

M2 October 2017 (MA)

Q1) $I = m(v - u)$

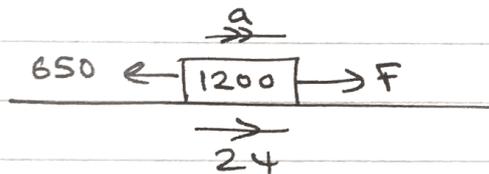
$$I = 0.2(5\mathbf{i} + 8\mathbf{j} - 10\mathbf{i} + 17\mathbf{j})$$

$$I = 0.2(-5\mathbf{i} + 25\mathbf{j}) = -\mathbf{i} + 5\mathbf{j}$$

$$\therefore |I| = \sqrt{1^2 + 5^2} = \sqrt{26} \text{ N s}$$

$$= \boxed{5.1} \text{ N s}$$

Q2a)



$$P = 30000 = Fv$$

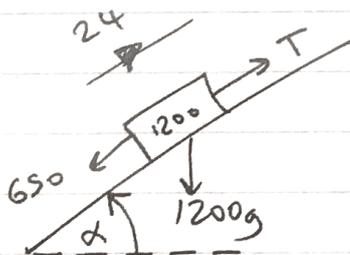
$$30000 = 24F$$

$$\therefore F = 1250 \text{ N} //$$

$$\text{N2L (var)} : F - 650 = 1200 a$$

$$\frac{1250 - 650}{1200} = a = \boxed{0.5 \text{ m s}^{-2}}$$

b)



$$\sin \alpha = \frac{1}{12}$$

$$P = Tv \rightarrow T = \frac{P}{24} //$$

$$\text{N2L (var)} : T - 650 - \frac{1200g}{12} = 1200 (0)$$

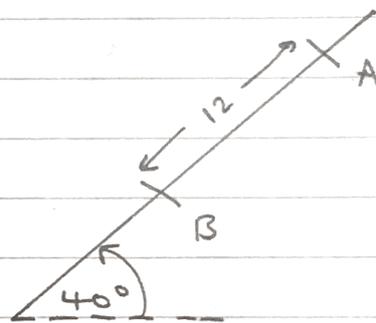
$$\therefore \frac{P}{24} = 650 + \frac{1200g}{12} = 1630 //$$

$$\begin{aligned} \therefore P &= 24 \times 1630 = 39120 \text{ W} \\ &= \boxed{39.1 \text{ kW}} \end{aligned}$$

Q3a) W.D due to friction = $F \times d$
 $= \mu R \times d$
 $= 0.5(4g \cos 40) \times 12 = \boxed{180 \text{ J}}$

b) at A : $KE = 2v^2$
 $GPE = 4g(12 \sin 40)$

at B : $KE = 2(24)^2$
 $GPE = 0$

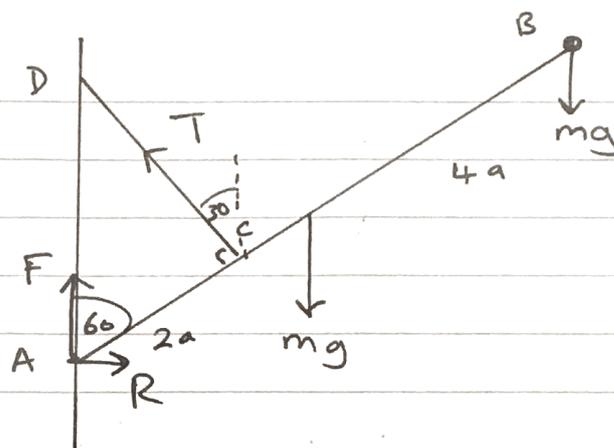


W.D due to friction $\leq 180 \text{ J}$.

