Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Tuesday 22 January 2019

Morning (Time: 1 hour 30 minutes) | Paper Reference **WME02/01**

Mechanics M2
Advanced/Advanced Subsidiary

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$, and give your answer to either two significant figures or three significant figures.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information
- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

Jan 2019.

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1. Three particles of masses $3m$, m and $2m$ are positioned at the points with coordinates $(a, 8)$, $(-4, 0)$ and $(5, -2)$ respectively.

Given that the centre of mass of the three particles is at the point with coordinates $(k, 2k)$, where k is a constant, find the value of a.

(5)

$3\cancel{m}\begin{pmatrix} a \\ 8 \end{pmatrix} + \cancel{m}\begin{pmatrix} -4 \\ 0 \end{pmatrix} + 2\cancel{m}\begin{pmatrix} 5 \\ -2 \end{pmatrix} = 6\cancel{m}\begin{pmatrix} k \\ 2k \end{pmatrix}$

$\begin{pmatrix} 3a - 4 + 10 \\ 24 - 4 \end{pmatrix} = 6\begin{pmatrix} k \\ 2k \end{pmatrix}$

$3a + 6 = 6k.$

$20 = 12k.$

$\therefore k = 5/3.$

$3a + 6 = 6(5/3)$

$a = 4/3.$

Leave blank

2. A particle of mass 0.75 kg is moving with velocity $(4\mathbf{i} + \mathbf{j})\,\mathrm{m\,s^{-1}}$ when it receives an impulse $(-6\mathbf{i} + 4\mathbf{j})$ N s.

Find

(a) the velocity of the particle immediately after receiving the impulse,

(3)

(b) the size of the angle through which the path of the particle is deflected as a result of the impulse.

(3)

$0.75\left(v-\begin{pmatrix}4\\1\end{pmatrix}\right)=\begin{pmatrix}-6\\4\end{pmatrix}$

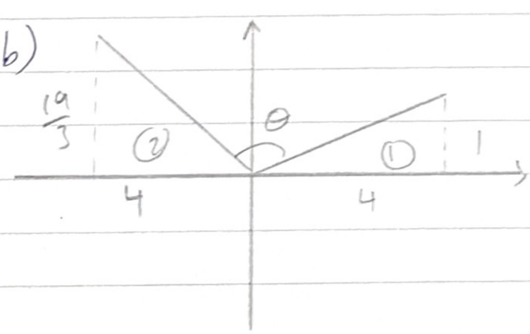
$v-\begin{pmatrix}4\\1\end{pmatrix}=\begin{pmatrix}-8\\16/3\end{pmatrix}$

$v=\begin{pmatrix}-8\\16/3\end{pmatrix}+\begin{pmatrix}4\\1\end{pmatrix}$

$v=\begin{pmatrix}-4\\19/3\end{pmatrix}$

$\therefore v=-4i+\frac{19}{3}j.$

(b)



① $\tan^{-1}\left(\frac{1}{4}\right)=14.03624347.$

② $\tan^{-1}\left(\frac{19/3}{4}\right)$

$=57.72435599$

$\theta=180-(14.03\ldots+57.72\ldots)$

$=108°$ (3sf)

Leave blank

3. A car of mass 900 kg is moving on a straight road that is inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{49}$. When the car is moving up the road, with the engine of the car working at a constant rate of 10.8 kW, the car has a constant speed of $v\,\text{m s}^{-1}$. The resistance to the motion of the car from non-gravitational forces is modelled as a constant force of magnitude R newtons.

When the car is moving down the road, with the engine of the car working at a constant rate of 10.8 kW, the car has a constant speed of $2v\,\text{m s}^{-1}$. The resistance to the motion of the car is still modelled as a constant force of magnitude R newtons.

Find

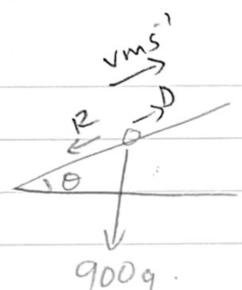
(i) the value of R,

(ii) the value of v.

