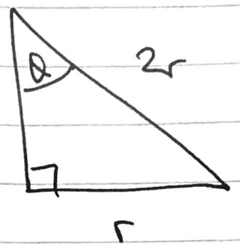
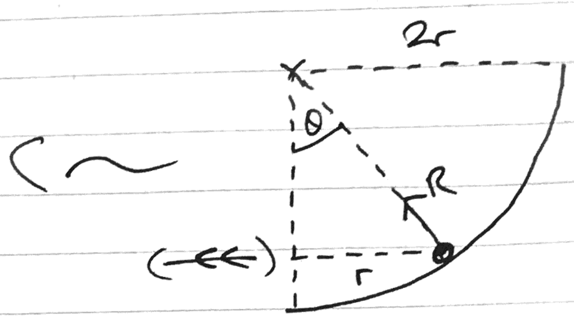
M3 Jan 2016 IAL (MA)

Q1)





$\therefore \sin\theta = \frac{r}{2r} = \frac{1}{2}$

$\therefore \theta = 30^\circ$

$R(\updownarrow): \quad R\cos 30 = mg$

$\overleftarrow{+}$ N2L(P) : $R\sin 30 = m(r)\omega^2$.

but $R = \frac{mg}{\cos 30}$ $\therefore$ $\cancel{m}g\tan 30 = \cancel{m}r\omega^2$

$\Rightarrow \omega^2 = \frac{g\tan 30}{r} = \frac{g\sqrt{3}}{3r}$

$\Rightarrow \omega = \sqrt{\frac{g\sqrt{3}}{3r}}$

Q2a) $a = 6 - 2t = \frac{dv}{dt}$

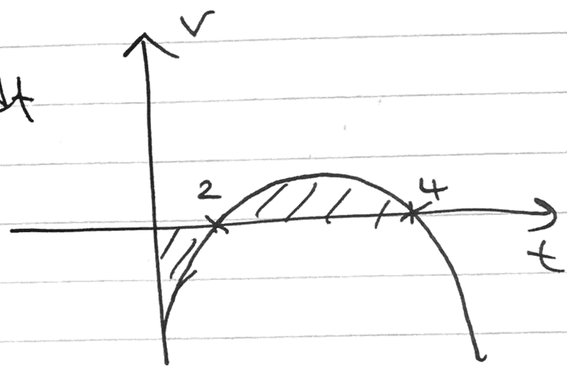
$$\int (6-2t)\,dt = \int (1)\,dv.$$

$$v = 6t - t^2 + c.$$

$t=0, v=-8$: $-8 = c$

$$\therefore \boxed{v = 6t - t^2 - 8}$$

b) $x_{TOTAL} = \left|\int_0^2 [v]\,dt\right| + \int_2^4 [v]\,dt$



$\Rightarrow \int_0^2 [6t - t^2 - 8]\,dt + \ldots$

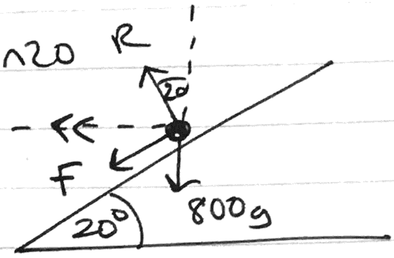
$= \left[\cancel{12}\, 3t^2 - \frac{t^3}{3} - 8t\right]_0^2 = -\frac{20}{3} \quad \therefore x = \frac{20}{3}$ //

$\Rightarrow \int_2^4 (v)\,dt = \left[3t^2 - \frac{t^3}{3} - 8t\right]_2^4$

$= \frac{4}{3}$ //

$\therefore$ Distance $= \frac{4}{3} + \frac{20}{3} = \boxed{8}$ m.

Q3) $R(\updownarrow)$: $R\cos 20 = 800g + F\sin 20$



$\underline{N2L(car)}$: $R\sin 20 + F\cos 20 = \frac{800v^2}{20} = 40v^2$

$F = \mu R \longrightarrow F_{max} = 0.5R$.

