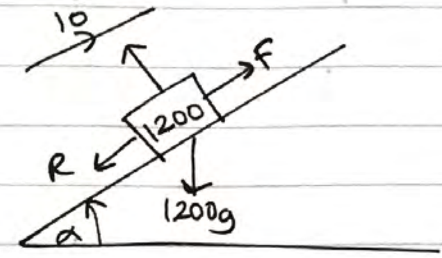
M2 Jan 17 IAL (MA)

Q1a)

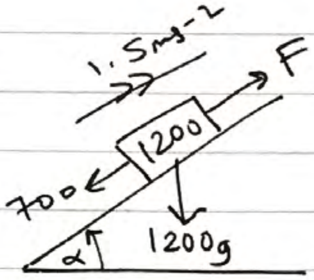


$$P = Fv$$
$$11760 = F(10)$$
$$F = 1176\text{ N}$$

N2L $\nearrow^{+}$ (Parallel to slope): $F - R - 1200g\sin\alpha = 1200(0)$

$$R = 1176 - \frac{1200g}{15} = \boxed{392\text{N}}$$

b)



$$P = Fv$$
$$50000 = Fv$$
$$F = \frac{50000}{v}$$

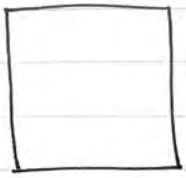
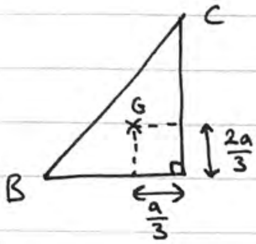
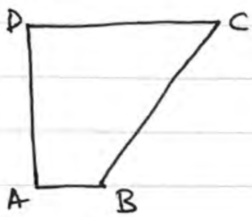
N2L $\nearrow^{+}$ (Parallel to slope): $F - 1200g\sin\alpha - 700 = 1200(1.5)$

$$\frac{50000}{v} = 1200(1.5) + 700 + \frac{1200g}{15}$$

$$\frac{50000}{v} = 3284$$

$$\therefore v = \frac{50000}{3284} = \boxed{15.2\text{ m/s}}$$

Q2ai/ii)

	Shape	Area	Distance of c.o.m from... AD	AB
+		$\boxed{4a^2}$	$\boxed{a}$	$\boxed{a}$
−		$\frac{1}{2} \times a \times 2a = \boxed{a^2}$	$2a - \frac{a}{3} = \boxed{\frac{5a}{3}}$	$\boxed{\frac{2a}{3}}$
=		$\boxed{3a^2}$	$\boxed{\bar{x}}$	$\boxed{\bar{y}}$

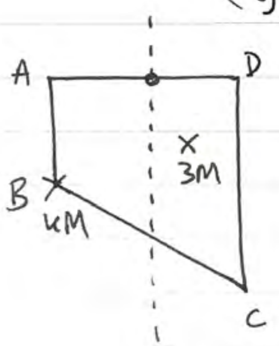
$$\underline{\sum m_i x_i = \bar{x} \sum m}$$

$$\Rightarrow \quad 4a^2 \begin{pmatrix} a \\ a \end{pmatrix} - a^2 \begin{pmatrix} 5a/3 \\ 2a/3 \end{pmatrix} = 3a^2 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

$$\Rightarrow \quad \begin{pmatrix} 4a - 5a/3 \\ 4a - 2a/3 \end{pmatrix} = 3 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 7a/3 \\ 10a/3 \end{pmatrix}$$

$$\therefore \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 7a/9 \\ 10a/9 \end{pmatrix} \quad //$$

so distance from... $\boxed{\begin{matrix} AB = 10a/9 \\ AD = 7a/9 \end{matrix}}$

b) 

<u>taking moments about midpoint of AD</u>

$$kM(a) = 3M\left(\frac{10a}{9} - a\right)$$

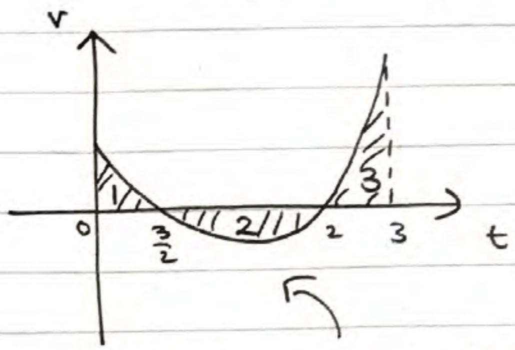
$$k(\cancel{a}) = 3\cancel{a}\left(\frac{1}{9}\right)$$

$$\therefore k = \frac{3}{9} = \boxed{\frac{1}{3}}$$

Q3) $v = (2t-3)(t-2) \rightarrow$ at $v=0$: $t = \frac{3}{2}$, $t = 2$ //

$v = 2t^2 - 7t + 6$

$x = \int (v)\, dt$.