(8)

3.

scenario 1



$\sin\theta = \frac{1}{49}$

Power = 10800W.

$10800 = D \times V$.

where D = driving force.

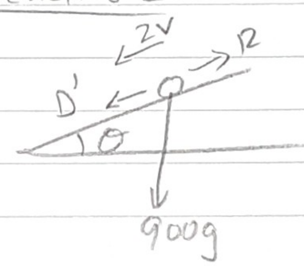
$R(\nearrow)$ $D - 900g \times \frac{1}{49} - R = 0$.

$D = \frac{900g}{49} + R$ -- (1).

$10800 = \left(\frac{900g}{49} + R\right) V$

$10800 = (180 + R)V$. -- (1)

scenario 2



Power = 10800W.

$10800 = 2V \times D'$

$D' + 900g \times \frac{1}{49} - R = 0$. $R(\swarrow)$

$D' = R - \frac{900g}{49}$.

$10800 = 2V \times \left(R - \frac{900g}{49}\right)$

$10800 = 2V \times (R - 180)$ -- (2).

From eqn (1)

$V = \frac{10800}{180 + R}$.

Leave blank

Question 3 continued

$$10800 = 2 \times \left(\frac{10800}{180+R}\right)(R-180)$$

From eqn ②

$$10800 = \frac{21600R - 388800}{180+R}$$

$$1944000 + 10800R = 21600R - 388800$$

$$10800R = 583200$$

$$R = 540$$

Replace into eqn ①.

$$V = \frac{10800}{180+540}$$

$$= 15$$

Leave blank

4.

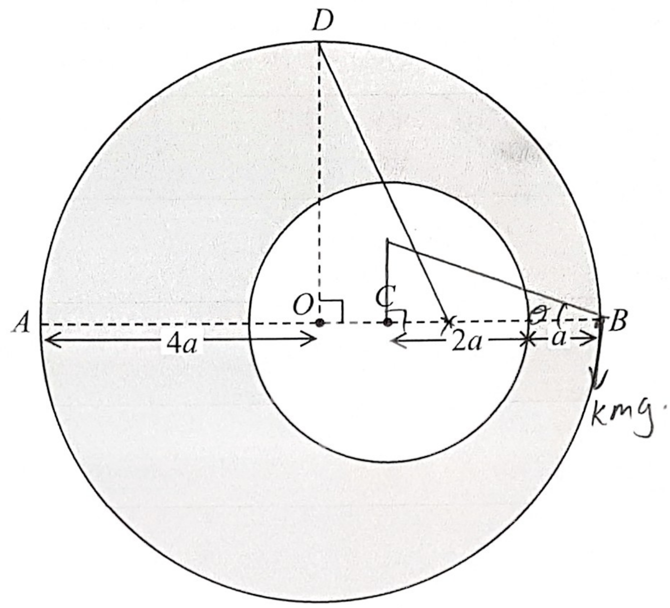


Figure 1

The uniform lamina L, shown shaded in Figure 1, is formed by removing a circular disc of radius $2a$ from a uniform circular disc of radius $4a$. The larger disc has centre O and diameter AB. The radius OD is perpendicular to AB. The smaller disc has centre C, where C is on AB and $BC = 3a$

(a) Show that the centre of mass of L is $\frac{13}{3}a$ from B.

(4)

The mass of L is M and a particle of mass kM is attached to L at B. When L, with the particle attached, is freely suspended from point D, it hangs in equilibrium with A higher than B and AB at an angle θ to the horizontal, where $\tan\theta = \frac{3}{4}$

(b) Find the value of k.

(5)

(a) Bigger circular disc.	Smaller circular disc.
Area $= \pi(4a)^2$	Area $= \pi(2a)^2$
$= 16\pi a^2$	$= 4\pi a^2$
COM	COM
$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} a \\ 0 \end{pmatrix}$.

