M2 January 2018 (IAL) (MA)

(Q1) $I = m(v - u)$

$4\underline{i} + 5\underline{j} = 0.5(v - (2\underline{i} - 3\underline{j}))$

$\underline{\times 2}$: $8\underline{i} + 10\underline{j} = v - 2\underline{i} + 3\underline{j}$

$v = 10\underline{i} + 7\underline{j}$

$\therefore$ KE after $= \frac{1}{2}(0.5)(10^2 + 7^2) = \boxed{37.25}$

and KE before $= \frac{1}{2}(0.5)(2^2 + 3^2) = \underline{\underline{3.25}}$

$\therefore$ Increase in KE $= 37.25 - 3.25$

$= \boxed{34\,J}$

(Q2a) $v = 2t - 2t^2 - 1 + t$

$v = -2t^2 + 3t - 1$

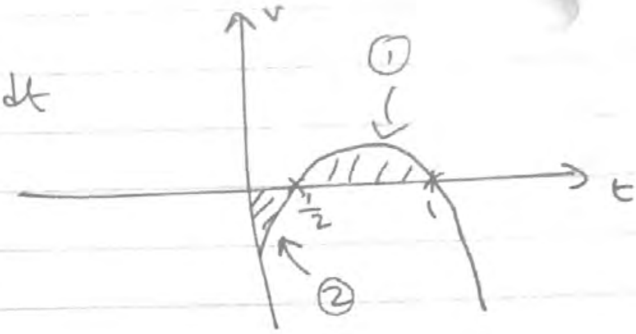
$a = \frac{dv}{dt} = -4t + 3$

$\underline{t = \frac{1}{2}}$: $a = -4(\frac{1}{2}) + 3 = \boxed{1\ ms^{-2}}$

b) P is at rest at $t = \frac{1}{2}$ and $t = 1$

$[(2t - 1)(1 - t) = 0]$

$\therefore$ distance (1) $= \displaystyle\int_{\frac{1}{2}}^{1} \left[-2t^2+3t-1\right] dt$



$$= \left[-\frac{2t^3}{3}+\frac{3t^2}{2}-t\right]_{\frac{1}{2}}^{1}$$

$$= \left[-\frac{2}{3}+\frac{3}{2}-1\right] - \left[-\frac{1}{12}+\frac{3}{8}-\frac{1}{2}\right]$$

$$= \boxed{\frac{1}{24}} = 1$$

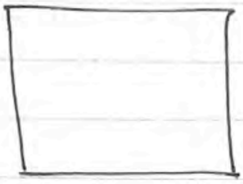
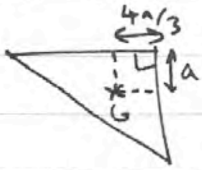
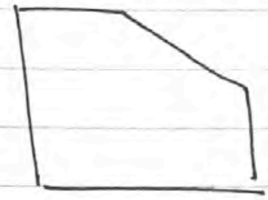
distance (2) $= \left|\displaystyle\int_{0}^{\frac{1}{2}} \left[-2t^2+3t-1\right] dt\right| = \left|\left[\frac{-2t^3}{3}+\frac{3t^2}{2}-t\right]_{0}^{\frac{1}{2}}\right|$

$$= \left|-\frac{5}{24}\right| = \frac{5}{24}$$

$\therefore$ total required distance $= \frac{5}{24}+\frac{1}{24}$

$$= \boxed{\frac{1}{4}}$$

Q3ai)/ii

	Shape	Mass (area)	Distance of c.o.m from... OD	OA
(+)		$\boxed{36a^2}$	$\boxed{3a}$	$\boxed{3a}$
(−)		$\frac{1}{2}\cdot 4a \cdot 3a = \boxed{6a^2}$	$\boxed{\frac{14a}{3}}$	$\boxed{5a}$
(=)		$\boxed{30a^2}$	$\boxed{\bar{x}}$	$\boxed{\bar{y}}$