Area 1 $= \int_0^{\frac{3}{2}} (2t^2 - 7t + 6)\, dt$

total distance = Area 1 + Area 2 + Area 3

$= \left[\frac{2t^3}{3} - \frac{7t^2}{2} + 6t\right]_0^{\frac{3}{2}} = \frac{27}{8} - 0 = \frac{27}{8}$ //

Area 2 $= \int_{\frac{3}{2}}^{2} (2t^2 - 7t + 6)\, dt = \left[\frac{2t^3}{3} - \frac{7}{2}t^2 + 6t\right]_{\frac{3}{2}}^{2}$

$= \left[\frac{10}{3}\right] - \left[\frac{27}{8}\right] = -\frac{1}{24}$ // $\rightarrow$ distance $= \frac{1}{24}$ //

Area 3 $= \int_{2}^{3} \left[2t^2 - 7t + 6\right] dt = \left[\frac{2t^3}{3} - \frac{7}{2}t^2 + 6t\right]_2^3$

$= \left[\frac{9}{2}\right] - \left[\frac{10}{3}\right] = \frac{7}{6}$ //

Total distance $= \frac{27}{8} + \left(\frac{1}{24}\right) + \frac{7}{6} = \boxed{\frac{55}{12}\text{ m}}$

Q4a) $I = m(v-u)$

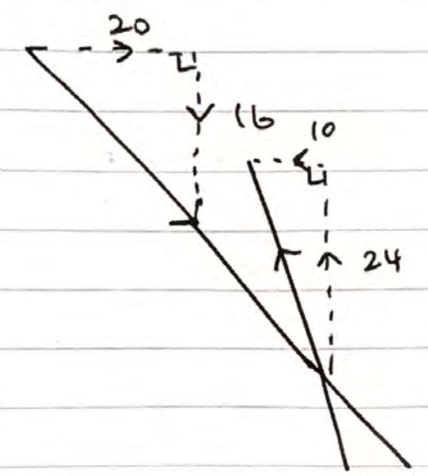
$-6\underline{i} + 8\underline{j} = 0.2(v - (20\underline{i} - 16\underline{j}))$

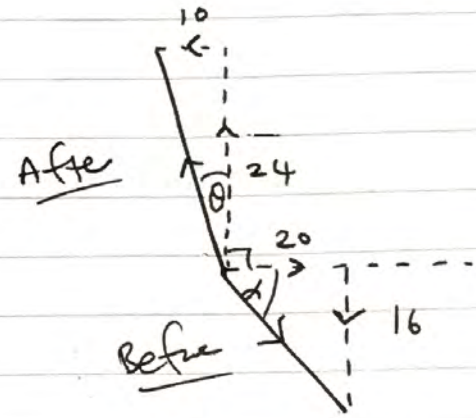
$-6\underline{i} + 8\underline{j} = 0.2(v - 20\underline{i} + 16\underline{j})$

$\times 5$: $-30\underline{i} + 40\underline{j} = v - 20\underline{i} + 16\underline{j}$

$v = -10\underline{i} + 24\underline{j}$ $\therefore$ speed $= \sqrt{10^2 + 24^2}$
$= \boxed{26\text{ms}^{-1}}$

b)





$\tan\alpha = \frac{16}{20} = \frac{4}{5}$

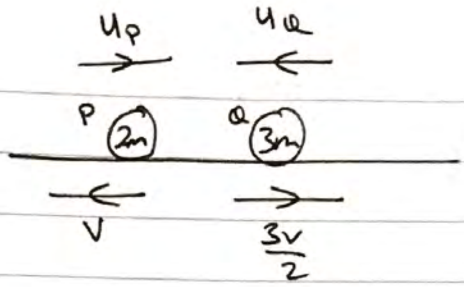
$\alpha = \arctan\left(\frac{4}{5}\right)$

$\tan\theta = \frac{10}{24} = \frac{5}{12}$

$\theta = \arctan\left(\frac{5}{12}\right)$

$\therefore$ angle required
$= \arctan\frac{4}{5} + \arctan\frac{5}{12} + 90^\circ$
$= \boxed{151.3^\circ}$
$= 151^\circ$ to 3s.f.