Leave blank

Question 4 continued

$$16\pi a^2\begin{pmatrix}0\\0\end{pmatrix} - 4\pi a^2\begin{pmatrix}a\\0\end{pmatrix} = 12\pi a^2\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\begin{pmatrix}0-4a\\0+0\end{pmatrix} = 12\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$\bar{x} = -\frac{1}{3}a$

$\bar{y} = 0.$

$\therefore$ distance from $B = 4a + \frac{1}{3}a$

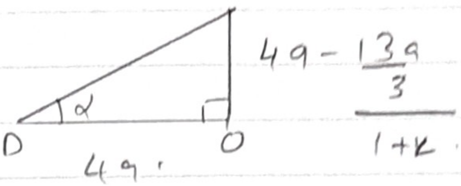
$= \frac{13}{3}a$ as required.

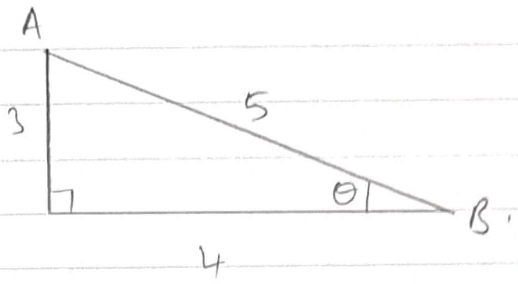
(b) From B

$$M\begin{pmatrix}13/3\,a\\0\end{pmatrix} + kM\begin{pmatrix}0\\0\end{pmatrix} = M(1+k)\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\begin{pmatrix}\frac{13a}{3}\\0\end{pmatrix} = (1+k)\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$\bar{x} = \dfrac{13/3\,a}{1+k}$





Using similar triangles.

$$\frac{4a - \frac{13/3\,a}{1+k}}{4a} = \frac{3}{4}$$

$4 - \dfrac{13}{3(1+k)} = 3.$

$1 = \dfrac{13}{3+3k}.$

$3 + 3k = 13.$

$3k = 10$

$k = \dfrac{10}{3}.$

Leave blank

5. A particle moves along the x-axis. At time t seconds, $t \geqslant 0$, the velocity of the particle is $v\,\mathrm{m\,s^{-1}}$ in the direction of x increasing, where $v = 2t^{\frac{3}{2}} - 6t + 2$

At time $t = 0$ the particle passes through the origin O. At the instant when the acceleration of the particle is zero, the particle is at the point A.

Find the distance OA.

(8)

$a = \frac{dv}{dt}$

$= \frac{3}{2}(2)t^{\frac{1}{2}} - 6$

$= 3t^{\frac{1}{2}} - 6$

when $a = 0$

$3t^{\frac{1}{2}} - 6 = 0$

$t^{\frac{1}{2}} = \frac{6}{3}$

$= 2$

$t = 4$

$s = \int v\,dt$

$= \int 2t^{\frac{3}{2}} - 6t + 2 \; dt$

$= \frac{2t^{\frac{5}{2}}}{\frac{5}{2}} - \frac{6t^2}{2} + 2t + c$

$= \frac{4}{5}t^{\frac{5}{2}} - 3t^2 + 2t + c$

when $t = 0$, $s = 0$

$\frac{4}{5}t^{\frac{5}{2}} - 3t^2 + 2t + c = 0$

$\therefore \; c = 0$

when $t = 4$, $s = \frac{4}{5}(4)^{\frac{5}{2}} - 3(4)^2 + 2(4)$

$= -14.4$

distance $= 14.4\,\mathrm{m}$

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6.

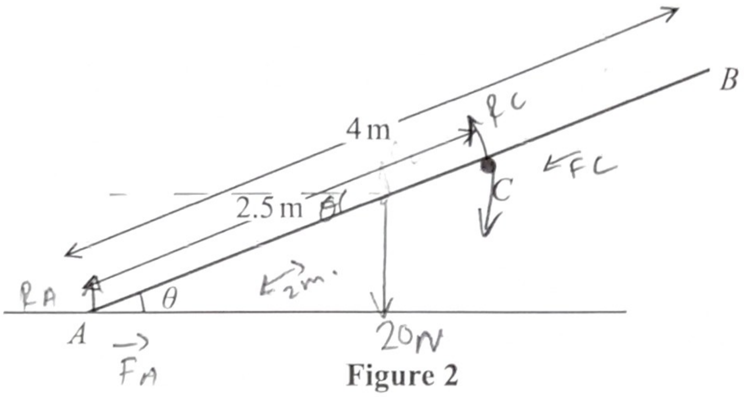

Figure 2

A plank AB rests in equilibrium against a fixed horizontal pole. The plank has length 4 m and weight 20 N and rests on the pole at C, where $AC = 2.5$ m. The end A of the plank rests on rough horizontal ground and AB makes an angle θ with the ground, as shown in Figure 2. The coefficient of friction between the plank and the ground is $\frac{1}{4}$.