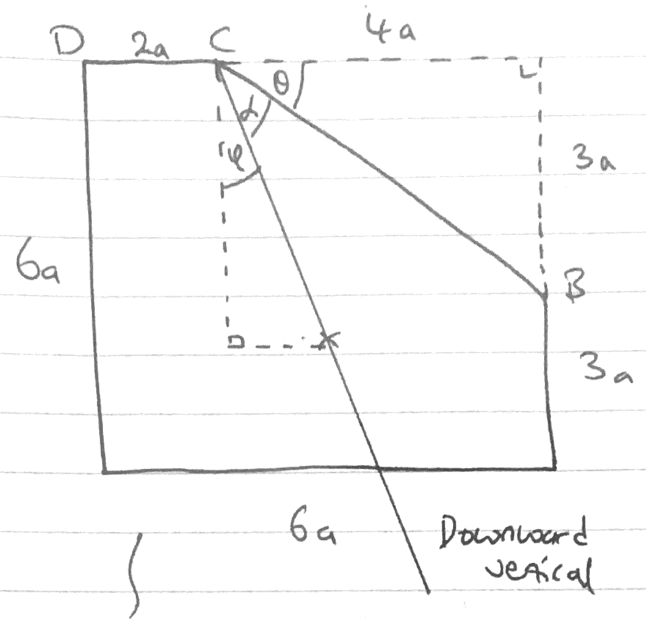
$$\underline{\bar{x}\sum M_i = \sum m_i x_i}$$

$$36\cancel{a^2}\begin{pmatrix}3a\\3a\end{pmatrix} - 6\cancel{a^2}\begin{pmatrix}14a/3\\5a\end{pmatrix} = 30\cancel{a^2}\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\begin{pmatrix}108a - 28a\\108a - 30a\end{pmatrix} = 30\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\therefore \begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix} = \begin{pmatrix}8a/3\\13a/5\end{pmatrix}$$

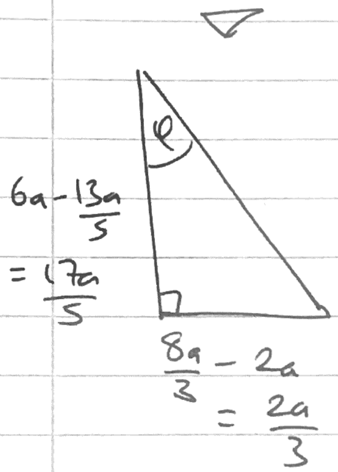
b)



$$\alpha = 90 - \theta - \varphi$$

$$\tan\theta = \frac{3}{4}$$

$$\therefore \theta = \arctan\frac{3}{4}.$$



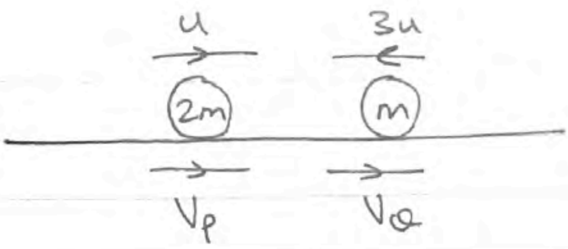
$$\therefore \tan\varphi = \frac{\frac{2}{3}}{\frac{17}{5}} = \frac{10}{51}$$

$$\therefore \varphi = \arctan\frac{10}{51}$$

$$\therefore \alpha = 90 - \arctan\frac{10}{51} - \arctan\frac{3}{4}$$

$$\boxed{\alpha \simeq 42^\circ}$$

(Q4ai)