Q5a i)



$e = \frac{1}{5}$

$\overrightarrow{\text{C.L.M}}$: $2m(u_P) - 3m(u_Q) = 3m\left(\frac{3v}{2}\right) - 2m(v)$

$2u_P - 3u_Q = \frac{5v}{2}$ ~ (1)

$\overrightarrow{\text{N.L.L}}$: $\frac{1}{5} = \frac{\frac{3v}{2} + v}{u_P + u_Q}$ ← speed of seperation ← speed of approach

$\Rightarrow \frac{1}{5}(u_P) + \frac{1}{5}(u_Q) = \frac{3v}{2} + v$

$\Rightarrow u_P + u_Q = \frac{25v}{2}$

$\xRightarrow{\times 3}$ $3u_P + 3u_Q = \frac{75v}{2}$ ~ (2)

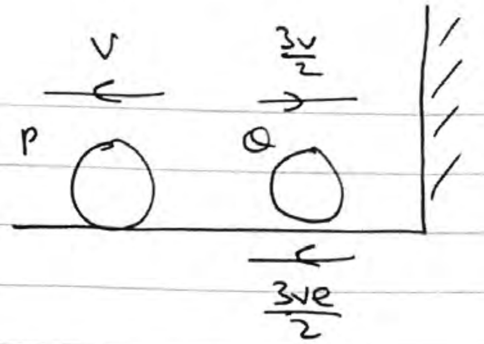
(1) + (2) : $2u_P + 3u_P + 3u_Q - 3u_Q = \left(\frac{75}{2} + \frac{5}{2}\right)v$

$5u_P = 40v$

$\therefore u_P = \boxed{8v}$ → now sub into (1) or (2)

from (2), $u_P + u_Q = \frac{25v}{2}$

$\therefore u_Q = \frac{25v}{2} - 8v = \boxed{\frac{9v}{2}}$

b) 

By N.L.L, Q rebounds with speed $\frac{3v}{2} \times e$ //

Because there is a further collision, $\frac{3ve}{2} > v$

$\Rightarrow \frac{3e}{2} > 1 \quad \therefore e > \frac{2}{3}$ //

$\Rightarrow \boxed{1 \geq e > \frac{2}{3}}$

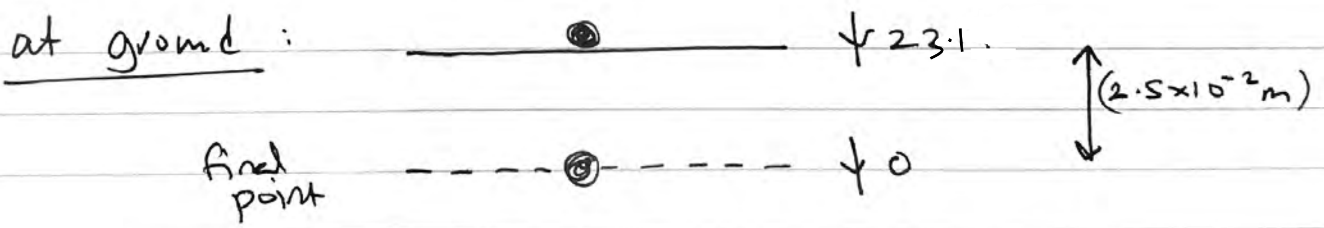
Q6) from point of projection to ground,

at point of projection: $KE = \frac{1}{2}(0.6)(22.4^2) = 150.53J$
$GPE = 0.6g(1.5) = 8.82J$

at ground: $KE = \frac{1}{2}(0.6)v^2 = 0.3v^2$
$GPE = 0$

(taking ground as 0 GPE level)

Conservation of Energy: $150.53 + 8.82 = 0.3v^2$
$v^2 = 531.167..$
$v = 23.05$ // = speed at ground.



Q6 cont) at ground : $KE = \frac{1}{2}(0.6)(23.05)^2 = 159.39$

(not final position) $GPE = 0.6g(2.5\times10^{-2}) = 0.147$

$\therefore$ Work Done by $R = 159.39 + 0.147$

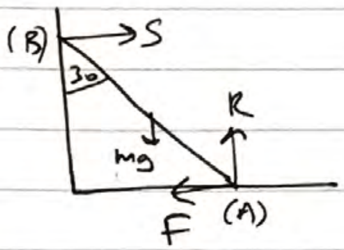
$= 159.537$ //

$\therefore \quad R \times (2.5\times10^{-2}) = 159.537$

$\Rightarrow \quad R = \dfrac{159.537}{0.025} = \boxed{6380N}$ (3s.f)

[Work Done = F×d]

Q7a)



