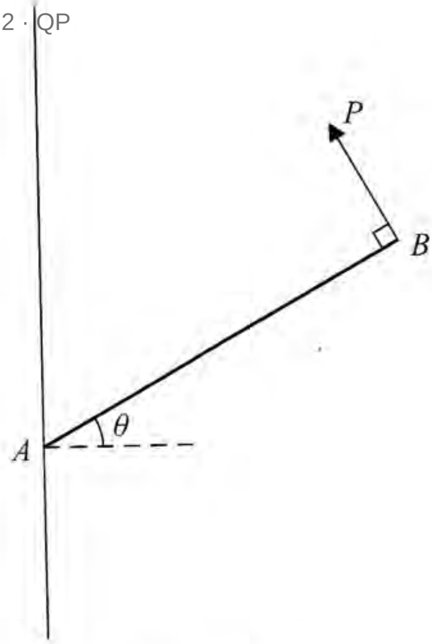


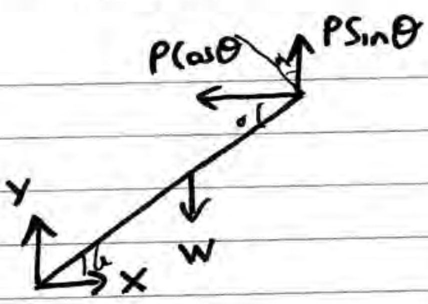
Figure 3

A uniform rod AB of weight W has its end A freely hinged to a point on a fixed vertical wall. The rod is held in equilibrium, at angle θ to the horizontal, by a force of magnitude P. The force acts perpendicular to the rod at B and in the same vertical plane as the rod, as shown in Figure 3. The rod is in a vertical plane perpendicular to the wall. The magnitude of the vertical component of the force exerted on the rod by the wall at A is Y.

(a) Show that $Y = \frac{W}{2}(2 - \cos^2\theta)$.
(6)

Given that $\theta = 45°$

(b) find the magnitude of the force exerted on the rod by the wall at A, giving your answer in terms of W.
(6)



$\uparrow = \downarrow \Rightarrow Y + P\sin\theta = W$

$\rightarrow = \leftarrow \Rightarrow X = P\cos\theta$

$A \circlearrowleft \quad W \times \frac{a}{2}\cos\theta = P\sin\theta \times a\cos\theta + P\cos\theta \times a\sin\theta$

$W \times \frac{\cancel{a}}{2}\cancel{\cos\theta} = 2\cancel{a}P\sin\theta\cancel{\cos\theta}$

$\frac{1}{2}W = 2P\sin\theta$

$\therefore P\sin\theta = \frac{1}{4}W$

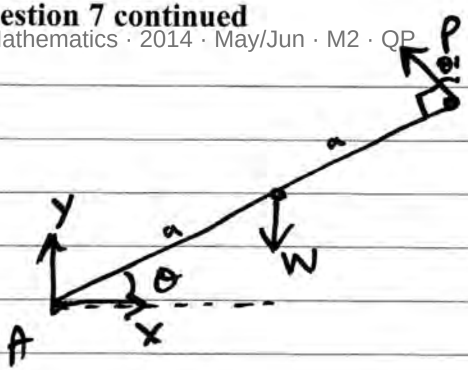
$a W\cos\theta = P \times 2a$

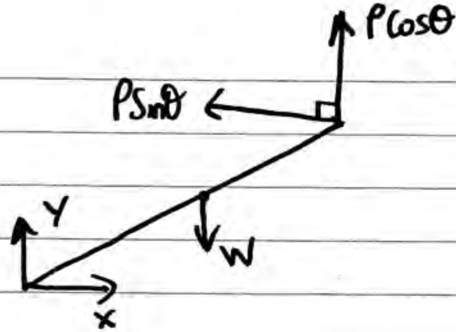
$P = \frac{1}{2}W\cos\theta$

$y + \frac{1}{2}W\cos\theta\sin\theta = \frac{2W}{2}$

$y = \frac{1}{2}W(2 - \sin\theta\cos\theta)$

Question 7 continued





$\uparrow = \downarrow = Y + P\cos\theta = W$

$Y = W - P\cos\theta$

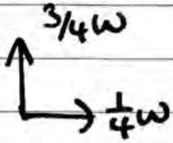
$A\circlearrowright = W \times \cancel{a}\cos\theta = P \times 2\cancel{a}$

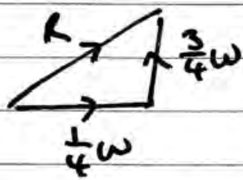
$P = \frac{1}{2}W\cos\theta \quad \Rightarrow \quad Y = W - \frac{1}{2}W\cos^2\theta$

$\therefore Y = \frac{1}{2}W(2 - \cos^2\theta)$ #

b) $\theta \Rightarrow \cos 45 = \frac{\sqrt{2}}{2} \quad \cos^2\theta = \frac{2}{4} = \frac{1}{2} \quad \Rightarrow \quad Y = \frac{3}{4}W$

$\rightarrow = \leftarrow \quad X = P\sin 45 = \left[\frac{1}{2}W\left(\frac{\sqrt{2}}{2}\right)\right]\left(\frac{\sqrt{2}}{2}\right) = \frac{1}{4}W$





$R^2 = \left(\frac{1}{4}W\right)^2 + \left(\frac{3}{4}W\right)^2$

$R^2 = \frac{5}{8}W^2 \quad \therefore R = \frac{1}{4}W\sqrt{10}$

8. The points A and B are 10 m apart on a line of greatest slope of a fixed rough inclined plane, with A above B. The plane is inclined at 25° to the horizontal. A particle P of mass 5 kg is released from rest at A and slides down the slope. As P passes B, it is moving with speed 7 m s^{-1}.

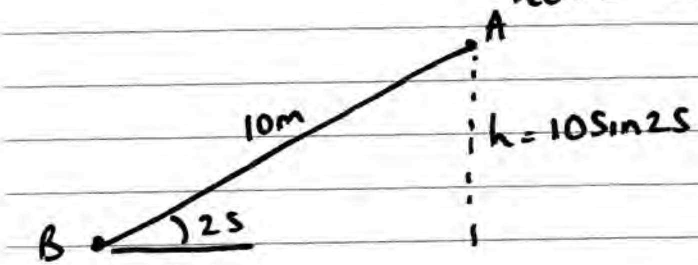
(a) Find, using the work-energy principle, the work done against friction as P moves from A to B.

(4)

(b) Find the coefficient of friction between the particle and the plane.

(5)

$KE = 0 \quad PE = 5g(10\sin 25)$



$PE = 0$

$KE = \frac{1}{2}(5)(7)^2$

$= 122.5$

Total A − Wd against f = total B

$207.0829\ldots - Wd_f = 122.5$

$\therefore Wd_f = 84.6N$

b) $Wd_f = f_{max} \times 10 \quad \therefore f_{max}\ 8.46\ldots$

$\therefore f_{max} = \mu NR$

$8.45829\ldots = \mu \times 5g\cos 25$

$\therefore \mu = 0.19$