$\overrightarrow{C.L.M}$: $2mu - 3mu = 2mV_P + mV_Q$

$-u = 2V_P + V_Q \sim (1)$

$\overrightarrow{N.I.L}$: $e = \dfrac{V_Q - V_P}{4u}$

$\therefore 4ue = V_Q - V_P$

$-(1)$: $-[-u = 2V_P + V_Q]$

$4ue + u = -2V_P - V_P + V_Q - V_Q$

$u(4e+1) = -3V_P$

$\therefore V_P = -\dfrac{u}{3}(4e+1)$ //

$\therefore$ speed $= \left|-\dfrac{u}{3}(4e+1)\right|$

$= \boxed{\dfrac{u}{3}(4e+1)}$ //

ii) and $V_Q = 4ue + V_P$

$= 4ue + -\left(\dfrac{4ue}{3} + \dfrac{u}{3}\right)$

$= \dfrac{8ue}{3} - \dfrac{u}{3} = \boxed{\dfrac{u}{3}(8e-1)}$ //

$8e - 1 > 0$ since $e > \frac{1}{8}$

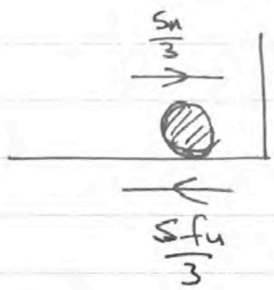
So no need for modulus sign

b) $V_P = -\frac{u}{3}(1+4e)$

$V_P < 0$ for all values of e $(e > \frac{1}{8})$

so P has its direction reversed.

c)



$V_Q = \frac{u}{3}(5)$

final speed $= \frac{5u}{3} \times f$

theres another collision ...

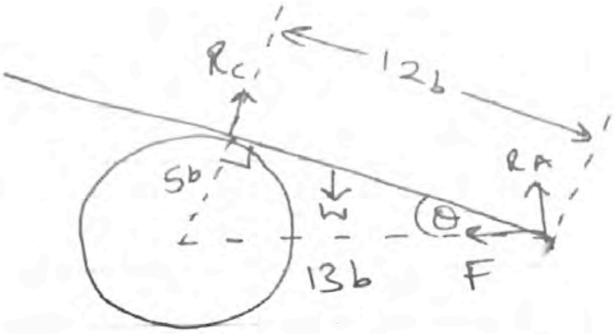
So $\frac{5fu}{3} \geq \frac{u}{3}(4)$

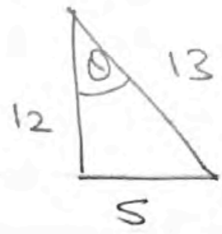
$\Rightarrow 5f \geq 4$

$\Rightarrow f \geq \frac{4}{5}$ //

So $\boxed{1 \geq f \geq \frac{4}{5}}$

(Q5 a)





$$M(A): \quad R_C(12b) = W\cos\theta(8b) \qquad \cos\theta = \frac{12}{13}$$

$$R_C = \frac{8 \times W \times \frac{12}{13}}{12} \qquad \sin\theta = \frac{5}{13}$$

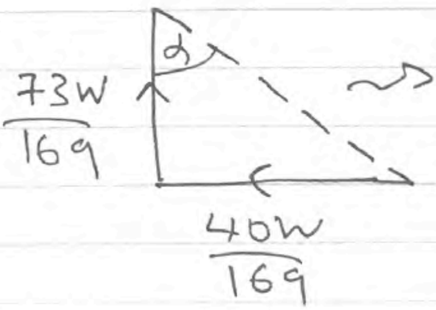
$$R_C = \boxed{\frac{8W}{13}}$$

b) $R(\updownarrow)$: $\quad R_A + R_C\cos\theta = W$

$$R_A = W - \frac{8W}{13}\left(\frac{12}{13}\right)$$

$$R_A = \frac{73W}{169} \;//$$

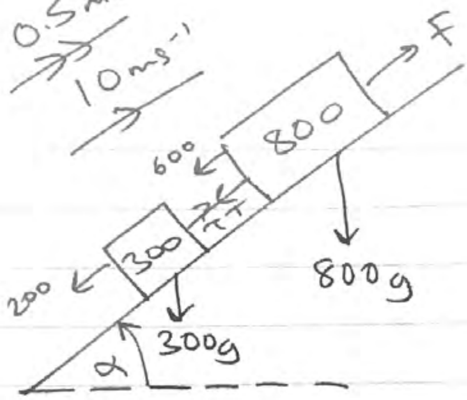
$R(\leftrightarrow)$: $\quad F = R_C\sin\theta = \frac{8W}{13} \times \frac{5}{13} = \frac{40W}{169} \;//$



$$\tan\alpha = \frac{\frac{40W}{169}}{\frac{73W}{169}}$$

$$\boxed{= \frac{40}{73}} \;//$$

Q6a)



