M2 October 2016 (MA)

Q1a) $\sum m_i x_i = \bar{x} \sum m_i$

$m\begin{pmatrix}-3\\2\end{pmatrix} + 4m\begin{pmatrix}4\\3\end{pmatrix} + um\begin{pmatrix}6\\-4\end{pmatrix} = (5+u)m\begin{pmatrix}c\\0\end{pmatrix}$

consider $\underline{j}$ components: $2m + 12m - 4um = 0$

$14 = 4u$

$\boxed{u = \frac{7}{2}}$

b) Consider $\underline{i}$: $-3\cancel{m} + 16\cancel{m} + 6u\cancel{m} = (5+\frac{7}{2})\cancel{m}\, c$

$\frac{16 - 3 + 6(\frac{7}{2})}{5 + \frac{7}{2}} = \boxed{c = 4}$

Q2) $I = m(v-u)$

$\lambda \underline{i} - 2\lambda \underline{j} = 2(v - 3\underline{i})$

$\lambda \underline{i} - 2\lambda \underline{j} = 2v - 6\underline{i}$

$(6+\lambda)\underline{i} - 2\lambda \underline{j} = 2v$

$\therefore v = (3 + \frac{\lambda}{2})\underline{i} + (-\lambda)\underline{j}$ //

but $|v| = 6$. $\therefore \sqrt{(3+\frac{\lambda}{2})^2 + (-\lambda)^2} = 6$

$$(3+\tfrac{\lambda}{2})^2 + \lambda^2 = 36$$

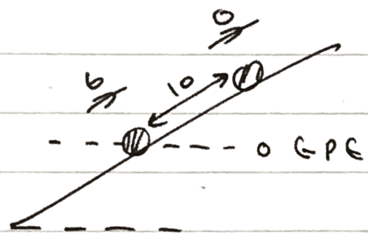
$$\lambda^2 + \frac{\lambda^2}{4} + 3\lambda + 9 = 36$$

$$\frac{5}{4}\lambda^2 + 3\lambda - 27 = 0$$

By Quadratic formula :

$$\boxed{\lambda = \frac{18}{5} \quad \lambda = -6}$$

Q3a)



$\sin\alpha = \frac{1}{7}$

$\therefore \cos\alpha = \sqrt{1-(\frac{1}{7})^2} = \frac{4\sqrt{3}}{7}$

Initial energy : $KE = \frac{1}{2}(4)(6)^2 = 72\text{ J}$
$GPE = 0$

Final energy : $KE = 0$
$GPE = 4g(10\sin\alpha) = \frac{40g}{7}$.

Initial energy − Final energy = W.D due to friction

$72 - \frac{40g}{7}$ = W.D against friction = $\boxed{16\text{ J}}$

b) At B : $KE = 0$
$GPE = \frac{40g}{7}$

At A : $KE = 2v^2$
$GPE = 0$

£ W.D due to friction = 16

so $\frac{40g}{7} = 2v^2 + 16$

$v^2 = \frac{\frac{40g}{7} - 16}{2} = 20$

so $v = \sqrt{20} = \boxed{4.47 ms^{-1}}$

Q4a) $\frac{dr}{dt} = v = (3t^2 - 9t - 24)\underline{i} + (-3t^2 + 6t + 12)\underline{j}$

When P moves parallel to $-\underline{i} - \underline{j}$, components of velocity (i and j) are equal.