① : $R\cos 20 - F\sin 20 = 800g$

② : $R\sin 20 + F\cos 20 = 40v^2$

(V_{max} occurs when F is max.
ie when $F = 0.5R$)

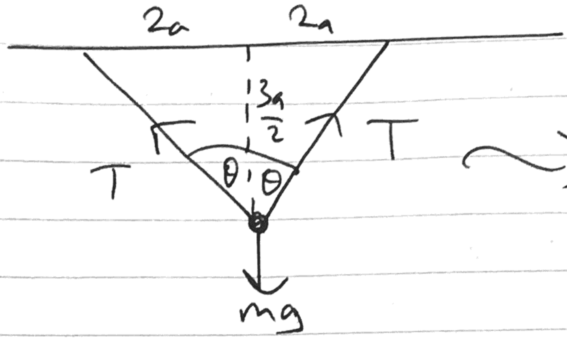
$\frac{②}{①}$: $\frac{\cancel{R}\sin 20 + 0.5\cancel{R}\cos 20}{\cancel{R}\cos 20 - 0.5\cancel{R}\sin 20} = \frac{40v^2}{800g}$.

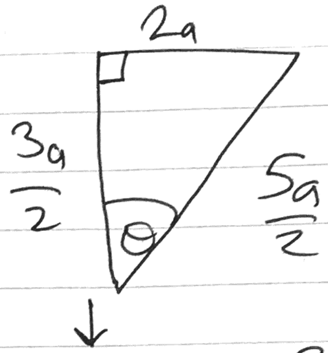
$\therefore \frac{40v^2}{800g} = 1.05618...$

$\therefore v^2 = \frac{800g(1.05618..)}{40} = 207..$

$\therefore \boxed{V = 14.4 \text{ m/s}}$

Q4a)





$\cos\theta = \frac{3}{5}$.

$\underline{N2L(P)\uparrow^{+}}$: $2T\cos\theta = mg$

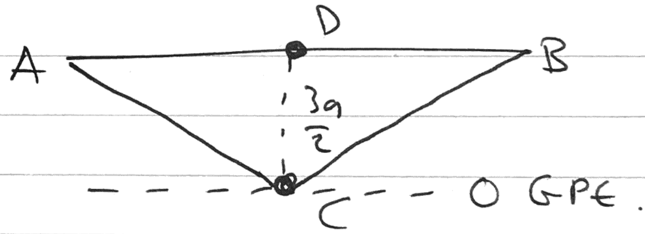
$$T = \frac{\lambda x}{l} = \frac{\lambda}{\frac{3a}{2}}\left(\frac{5a}{2} - \frac{3a}{2}\right) = \frac{2\lambda}{3}$$

for one half of the string.

$$\therefore 2 \times \frac{2\lambda}{3} \times \frac{3}{5} = mg$$

$$\Rightarrow \lambda = \frac{15mg}{2\times2\times3} = \boxed{\frac{5mg}{4}}$$

b) $\underline{A+D}$: $KE = 0$
$GPE = mg\left(\frac{3a}{2}\right)$
$EPE = \frac{5mg}{4(6a)}(a)^2$



$\underline{A+C}$: $KE = \frac{1}{2}mv^2$
$GPE = 0$
$EPE = \frac{5mg}{4}\left(\frac{4a^2}{6a}\right) = \frac{5mg}{4} \times \frac{2}{3}a = \frac{10amg}{12}$