So, ... $2v^2 + 4g(12 \sin 40) = 2(24)^2 + 180$

$$v^2 = \frac{2(24)^2 + 180 - 4g(12 \sin 40)}{2} \leq 515$$

so $v \leq \boxed{22.7 \text{ m/s}}$



$$M(A): T(2a) = mg(3a \sin 60) + mg(6a \sin 60)$$

$$T = \frac{9 \cancel{a} mg \sin 60}{2 \cancel{a}} = 9mg \times \frac{\sqrt{3}}{4} = \boxed{\frac{9\sqrt{3}mg}{4}}$$

b) $R(\leftrightarrow): R = T \cos 60 //$

$$\text{so } R = \frac{9\sqrt{3}}{4} \times \frac{1}{2} \times mg = \boxed{\frac{9\sqrt{3}mg}{8}}$$

c) $R_F(\updownarrow): F + T \cos 30 = 2mg$

$$F = 2mg - \frac{9\sqrt{3}}{4} \cos 30 (mg)$$

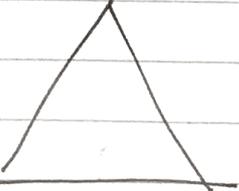
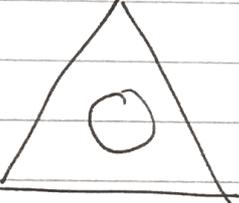
$$F = -\frac{11}{8} mg //$$

$F < 0$ so F acts downwards.

$$|F| = \frac{11}{8} mg.$$

$$F = \mu R \quad \leftarrow \text{limiting equilibrium}$$

$$\therefore \mu = \frac{F}{R} = \frac{\frac{11}{8}mg}{\frac{9\sqrt{3}mg}{8}} = \boxed{0.71}$$

(Q5a)	<u>Shape</u>	<u>Mass (area)</u>	<u>Distance of c.o.m from D</u>
(+)		$\boxed{4a^2\sqrt{3}}$	$\frac{4a\sin 60}{3} = \boxed{\frac{2a}{3}\sqrt{3}}$
(-)		$\boxed{\pi a^2}$	$\boxed{\frac{3a}{2}}$
(=)		$\boxed{a^2(4\sqrt{3} - \pi)}$	$\boxed{\bar{y}}$

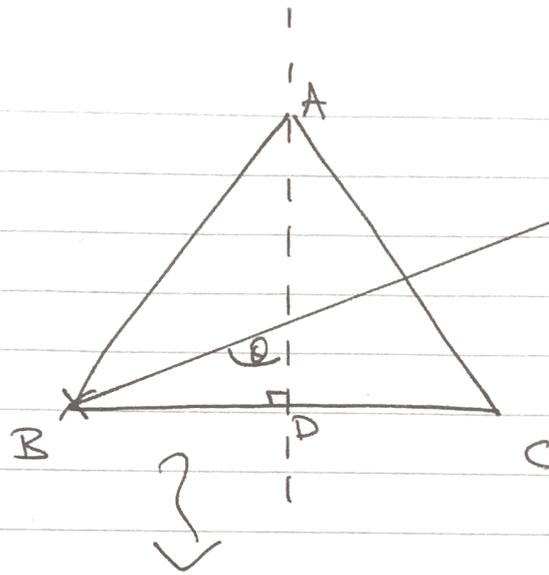
$$\bar{x} \sum m_i = \sum m_i x_i$$

$$4a^2\sqrt{3} \left(\frac{2a}{3}\sqrt{3} \right) - \pi a^2 \left(\frac{3a}{2} \right) = a^2(4\sqrt{3} - \pi)\bar{y}$$

$$8a - \frac{3\pi a}{2} = (4\sqrt{3} - \pi)\bar{y}$$

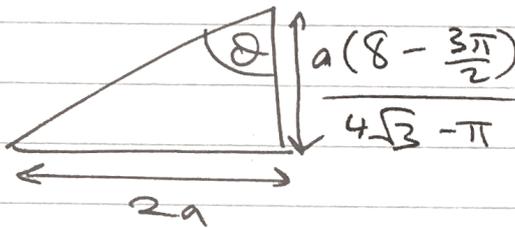
$$\therefore \bar{y} = \frac{a \left(8 - \frac{3\pi}{2} \right)}{4\sqrt{3} - \pi} = \text{distance from D of c.o.m}$$

b)



