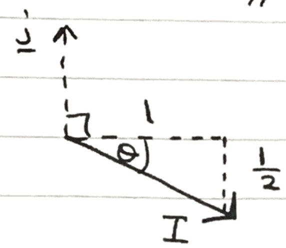
M2 June 2017 (MA)

Q1a) $I = m(v-u) = 0.5(2\underline{i}+3\underline{j} - 4\underline{j}) = 0.5(2\underline{i} - \underline{j})$

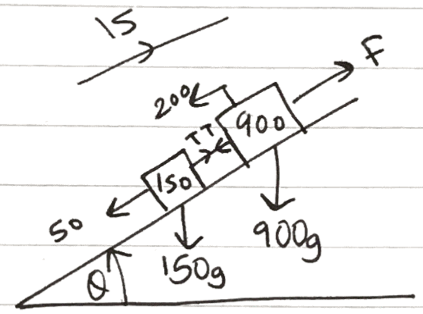
$= \underline{i} - \frac{1}{2}\underline{j}$ // $\quad \therefore |I| = \sqrt{1^2+(\frac{1}{2})^2} = \boxed{\frac{\sqrt{5}}{2}\text{ Ns}}$

b) 

$\tan\theta = \frac{\frac{1}{2}}{1} = \frac{1}{2}$

$\therefore \theta = \tan^{-1}(\frac{1}{2}) = 26.6^\circ$

so angle between I and $\underline{j}$ $= 90 + 26.6^\circ$
$= 116.6^\circ$
$= \boxed{117^\circ}$

Q2a) 

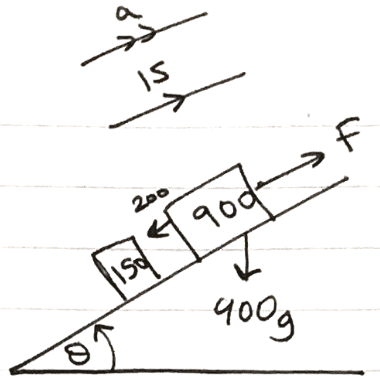
$P = Fv$
$P = F \cdot 15$

$\nearrow +$

N2L (system): $F - 900g\sin\theta - 150g\sin\theta - 250 = 1050(0)$

$(\sin\theta = \frac{1}{9})$ $\quad F = \frac{900g + 150g}{9} + 250 = \frac{4180}{3}\text{ N}$ //

$\therefore P = 15 \cdot \frac{4180}{3} = 20900\text{W} = \boxed{20.9\text{kW}}$



from (a), $F = \frac{4180}{3}$.

N2L (Truck): $\frac{4180}{3} - 200 - \frac{900g}{9} = 900a$

$\therefore a = \dfrac{\frac{4180}{3} - 200 - 100g}{900} = \boxed{0.24\,ms^{-2}}$

c) Trailer: Resistance $= 50\,N$.
Total Initial Energy $= \frac{1}{2} \cdot 150 \cdot (15)^2 = 16875\,J$

Initially
Energy (KE) $= 16875\,J$

Finally
$KE = 0$
$GPE = 150 g d \sin\theta$

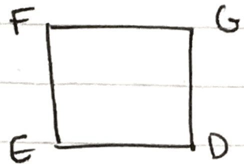
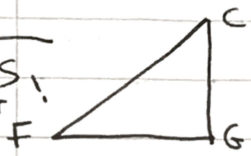
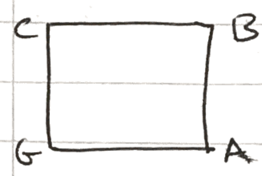
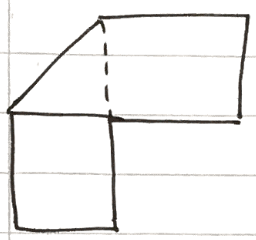
$+$ W.D by Resistance $= 50d$. $(F \times d)$

By C.O.E: $16875 = \frac{150gd}{9} + 50d$

$d\left(\frac{150g}{9} + 50\right) = 16875$

$d = \dfrac{16875}{50 + \frac{150g}{9}} = \boxed{79.1\,m}$

Q3a)

Shape	Area	Distance of c.o.m from EF	Distance of c.o.m from ED
	$4a^2$	a	a
	$2\times 2a^2$	$\frac{2}{3}\times 2a = \frac{4a}{3}$	$2a + 2a/3 = \frac{8a}{3}$
	$4a^2$	$3a$	$3a$
	$12a^2$	$\bar{x}$	$\bar{y}$

x2 because this bit is folded over!