$\sin\alpha = \frac{1}{14}$

$P = Fv$
$P = F(10)$

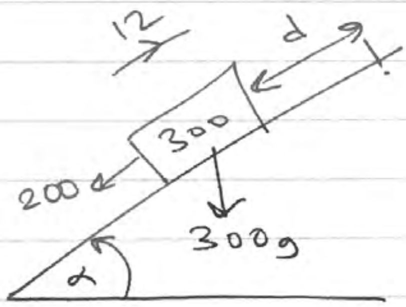
N2L (system): $F - 600 - 200 - 1100g\sin\alpha = 1100\left(\frac{1}{2}\right)$

$F = 800 + \frac{1100g}{14} + 550 = \underline{\underline{2120}}$

$P = F \times 10 = 21200\text{ W} = 21.2\text{kW}$

so $\boxed{P = 21.2}$

b)



Initial energy: $KE = \frac{1}{2}(300)(12^2) = 21600$
$GPE = 0$

Final energy: $KE = 0$
$GPE = 300gd\sin\alpha = 210d$

& W.D by resistence $= 200 \times d$

So... $21600 = 210d + 200d$

$d = \frac{21600}{210 + 200} = \boxed{52.7\text{m}}$

$(\longrightarrow)$

(Q7a)

$s = x$
$u = u\cos\alpha$
$v =$
$a = 0$
$t = t$

$\underline{s = ut}$
$x = u\cos\alpha \times t$

so $x = ut\cos\alpha$ //
$\therefore$ $t = \dfrac{x}{u\cos\alpha}$ //

$(\uparrow)$

$s = y$
$u = u\sin\alpha$
$v =$
$a = -g$
$t = t$

$s = ut + \frac{1}{2}at^2$

$y = ut\sin\alpha - \frac{g}{2}t^2$

but $t = \dfrac{x}{u\cos\alpha}$

so $y = u\sin\alpha\left(\dfrac{x}{u\cos\alpha}\right) - \dfrac{g}{2}\left(\dfrac{x}{u\cos\alpha}\right)^2$

$y = x\tan\alpha - \dfrac{gx^2}{2u^2\cos^2\alpha}$

$y = x\tan\alpha - \dfrac{gx^2}{2u^2}(\sec^2\alpha)$

$(\sec^2\alpha = 1+\tan^2\alpha)$: $y = x\tan\alpha - \dfrac{gx^2}{2u^2}(1+\tan^2\alpha)$

b) $\vec{a} = 0$ so $\vec{v} = u$

$(\downarrow)$ $\left.\begin{array}{l} s = h \\ u = 0 \\ v = v \\ a = g \\ t = t \end{array}\right\}$ $v = u + at$ $(t = T)$ here

$v = gt$ //

$\therefore \downarrow v = gT$

$\therefore$ speed at $B = \sqrt{\downarrow v^2 + \vec{v}^2}$

$= \sqrt{(gT)^2 + (u)^2}$

$= \boxed{\sqrt{g^2T^2 + u^2}}$

c) $\vec{s} = ut$; $\underline{\underline{d = uT}}$

from a, $y = x\tan\alpha - \frac{gx^2}{2u^2}(1 + \tan^2\alpha)$

$\underline{y = -h,\ x = d}$: $-h = d\tan\alpha - \frac{gd^2}{2u^2}(1 + \tan^2\alpha)$

we do not want h or u in our equation.

from (b): $(\downarrow)$ $\left.\begin{array}{l} s = h \\ u = 0 \\ v = v \\ a = g \\ t = t \end{array}\right\}$ $s = ut + \frac{1}{2}at^2$

$h = \frac{1}{2}gT^2$ //

now we know $d = uT$ and
$h = \frac{1}{2}gt^2$

so sub these in . . .

$$-\frac{1}{2}gt^2 = uT\tan\alpha - \frac{g}{2u^2}(u^2T^2)(\tan^2\alpha + 1)$$

leave as d.

$$-\frac{1}{2}gt^2 = d\tan\alpha - \frac{gt^2}{2}(\tan^2\alpha + 1)$$

$$-\frac{1}{2}\cancel{g}t^2 = d\tan\alpha - \cancel{\frac{gt^2}{2}} - \frac{gt^2}{2}\tan^2\alpha$$

$$\therefore\ d\tan\alpha = \frac{gt^2}{2}\tan^2\alpha$$