Downward vertical

$\theta =$ angle required.



$$\tan \theta = \frac{2a}{\frac{a(8 - \frac{3\pi}{2})}{4\sqrt{3} - \pi}} = \frac{8\sqrt{3} - 2\pi}{8 - \frac{3\pi}{2}}$$

$\approx 2.304\dots$

so $\theta = \tan^{-1}(2.304\dots) \approx \boxed{67^\circ}$

Q6a)

$a = 2t - 3$

$v = \int (a) dt = \int (2t - 3) dt = t^2 - 3t + c = v$

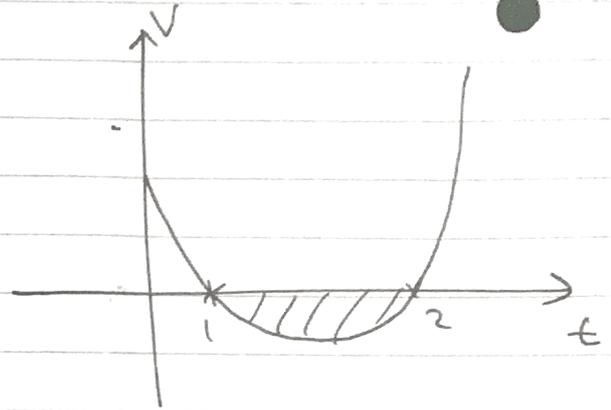
$t = 3, v = 2 : 2 = 9 - 3(3) + c$
 $\therefore c = 2$

so $v = t^2 - 3t + 2$

$$b) \quad V = t^2 - 3t + 2$$

$$V = (t-2)(t-1) = 0$$

so P is at A at $t=1$
and P is at B at $t=2$



$$\therefore AB = \left| \int_1^2 (t^2 - 3t + 2) dt \right|$$

$$\Rightarrow \int_1^2 (t^2 - 3t + 2) dt = \left[\frac{t^3}{3} - \frac{3t^2}{2} + 2t \right]_1^2$$

$$= \left[\frac{2}{3} \right] - \left[\frac{5}{6} \right] = -\frac{1}{6}$$

$$\therefore AB = \left| -\frac{1}{6} \right| = \boxed{\frac{1}{6}} \text{ m}$$

$$(Q7a) \quad \underline{\text{At A}}: \quad KE = \frac{1}{2} (m)(15)^2$$

$$GPE = mg(47.5)$$

$$\underline{\text{At B}}: \quad KE = \frac{1}{2} (m)(v)^2$$

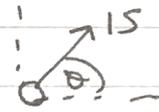
$$GPE = 0$$

$$\underline{\text{C.O.E}}: \quad \frac{225m}{2} + 47.5mg = \frac{mv^2}{2}$$

$$\therefore v^2 = 225 + 95g$$

$$v = \sqrt{225 + 95g} = \boxed{34 \text{ m/s}}$$

b)

$$\left. \begin{array}{l} s = s \\ u = 15 \sin \theta = 9 \\ v = 0 \\ a = -g \\ t = t \end{array} \right\} \begin{array}{l} v^2 = u^2 + 2as \\ 0^2 = 9^2 - 2gs \\ s = \frac{9^2}{2g} = \frac{405}{98} \text{ m} \end{array}$$


So greatest height = $\frac{405}{98} + 47.5$

$$= \boxed{51.6 \text{ m}}$$

c) least speed occurs at highest point (when vert component of speed = 0)

$$\text{least speed} = \vec{u} = 15 \cos \theta = 15 \times \frac{4}{5} = \boxed{12} \text{ m/s}$$