The plank is modelled as a uniform rod and the pole as a rough horizontal peg that is perpendicular to the vertical plane containing AB.

Given that $\cos\theta = \frac{4}{5}$ and that the friction is limiting at both A and C,

(a) find the magnitude of the normal reaction on the plank at C, **(3)**

(b) find the coefficient of friction between the plank and the pole. **(8)**

(a) M(A)

$R_C \times 2.5 = 20\cos\theta \times 2$

$R_C = \dfrac{20 \times 0.8 \times 2}{2.5}$

$R_C = 12.8\text{N}$

(b) Balancing forces

$R_C.\cos\theta + R_A = 20 + F_C.\sin\theta$

Given $R_C = 12.8$ and $F_C = \mu_C . R_C$.

$\cos\theta = \frac{4}{5}$ $\quad \mu_C = \mu @ C.$

$\sin\theta = 3/5.$

$R_A = 9.76\text{N} + 7.68.\mu_C$ --- ①.

$\frac{1}{4} \times R_A = 7.68 + 10.24\mu_C.$

$\frac{1}{4}(9.76 + 7.68\mu_C) = 7.68 + 10.24\mu_C.$

Leave blank

Question 6 continued

$2.44 + 1.92\mu_c = 7.68 + 10.24\mu_c$

$8.32\mu_c = 5.24$

$\mu_c = \underline{\underline{0.63}}$

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7. A particle P of mass $3m$ is moving in a straight line with speed u on a smooth horizontal table. A second particle Q of mass $2m$ is moving with speed $2u$ in the opposite direction to P along the same straight line. Particle P collides directly with Q. The coefficient of restitution between P and Q is e.

(a) Show that the direction of motion of P is reversed as a result of the collision with Q. **(3)**

(b) Find the range of values of e for which the direction of motion of Q is also reversed as a result of the collision. **(5)**

Given that $e = \frac{1}{2}$

(c) find, in terms of m and u, the kinetic energy lost in the collision between P and Q. **(5)**

(a) P Q

3m 2m

$\xrightarrow{u}$ $\xleftarrow{2u}$

$\xrightarrow{x}$ $\xrightarrow{y}$

$\xleftarrow{x}$

Law of conservation of momentum.

$3mu - 4mu = 3mx + 2my$

$-mu = 3mx + 2my.$

→ Since the total is negative,

(i) either $x < 0$ and $y < 0$

because $x < 0, y > 0$ is not possible.

∴ the direction of P's motion has definetly been reversed.

(b) Using Impact law;

$\frac{y + x}{3u} = e.$

$y + x = 3ue.$

$x = 3ue - y.$

$-u = -3(3ue - y) + 2y.$

$-u = -9ue + 3y + 2y.$

$5y = 9ue - u.$

$y = \frac{u}{5}(9e - 1)$

$\frac{u}{5}(9e - 1) > 0.$

$9e - 1 > 0$

$e > 1/9$ but $e \leq 1$

$1/9 < e \leq 1$

Leave blank

Question 7 continued

(c) given $e = 1/2$

KE Before

$\frac{1}{2} \times 3m \times u^2 = \frac{3}{2} mu^2$

$+ \frac{1}{2} \times 2m \times (2u)^2 = 4mu^2$

$= 5.5mu^2$

KE After

$y = \frac{u}{5}(4.5 - 1)$

$= \frac{7}{10} u$

$\therefore x = \frac{4}{5} u$

$\frac{1}{2} \times 3m \times \left(\frac{4}{5}u\right)^2 = \frac{24}{25} mu^2$

$+ \frac{1}{2} \times 2m \times \left(\frac{7}{10}u\right)^2 = \frac{49}{100} mu^2$

$1.45mu^2$

$5.5mu^2 - 1.45mu^2 =$
$4.05mu^2$

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8.