$$\sum m_i x_i = \bar{x}\sum m_i$$

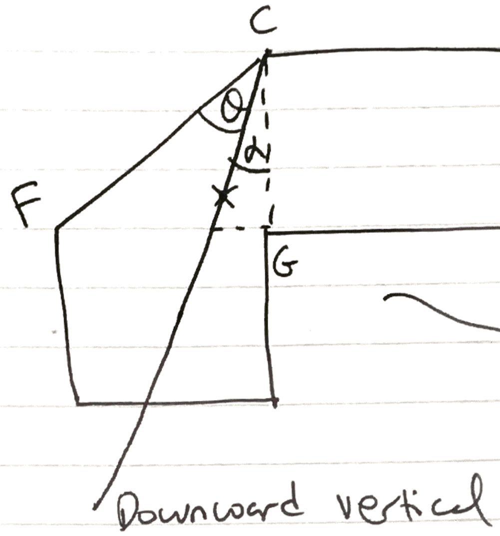
$$\Rightarrow 4\cancel{a^2}\begin{pmatrix} a \\ a \end{pmatrix} + 2\times 2\cancel{a^2}\begin{pmatrix} 4a/3 \\ 8a/3 \end{pmatrix} + 4\cancel{a^2}\begin{pmatrix} 3a \\ 3a \end{pmatrix} = 12\cancel{a^2}\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 4a + \frac{16a}{3} + 12a \\ 4a + \frac{32a}{3} + 12a \end{pmatrix} = 12\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

$$\Rightarrow 12\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 64a/3 \\ 80a/3 \end{pmatrix} \quad \therefore \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 16a/9 \\ 20a/9 \end{pmatrix}$$

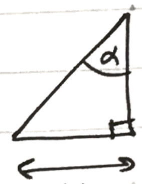
hence $\bar{x}$ = distance from EF $= \frac{16a}{9}$

b)



let θ be angle required,

let α be $\angle GCF - \theta$.



$4a - 20a/9 = \frac{16a}{9}$

$2a - \frac{16a}{9} = \frac{2a}{9}$

[notice $\theta + \alpha = 45$.]

$\therefore \tan\alpha = \dfrac{\frac{2}{9}}{\frac{16}{9}} = \frac{1}{8}$

$\therefore \alpha = \tan^{-1}\left(\frac{1}{8}\right)$

$\Rightarrow \theta = 45 - \alpha = 45 - \tan^{-1}\left(\frac{1}{8}\right) = \boxed{37.9^\circ}$

Q4a) $v = 3t^2 - 16t + 21 = 0$

$(3t-7)(t-3) = 0$

$t = \frac{7}{3} \quad , t = 3$

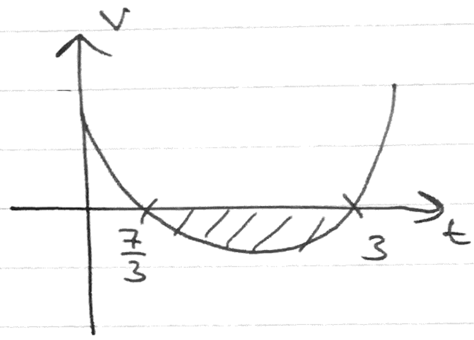
$\therefore \boxed{t_1 = \frac{7}{3}}$ and $\boxed{t_2 = 3}$

b) $a = \frac{dv}{dt} = 6t - 16$

$\underline{t = \frac{7}{3}}$: $a = 6\left(\frac{7}{3}\right) - 16) = -2 ms^{-2}$

$\therefore |a| = \boxed{2 ms^{-2}}$

c) distance $= \left| \int_{\frac{7}{3}}^{3} [v]\, dt \right|$



$\int_{\frac{7}{3}}^{3} (v) dt = \int_{\frac{7}{3}}^{3} [3t^2 - 16t + 21] dt$

$= \left[t^3 - 8t^2 + 21t \right]_{\frac{7}{3}}^{3} = [27 - 72 + 63] - \left[\frac{490}{27}\right]$