So $3t^2 - 9t - 24 = -3t^2 + 6t + 12$

$6t^2 - 15t - 36 = 0$

By Quadratic formula: $t = 4$, $t = -\frac{3}{2}$

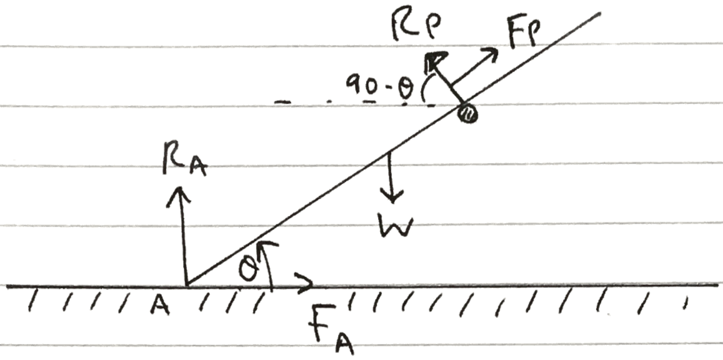
$t > 0$ so $t = \boxed{4 = T}$

b) $a = \frac{dv}{dt} = (6t - 9)\underline{i} + (-6t + 6)\underline{j}$

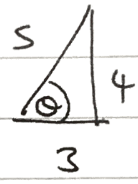
$|a| = \sqrt{(6 \times 4 - 9)^2 + (-6(4) + 6)^2} = 3\sqrt{61}$

$= \boxed{23.4 m/s}$

(Q5a)



$\tan\theta = \frac{4}{3}$



$\cos\theta = \frac{3}{5}$

$\sin\theta = \frac{4}{5}$

$M(A): \quad R_p(5) = W\cos\theta(4)$

$R_p = \frac{4W\cos\theta}{5} = \frac{4}{5} \times \frac{3}{5} \times W = \boxed{0.48W}$

b) $R(\updownarrow): \quad R_p\cos\theta + R_A + F_p\sin\theta = W \sim ①$

$R(\leftrightarrow): \quad F_A + F_p\cos\theta = R_p\sin\theta \sim ②$

$F_p = \frac{1}{4}R_p$

$②: \quad F_A + \frac{1}{4}R_p\left(\frac{3}{5}\right) = R_p\left(\frac{4}{5}\right)$

$F_A = \frac{13}{20}R_p$ //

$\downarrow$

$①: \quad R_p\cos\theta + \frac{F_A}{\mu} + F_p\sin\theta = W$

$(F_A = \mu R_A)$

$\Rightarrow \quad R_p\left(\frac{3}{5}\right) + \frac{13R_p}{20\mu} + \frac{1}{4}R_p\left(\frac{4}{5}\right) = W$

$\Rightarrow \quad R_p\left(\frac{3}{5} + \frac{13}{20\mu} + \frac{1}{5}\right) = W$

$$0.48\cancel{W}\left(\frac{4}{5}+\frac{13}{20\mu}\right)=\cancel{W}$$

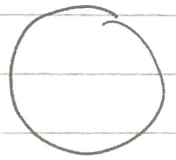
$$\frac{4}{5}+\frac{13}{20\mu}=\frac{25}{12}$$

$$\frac{13}{20\mu}=\frac{77}{60}$$

$$\mu\left(\frac{20}{13}\right)=\frac{60}{77}$$

$$\mu=\frac{\frac{60}{77}}{\frac{20}{13}}=\boxed{0.506}=\frac{39}{77}$$

Q6a)

	Shape	Mass (area)	Distance of c.o.m from R
(+)		$\pi(8a)^2 = \boxed{64\pi a^2}$	$\boxed{8a}$
(−)		$\boxed{\pi(a)^2}$	$\boxed{12a}$
(−)		$\boxed{4\pi a^2}$	$\boxed{4a}$
(=)		$\boxed{59\pi a^2}$	$\boxed{\bar{x}}$