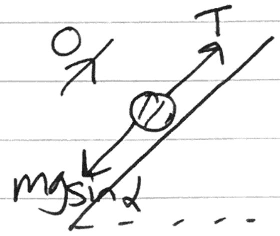
By C.O.E : $\frac{3amg}{2} + \frac{5amg}{24} = \frac{mv^2}{2} + \frac{5amg}{6}$

$$\left(\frac{3}{2} + \frac{5}{24} - \frac{5}{6}\right) \cancel{m}ag = \frac{\cancel{m}v^2}{2}$$

$$\frac{v^2}{2} = \frac{7ag}{8}$$

$$v^2 = \frac{7ag}{4} \quad \therefore \boxed{v = \frac{\sqrt{7ag}}{2}}$$

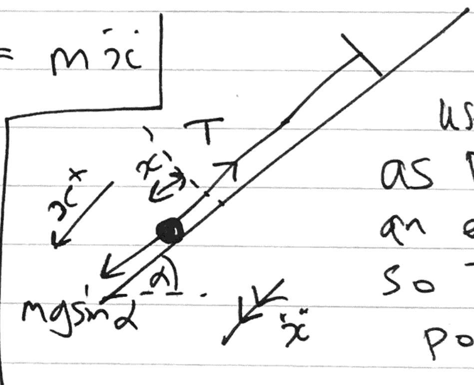
Q5a) $mg\sin\alpha = T$



$$\frac{3}{5}mg = \frac{\lambda\left(\frac{l}{5}\right)}{l}$$

$$\therefore \frac{3}{5}mg = \frac{\lambda}{5} \xrightarrow{\times 5} \lambda = 3mg$$

b) N2L(P) ↙+ : $mg\sin\alpha - T = m\ddot{x}$



use $x = a\cos\omega t$ as P starts at an endpoint. So a will be positive

$$T = \frac{\lambda x}{l} = \frac{3mg\left(x + \frac{l}{5}\right)}{l}$$

$$\therefore mg\left(\frac{3}{5}\right) - \frac{3mg}{l}\left(x + \frac{l}{5}\right) = m\ddot{x}$$

$$\Rightarrow \frac{3mg}{5} - \frac{3mgx}{l} - \frac{3mg}{5} = m\ddot{x} = -\frac{3mgx}{l}$$

$\therefore \ddot{x} = -\frac{3g}{l}x$ hence P moves with S.H.M, centre B.

c) $|\ddot{x}_{max}| = \frac{3g}{l} \times x_{max} = \frac{3g}{l} \times \frac{l}{2} = \boxed{\frac{3g}{2}}$ ms⁻²

amplitude

d) $x = \frac{l}{2}\cos(\omega t)$ $\omega = \sqrt{\frac{3g}{l}}$

time from C to slack point:

$$-\frac{l}{5} = \frac{l}{2}\cos\left(t\sqrt{\frac{3g}{l}}\right)$$

$$-\frac{2}{5} = \cos\left(t\sqrt{\frac{3g}{l}}\right)$$

$$t\sqrt{\frac{3g}{l}} = 1.982\ldots$$

$$t = \frac{1.982..}{\sqrt{3}}\sqrt{\frac{l}{g}}$$

time from C to D:

$$\frac{l}{4} = \frac{l}{2}\cos\left(t\sqrt{\frac{3g}{l}}\right)$$

$$\cos\left(t\sqrt{\frac{3g}{l}}\right) = \frac{1}{2}$$

$$\cos^{-1}\frac{1}{2} = \frac{\pi}{3} = t\sqrt{\frac{3g}{l}}$$

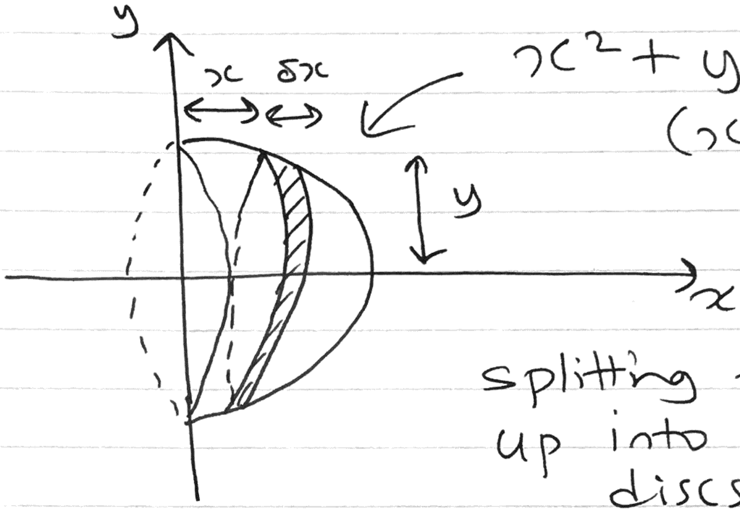
$$\therefore t = \left(\frac{\pi}{3\sqrt{3}}\right)\sqrt{\frac{l}{g}}$$