$= \frac{-4}{27}$

$\therefore$ distance $= \left| \frac{-4}{27} \right| = \boxed{\frac{4}{27} \text{ m}}$

d) $x = \int (v)\,dt = t^3 - 8t^2 + 21t + c$

$t = 0,\ x = 0$: $0 = c$

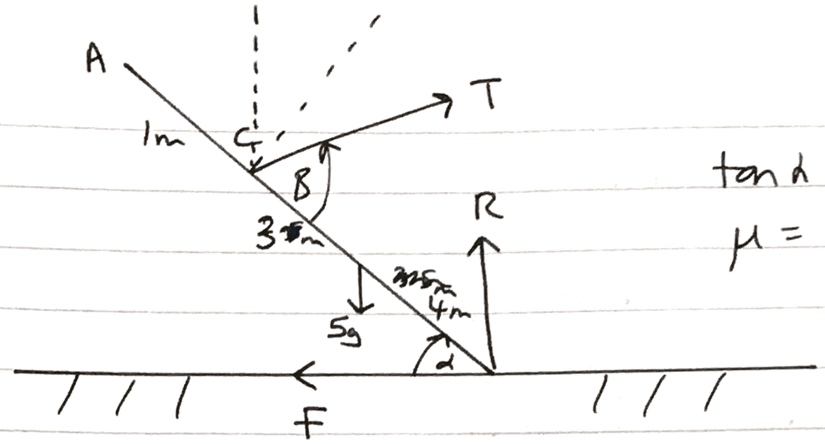
$\therefore\ x = t^3 - 8t^2 + 21t$

at $t = 3$: $x = (3)^3 - 8(3^2) + 21(3)$
$= 18$

$x > 0$ at $t = 3$.
and after $t = 3$, v is an increasing function indefinitely. (see graph on previous page).

$\therefore$ P does not return to O.

Q5a)



$\tan\alpha = \frac{3}{4}$

$\mu = \frac{2}{3} \longrightarrow F = \frac{2}{3}R$

$M(C): R\cos\alpha(7) = 5g\cos\alpha(3) + F\sin\alpha(7)$

$\xrightarrow{\div\cos\alpha} \quad 7R = 15g + 7F\tan\alpha$

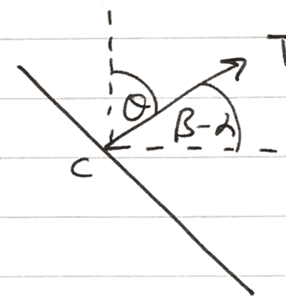
$\Rightarrow 7R = 15g + 7F\left(\frac{3}{4}\right)$

$F = \frac{2}{3}R: \quad 7R = 15g + \frac{7}{2}R$

$R = \frac{15g}{\frac{7}{2}} = \frac{30g}{7} = \boxed{42N}$

b) $R(\updownarrow): T\cos\theta = 5g - R$

$\therefore T\cos\theta = 7$ ~ 1



$R(\leftrightarrow): T\cos(\beta - \alpha) = F$

$\theta = 90 - (\beta - \alpha)$

$F = \frac{2}{3} \cdot 42 = 28N$

$\therefore T\cos(\beta - \alpha) = 28$ ~ 2

but $\theta = 90 - (\beta - \alpha)$: 1 becomes: $T\cos(90 - (\beta - \alpha)) = 7$

$\Rightarrow T\sin(\beta - \alpha) = 7$

Our 2 equations are:

$$T\sin(\beta-\alpha) = 7 \quad \text{— } 1$$

and $$T\cos(\beta-\alpha) = 28 \quad \text{— } 2$$

$$\frac{1}{2} : \frac{\cancel{T}\sin(\beta-\alpha)}{\cancel{T}\cos(\beta-\alpha)} = \frac{7}{28}$$

$$\Rightarrow \tan(\beta-\alpha) = \frac{7}{28}$$

$$\Rightarrow \tan^{-1}\left(\frac{7}{28}\right) = \beta-\alpha$$

$$\Rightarrow \beta = \tan^{-1}\left(\frac{7}{28}\right) + \alpha$$

$$\Rightarrow \beta = \tan^{-1}\left(\frac{7}{28}\right) + \tan^{-1}\left(\frac{3}{4}\right)$$

$$\boxed{\beta = 50.9^\circ}$$

Q6ai) $s = ut$ for both P and Q

P: $x = (30\cos 60)(2) = \overrightarrow{\text{distance}}$ to collision.

$\therefore$ Q travels $\overleftarrow{(40 - 60\cos 60)}$ m to collision.