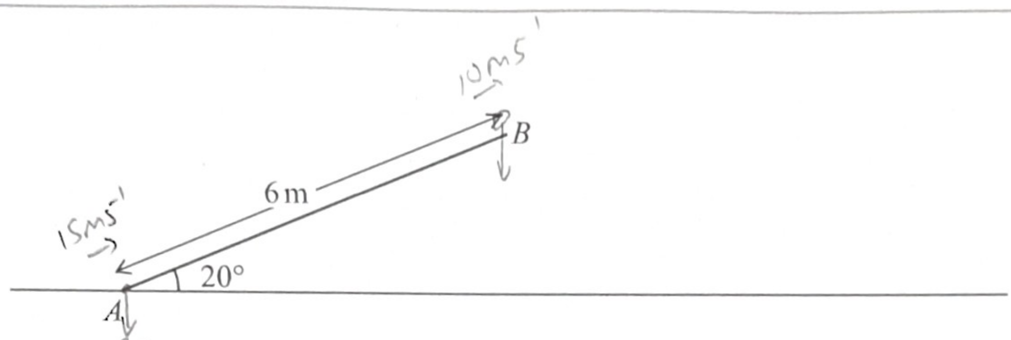

Figure 3

A rough ramp AB is fixed to horizontal ground at A. The ramp is inclined at 20° to the ground. The line AB is a line of greatest slope of the ramp and $AB = 6$ m. The point B is at the top of the ramp, as shown in Figure 3. A particle P of mass 3 kg is projected with speed 15 m s^{-1} from A towards B. At the instant P reaches the point B the speed of P is 10 m s^{-1}. The force due to friction is modelled as a constant force of magnitude F newtons.

(a) Use the work-energy principle to find the value of F.

(6)

After leaving the ramp at B, the particle P moves freely under gravity until it hits the horizontal ground at the point C. The speed of P as it hits the ground at C is w m s^{-1}.

Find

(b) (i) the value of w,

(ii) the direction of motion of P as it hits the ground at C,

(5)

(c) the greatest height of P above the ground as P moves from A to C.

(4)

(a) Gain in GPE.

$3g \times 6\sin 20 = 60.33235328.$

Loss in KE.

$\frac{1}{2} \times 3 \times (15^2 - 10^2)$

$= 187.5.$

$187.5 - 60.33 = F \times 6$

$F = 21.19460779.$

$= \underline{\underline{21.2N}}$

(b) $10\cos 20 \rightarrow U_x.$

$10\sin 20 \rightarrow V_y.$

$s = -6\sin 20$ ↑ +ve

$u = 10\sin 20$

$v = ?$

$a = -9.8.$

$t = x.$

$v^2 = u^2 + 2as$

$v^2 = (10\sin 20)^2 + 2(-9.8)(-6\sin 20)$

$v = 7.205508081.$

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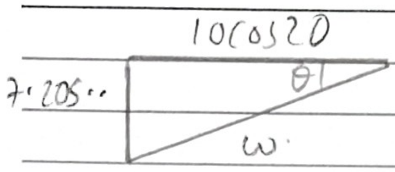
Question 8 continued



$w = \sqrt{(7 \cdot 205..)^2 + (10\cos 20)^2}$

$= \underline{11 \cdot 8 ms^{-1}}$

$\tan^{-1}\left(\frac{7 \cdot 205..}{10\cos 20}\right)$

$\theta = 37 \cdot 5°$ below the Horizontal.

(c) $s = ?$

$u = 10\sin 20$

$v = 0$

$a = -9 \cdot 8$

$t = x$

$v^2 = u^2 + 2as$

$0 = (10\sin 20)^2 + 2(-9 \cdot 8)(s)$

$s = 0 \cdot 596$

$0 \cdot 596 + 6\sin 20$

$= 2 \cdot 65m$ above the ground.