hence required time $= \left(\frac{1.982}{\sqrt{3}} - \frac{\pi}{3\sqrt{3}}\right)\sqrt{\frac{l}{g}}$

$= 0.54\sqrt{\frac{l}{g}}$

hence $\boxed{k = 0.54}$

(Q6a)



$x^2 + y^2 = r^2$
$(x \geq 0)$

splitting the hemisphere up into lots of thin discs ...

for one disc

$\delta m = \rho\pi y^2 \delta x$

$\delta\bar{x} = x$

$\delta m \cdot \delta x = (\rho\pi y^2 \delta x)x$
$= m \cdot x$

entire "mass" $= \frac{2}{3}\rho\pi r^3$

$\bar{x}\sum m_i = \sum m_i x_i$

$\bar{x}\left(\frac{2\rho\pi r^3}{3}\right) = \sum_{x=0}^{r}(\rho\pi x y^2)\delta x$

$\lim_{\delta x \to 0}\sum_{x=0}^{r}\rho\pi x y^2 \delta x = \rho\pi\int_0^r (x\ y^2)\,dx$

$$= p\pi \int_0^r (r^2 - x^2)x \, dx \quad = p\pi \int_0^r r^2 x - x^3 \, dx$$

$$= p\pi \left[\frac{r^2 x^2}{2} - \frac{x^4}{4} \right]_0^r = p\pi \left[\frac{r^4}{2} - \frac{r^4}{4} \right]$$

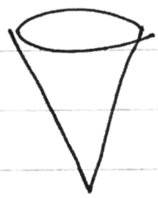
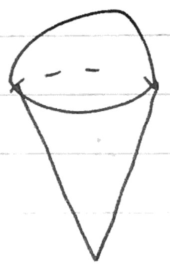
$$= \frac{p\pi r^4}{4} \ //$$

$$\therefore \ \bar{x} \sum m_i = \bar{x} \left(\frac{2 \cancel{p\pi} r^3}{3} \right) = \frac{\cancel{p\pi} r^4}{4}$$

$$\Rightarrow \ \bar{x} \left(\frac{2}{3} \right) = \frac{r}{4}$$

$$\Rightarrow \ \bar{x} = \frac{3r}{4 \times 2} = \boxed{\frac{3r}{8}}$$

b)

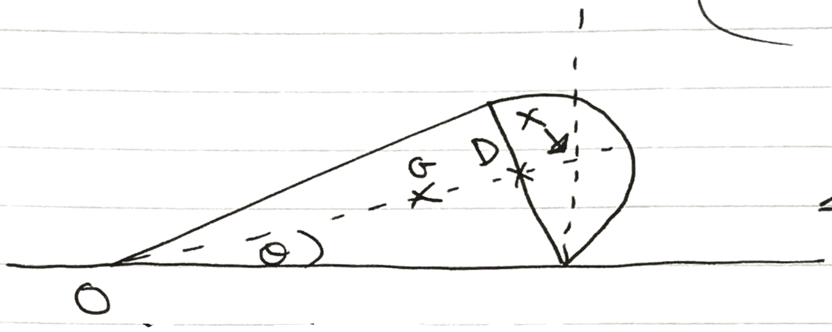
Shape	Mass (vol.)	Distance of c.o.m from O
	$\boxed{m}$	$4r + \frac{3r}{8} = \boxed{\frac{35r}{8}}$
	$\boxed{M}$	$\frac{3}{4}(4r) = \boxed{3r}$
	$\boxed{m+M}$	$\boxed{\bar{y}}$