$\Rightarrow (40 - 60\cos 60) = q\cos\theta\,(2)$

$\Rightarrow q\cos\theta = 20 - 30\cos 60$ ~ 1

both P and Q travel the same vertical distance.

$$\underbrace{(30\sin 60)(2) - \tfrac{g}{2}(4)}_{\left[s = ut + \frac{1}{2}at^2\right] \text{ for P}} = \underbrace{(q\sin\theta)(2) - \tfrac{g}{2}(4)}_{\text{for Q}}$$

$\Rightarrow 60\sin 60 = 2q\sin\theta$

$\Rightarrow \therefore q\sin\theta = 30\sin 60$ ~ 2

$\dfrac{2}{1}$: $\dfrac{q\sin\theta}{q\cos\theta} = \dfrac{30\sin 60}{20 - 30\cos 60} = \tan\theta = 3\sqrt{3}$

$\therefore \theta = \tan^{-1}(3\sqrt{3}) = \boxed{79.1°}$

$\therefore q = \dfrac{30\sin 60}{\sin(79.1)} = \boxed{26.5\,\text{m/s}}$

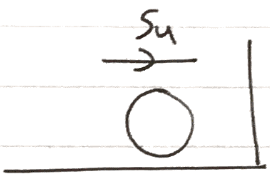
b) <u>For P</u>

$\uparrow^{+}$
$$\left.\begin{aligned} s &= \\ u &= 30\sin 60 \\ v &= v \\ a &= -g \\ t &= 2 \end{aligned}\right\} \qquad \begin{aligned} v &= u + at \\ \uparrow v &= 30\sin 60 - 2g \; // \end{aligned}$$

$$\therefore \ |\text{speed}_P| = \sqrt{(30\sin 60 - 2g)^2 + (30\cos 60)^2}$$

$$= 16.3 \text{ m/s} \; //$$

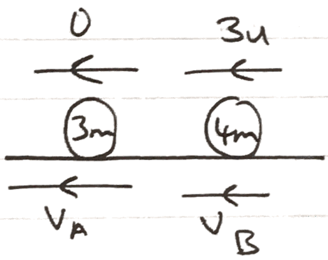
Q7a)



$5u \times \frac{3}{5} = 3u$

$$I = 4m(v - u)$$
$$= 4m(3u - -5u)$$
$$= \boxed{32mu}$$

b)



$\overleftarrow{\text{C.L.M}}$: $4\cancel{m}(3u) = 3\cancel{m}(V_A) + 4\cancel{m}(V_B)$

$$\Rightarrow \ 12u = 3V_A + 4V_B \quad (1)$$

$\overleftarrow{\text{N.I.L}}$: $e = \dfrac{V_A - V_B}{3u}$

$$\therefore \ 3ue = V_A - V_B$$

$$\Rightarrow 9ue = 3V_A - 3V_B \quad (2)$$

$$-\begin{bmatrix} 12u = 3V_A + 4V_B \\ 9ue = 3V_A - 3V_B \end{bmatrix}$$

$$12u - 9ue = 7V_B$$

$$7V_B = 3u(4-3e)$$

$$V_B = \frac{3u}{7}(4-3e)$$

$$V_A = 3ue + V_B = 3ue + \frac{12u}{7} - \frac{9ue}{7}$$

$$V_A = \frac{12ue}{7} + \frac{12u}{7} = \frac{12u}{7}(1+e)$$

both V_A and V_B are greater than 0.

(as $0 \leq e \leq 1$).

$\therefore$ They are travelling in the same dir.

c) $KE \text{ after} = \frac{1}{2}(4m)\left[\frac{3u}{7}(4-3e)\right]^2$

$KE \text{ before} = \frac{1}{2}(4m)\left[3u\right]^2$

$$\Rightarrow 2m\left[\frac{9u^2}{49}\right]\left[4-3e\right]^2 = \frac{1}{4}\left[2m \cdot 9u^2\right]$$

$$\Rightarrow \frac{18\cancel{mu^2}}{49}(16-24e+9e^2) = \frac{9}{2}\cancel{mu^2}$$

$$\Rightarrow 16 - 24e + 9e^2 = 49/4$$

$\Rightarrow \quad 9e^2 - 24e + \frac{15}{4} = 0$

By Quadratic formula, $e = \frac{5}{2}$, $e = \frac{1}{6}$
×

(e ≤ 1
∴ reject)

So $\boxed{e = \frac{1}{6}}$