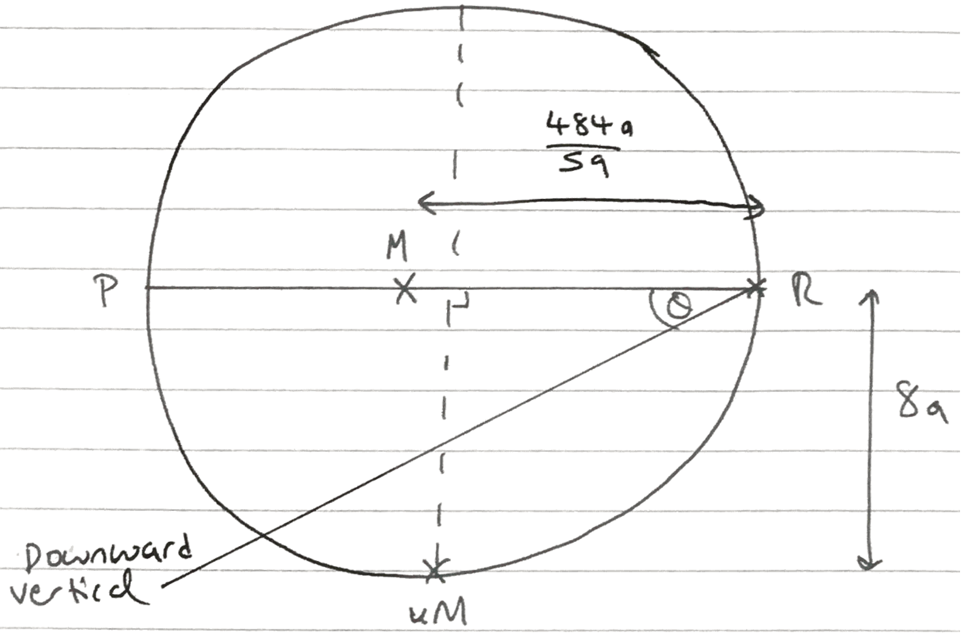
$$\sum_i m_i x_i = x\sum_i m_i$$

$$64(8a)-1(12a)-4(4a)=59(\bar{x})$$

$$484=59\bar{x}$$

hence $\bar{x} = \boxed{\frac{484}{59}a}$

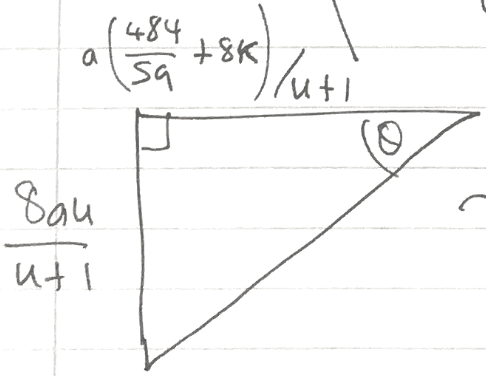
b) $\tan\theta = \frac{1}{4}$



$$\bar{x}\sum m_i = \sum m_i x_i$$

$$M\begin{pmatrix}\frac{484a}{59}\\ 0\end{pmatrix} + uM\begin{pmatrix}8a\\ -8a\end{pmatrix} = (u+1)M\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}$$

$$\begin{pmatrix}\dfrac{a\left(\frac{484}{59}+8u\right)}{u+1}\\[2ex] \dfrac{-8au}{u+1}\end{pmatrix} = \begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}$$



$$\rightsquigarrow \tan\theta = \frac{8u}{\frac{484}{59}+8u} = \frac{1}{4}$$

$\Rightarrow \frac{484}{59} + 8u = 324$

$\Rightarrow u = \frac{\frac{484}{59}}{24} = \boxed{\frac{121}{354}}$

Q7)

expressing position as a vector relative to O,

$r = \begin{pmatrix} 3t \\ \lambda t - \frac{9.8t^2}{2} \end{pmatrix}$ $\left(\uparrow\right)$ $\left.\begin{matrix} s = s \\ u = \lambda \\ v = \\ a = -g \\ t = t \end{matrix}\right\}$ $\underline{\underline{s = \lambda t - \frac{g}{2}t^2}}$

$\therefore v = \dot{r} = \begin{pmatrix} 3 \\ \lambda - 9.8t \end{pmatrix}$ //

KE at O $= \frac{1}{2}m(9 + \lambda^2)$

KE at A $= \frac{1}{2}m(25)$

KE at A $= \frac{1}{2} \times$ KE at O

$\therefore \frac{25\cancel{m}}{2} = \frac{\cancel{m}}{2}(9 + \lambda^2) \times \frac{1}{2}$

$$\frac{25}{2} = \frac{1}{4}(9+\lambda^2)$$

$$\lambda^2 + 9 = 50$$

$$\lambda^2 = 41$$

$$\lambda = \pm\sqrt{41}$$

$$\lambda > 0 \quad \therefore \quad \lambda = \sqrt{41}$$

at A, $\quad r = 3\underline{i} - 4\underline{j} \quad$ and $\quad \dot{r} = \begin{pmatrix} 3 \\ \lambda - 9.8t \end{pmatrix}$