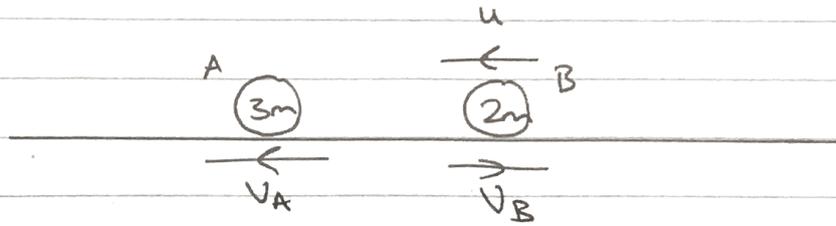
d)

$$\left. \begin{array}{l} s = -47.5 \\ u = 9 \\ v = \\ a = -g \\ t = t \end{array} \right\} \begin{array}{l} s = ut + \frac{1}{2}at^2 \\ -47.5 = 9t - 4.9t^2 \\ 4.9t^2 - 9t - 47.5 = 0 \end{array}$$

By Quadratic formula, $t = \underline{4.16 \dots}$
 $t > 0$ so reject other solution.

$$\vec{s} = ut = 12 \times t = 12 \times 4.16 \dots \approx \boxed{50 \text{ m}}$$

Q8ai)



$$\underline{\text{C.L.M}} : 2mu = 3mv_A - 2mv_B$$

$$2u = 3v_A - 2v_B \quad \text{--- (1)}$$

$$\underline{\text{N.I.L}} : e = \frac{v_A + v_B}{u}$$

$$\therefore ue = v_A + v_B$$

$$\times 2 : 2ue = 2v_A + 2v_B$$

$$+ (1) : + [2u = 3v_A - 2v_B]$$

$$2u(1+e) = 5v_A + 0$$

$$\therefore v_A = \boxed{\frac{2u}{5}(1+e)} = \text{speed}$$

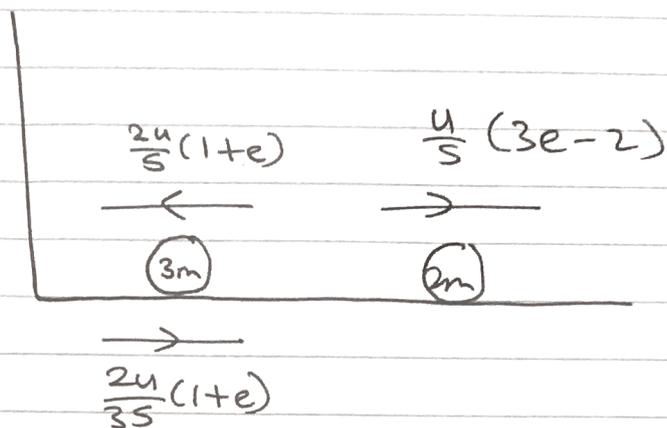
$$\text{ii) } v_B = ue - v_A = ue - \frac{2u}{5} - \frac{2ue}{5}$$

$$v_B = \frac{3ue}{5} - \frac{2u}{5} = \frac{u}{5}(3e-2) //$$

$$\therefore \text{speed}_B = \boxed{\left| \frac{u}{5}(3e-2) \right|}$$

Modulus not required
since we know B changes
direction so $\frac{u}{5}(3e-2) > 0$

b)



Second collision occurs

$$\text{So } \frac{2u}{35}(1+e) > \frac{u}{5}(3e-2)$$

$$\Rightarrow \frac{2}{35} + \frac{2e}{35} > \frac{3e}{5} - \frac{2}{5}$$

$$\Rightarrow \frac{19e}{35} < \frac{16}{35} \rightarrow e < \frac{16}{19}$$

$$\Rightarrow e < \frac{16}{19} //$$

and $V_B > 0$ since we are told B's direction is reversed...

$$\Rightarrow \frac{u(3e-2)}{5} > 0 \therefore 3e-2 > 0 //$$

$$\Rightarrow 3e > 2 \therefore e > \frac{2}{3} //$$

hence $\boxed{\frac{2}{3} < e < \frac{16}{19}}$