$M(A): \quad mg\sin30(d) = S\sin60(2\cancel{d})$

$S = \dfrac{mg\sin30}{2\sin60} = \boxed{\dfrac{\sqrt{3}\,mg}{6}}$

b) $Rf(\updownarrow): \quad R = mg$

$Rf(\leftrightarrow): \quad S = F$

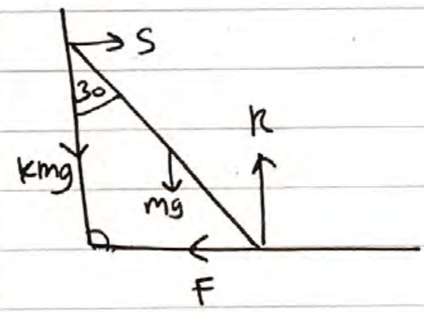
$F \leq \mu R$

$S \leq \mu mg$

$\cancel{mg}\dfrac{\sqrt{3}}{6} \leq \mu \cancel{mg}$

$\therefore \quad \mu \geqslant \dfrac{\sqrt{3}}{6}$

c)



$Rf(\updownarrow): \quad R = mg + kmg = mg(k+1)$

$Rf(\leftrightarrow): \quad S = F$

$M(A): \quad mg\sin30(d) + kmg\sin30(2d) = S\sin60(2d)$

$\xrightarrow{\times 2}: \quad mg + 2kmg = (2\sqrt{3})F \qquad (F = S)$

$F = \mu R$ (limiting equilibrium)

$\Rightarrow \quad F = \dfrac{mg(1+2u)}{2\sqrt{3}} = \mu R$

but $R = mg(u+1)$ and $\mu = \dfrac{\sqrt{3}}{5}$

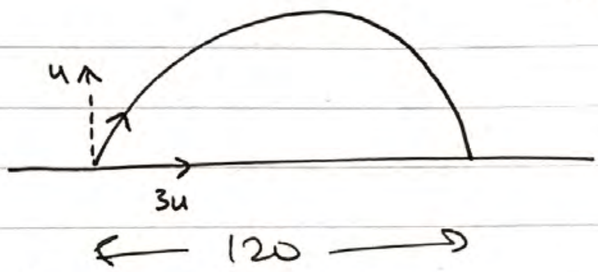
$\Rightarrow \quad \dfrac{\cancel{mg}(1+2u)}{2\sqrt{3}} = \dfrac{\sqrt{3}}{5}\cancel{mg}(1+u)$

$\xrightarrow{\times 2\sqrt{3}} \quad 1 + 2u = \frac{6}{5}(1+u)$

$\Rightarrow \quad 1 + 2u = \frac{6}{5} + \frac{6}{5}u$

$\Rightarrow \quad \frac{4}{5}u = \frac{1}{5} \quad \therefore u = \dfrac{\frac{1}{5}}{\frac{4}{5}} = \boxed{\frac{1}{4}}$

Q8a)



until highest point

$(\uparrow)$ $s =$, $u = u$, $v = 0$, $a = -g$, $t = \frac{t}{2}$

$v = u + at$

$0 = u - \frac{gt}{2}$

$\therefore t = \frac{2u}{g}$ = time for total journey

$(\rightarrow)$ $s = ut$: $120 = 3u \times \frac{2u}{g}$

$u^2 = \dfrac{120}{\frac{6}{9.8}} = 196$

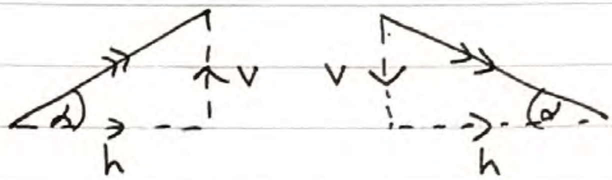
$\therefore u = \sqrt{196} = \boxed{14}$

b) $\vec{v} = \vec{u} = 3u$ (no acceleration $\rightarrow$)

$\downarrow v = u \uparrow$ due to symmetry of the motion

$\therefore$ speed $= \sqrt{(u)^2 + (3u)^2} = 14\sqrt{10} = \boxed{44.3}$ m/s

c) $\tan\alpha = \frac{1}{4}$



$\Rightarrow \frac{v}{h} = \frac{1}{4} \quad \therefore v = \frac{1}{4}h$ or $4v = h$

[So $\tan\alpha = \frac{1}{4}$ means that the horizontal component of velocity must have a magnitude 4 times that of the vertical component.]

expressing the velocity as a vector,

$$v = \begin{pmatrix} 3u \\ u - gt \end{pmatrix}$$

$\sim \vec{speed}$ is constant

$\sim$ initial speed $u\uparrow$ under influence of g.

$$|u - gt| = \frac{1}{4}(3u)$$

$u - gt = \frac{3u}{4}$ and $-(u - gt) = \frac{3u}{4}$

$t = \frac{u}{4g} = \boxed{\frac{5}{14}}$ $\qquad -u + gt = \frac{3u}{4}$

$$t = \frac{7u}{4g} = \boxed{\frac{5}{2}}$$