$$\therefore -4 = \lambda - 9.8t$$

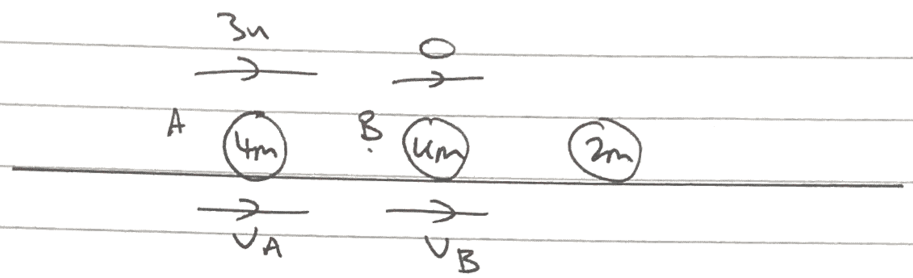
$$t = \frac{\lambda+4}{9.8} = \frac{4+\sqrt{41}}{9.8} = 1.062\ldots$$

at $t = 1.062$, $\quad r = \begin{pmatrix} 3 \times 1.062 \\ \sqrt{41}(1.062) - 4.9(1.062)^2 \end{pmatrix}$

$$r = \begin{pmatrix} 3.18 \\ 1.27 \end{pmatrix}$$

$$r = \boxed{\begin{pmatrix} 3.2 \\ 1.3 \end{pmatrix}}$$

Q8a)



$\overrightarrow{\text{C.L.M}}$: $4m(3u) = 4mV_A + umV_B$

$12u = 4V_A + uV_B \sim ①$

$\overrightarrow{\text{N.T.L}}$: $e = \frac{2}{3} = \frac{V_B - V_A}{3u}$

$\therefore 2u = V_B - V_A \sim ②$

$\times 4$: $8u = 4V_B - 4V_A$

$+①$: $+[12u = 4V_A + uV_B]$

$20u = (4+u)V_B$

$\therefore V_B = \frac{20u}{4+u}$

② : so $V_A = V_B - 2u = \frac{20u}{4+u} - 2u$

$V_A = \frac{20u - 2u(4+u)}{4+u}$

$V_A = \frac{12u - 2uu}{u+4}$

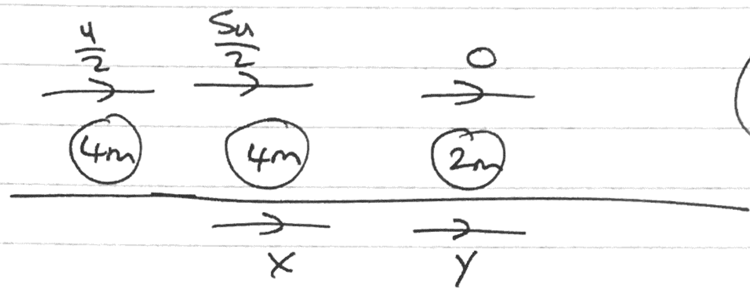
hence $V_A = \dfrac{2u(6-u)}{u+4}$

so speed $= \left| \dfrac{2u(6-u)}{4+k} \right|$

b) $V_A > 0$: $2u(6-u) > 0$

$0 < \boxed{u < 6}$

c)



(for B/C)

$\overrightarrow{C \cdot L \cdot M}$: $4m\left(\frac{5u}{2}\right) = 4mx + 2my$

$10u = 4x + 2y$

$5u = 2x + y$ // — ①

$\overrightarrow{N \cdot L \cdot L}$: $\dfrac{2}{3} = \dfrac{y-x}{\frac{5u}{2}}$

$\therefore \frac{5}{3}u = y - x$ // — ②

solve ① and ② for x.

① – ② : $\left(5u - \frac{5u}{3}\right) = 3x$

$x = \frac{10u}{9}$ //

$\frac{10u}{9} > \frac{u}{2}$ hence A won't collide with B.