M3 18th May UK.

Q1

a) $F = mv\frac{dv}{dx}$ $\qquad v = \frac{12}{x+3} \qquad \frac{dv}{dx} = \frac{-12}{(x+3)^2}$

$$= \tfrac{1}{2}\left(\frac{12}{x+3}\right)\left(-\frac{12}{(x+3)^2}\right) = \frac{-72}{(x+3)^3}. \qquad x = 3$$

$$F = \frac{-72}{(6)^3} = -\tfrac{1}{3}N \quad \text{so}$$

Magnitude $\tfrac{1}{3}N$ or $0.333N$ 3s.f.

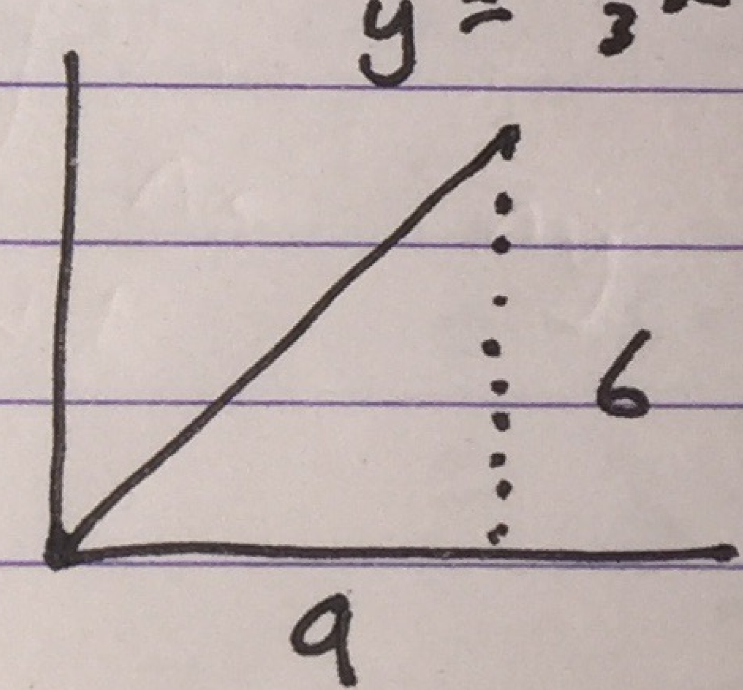
b) $$\frac{dx}{dt} = \frac{12}{x+3} \implies \frac{(x+3)^2}{2} = 12t + c$$

$$\frac{7^2}{2} - 24 = c = \tfrac{1}{2}$$

So $$\frac{(x+3)^2}{2} = 12t + \tfrac{1}{2}$$

$$\frac{(13)^2}{2} - \tfrac{1}{2} = 12t = 84 \implies t = 7s.$$

Q2 Put C at O.



So $M = \tfrac{1}{2}(6)(9)\rho = 27\rho$

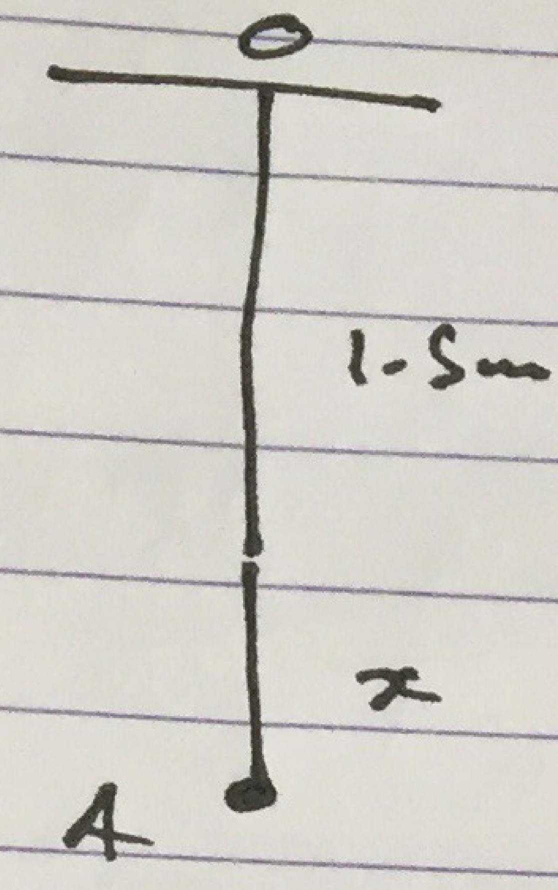
$$27\rho\,\bar{x} = \rho\int_0^9 \tfrac{2}{3}x^2\,dx = \rho\left[\frac{2x^3}{9}\right]_0^9 = 162\rho$$

$$\implies \bar{x} = \frac{162\rho}{27\rho} = 6\text{cm} \quad \text{from C}$$

So $9 - 6 = 3$cm from AB.