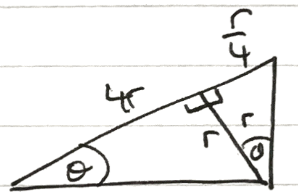
Moments about vertex O

$$\frac{35mr}{8} + 3Mr = (m+M)\bar{y}$$

$$\frac{(35m + 24M)r}{8(M+m)} = \bar{y}$$

c)





=> in equilibrium ∴ $OG \le OX$

ie $\bar{y} \le 4r + \frac{r}{4}$ ← similar triangles (ODA and ADX)

$$\bar{y} \le \frac{17r}{4}$$

$$\frac{(35m + 24M)\cancel{r}}{8(M+m)} \le \frac{17\cancel{r}}{4}$$

$$35m + 24M \le 34M + 34m$$

$$(35-34)m \le (34-24)M$$

$$10M \geq m$$

hence $M \geq \frac{m}{10}$

Q7a) $v > 0$ for complete circle (at top)

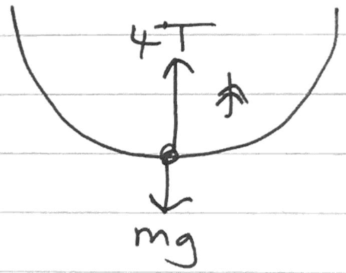
C.O.E : Total energy at A > GPE at top.

$$\frac{1}{2}mu^2 + mgl\left(\frac{4}{5}\right) > mgl$$

$$\frac{1}{2}\cancel{m}u^2 > \frac{\cancel{m}gl}{5}$$

$$u^2 > \frac{2gl}{5} \quad \therefore \quad u > \sqrt{\frac{2gl}{5}}$$

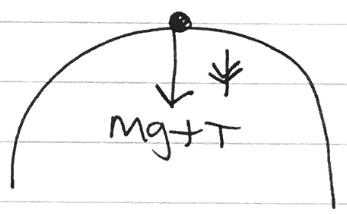
b) Tension max at bottom.



N2L $\uparrow^+$ (P) : $4T - mg = \frac{mv^2}{l}$

$$4T = mg + \frac{mv^2}{l} \sim (1)$$

Tension minimum at top.



N2L(P) $\downarrow_+$: $mg + T = \frac{mv^2}{l}$

×4 : $4mg + 4T = \frac{4mv^2}{l}$

ΔKE ↓ ΔGPE ↓

Energy to bottom : $\frac{mv^2}{2} - \frac{mu^2}{2} = mgl(1+\cos\alpha)$

$\therefore v^2 = 2gl(1+\cos\alpha) + u^2$

$v^2 = \frac{18gl}{5} + u^2$

at top ; $\frac{mu^2}{2} - \frac{mv^2}{2} = mgl(1-\cos\alpha)$

$u^2 - v^2 = 2gl\left(\frac{1}{5}\right)$

$v^2 = u^2 - \frac{2gl}{5}$

now equate (1) and (2) and sub v^2:

$mg + \frac{m}{l}\left(u^2 + \frac{18gl}{5}\right) = \frac{4m}{l}\left(u^2 - \frac{2gl}{5}\right) - 4mg$

$mg + \frac{mu^2}{l} + \frac{18mg}{5} = \frac{4mu^2}{l} - \frac{8mg}{5} - 4mg$

$\frac{3mu^2}{l} = \left(\frac{18}{5} + \frac{8}{5} + 5\right)mg$

$\frac{3u^2}{l} = \frac{51}{5}g$

$\therefore u^2 = \frac{51gl}{5\times 3} = \frac{17gl}{5}$ hence $u = \sqrt{\frac{17gl}{5}}$