Q3

a)



$$EPE \text{ at } A = \frac{14.7x^2}{3} = 4.9x^2$$

$$GPE \text{ At } O = 0.6g(1.5+x)$$

$$\text{So } \frac{g}{2}x^2 = 0.6g(1.5+x)$$

$$x^2 = 1.2(1.5+x)$$

$$x^2 - 1.2x - 1.8 = 0$$

$$x = \frac{1.2 \pm \sqrt{1.2^2 + 4(1.8)}}{2} = \frac{3 \pm 3\sqrt{6}}{5} \qquad x \neq \frac{3-3\sqrt{6}}{5}$$

$$\text{So } \quad x = \frac{3+3\sqrt{6}}{5} \implies OA = \frac{3+3\sqrt{6}}{5} + 1.5$$

$$= 3.5696938\text{m}$$

$$= \underline{3.57\text{m}}.$$

b) $|T - mg| = m|a|$ Modulus signs not required

$$\frac{14.7}{1.5}\left(\frac{3+3\sqrt{6}}{5}\right) - 0.6g = 0.6a$$

$$0.6a = 14.4029969.$$

$$a = 24.00499949\ ms^{-2}$$

$$\underline{a = 24.0ms^{-2}(3sf)}$$

Q4

Object	Mass	Dist of com. O	Moment
(1) cone, length kh, radius $2r$	$\frac{1}{3}\pi(r)^2 kh\rho$	$\frac{kh}{4}$	$\frac{\pi r^2h^2}{12}\rho(k^2)$
(2) cone, length $2h$	$\frac{1}{3}\pi(r^2)(2h)\rho$	$-\frac{h}{2}$	$\frac{-\pi r^2h^2\rho}{3}$
	$\frac{\pi r^2\rho h}{3}(k+2)$	$\bar{x}$	$\pi r^2h^2\rho\left(\frac{k^2}{12}-\frac{1}{3}\right)$

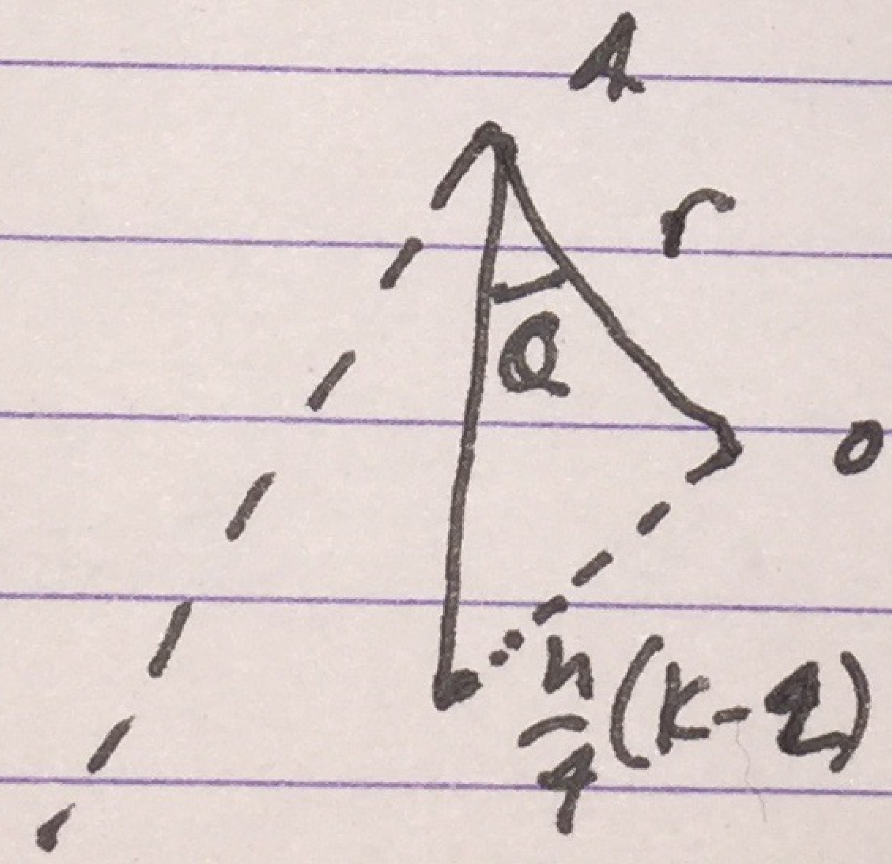
So $\frac{\pi r^2\rho h}{3}(k+2)\bar{x} = \pi r^2h^2\rho\left(\frac{k^2-4}{12}\right)$

$\frac{h}{3}(k+2)\bar{x} = h^2\left(\frac{(k-2)(k+2)}{12}\right)$

$\frac{h}{3}\bar{x} = h^2\left(\frac{k-2}{12}\right)$

$\bar{x} = \frac{3}{12}h(k-2) = \frac{h}{4}(k-2)$

6)

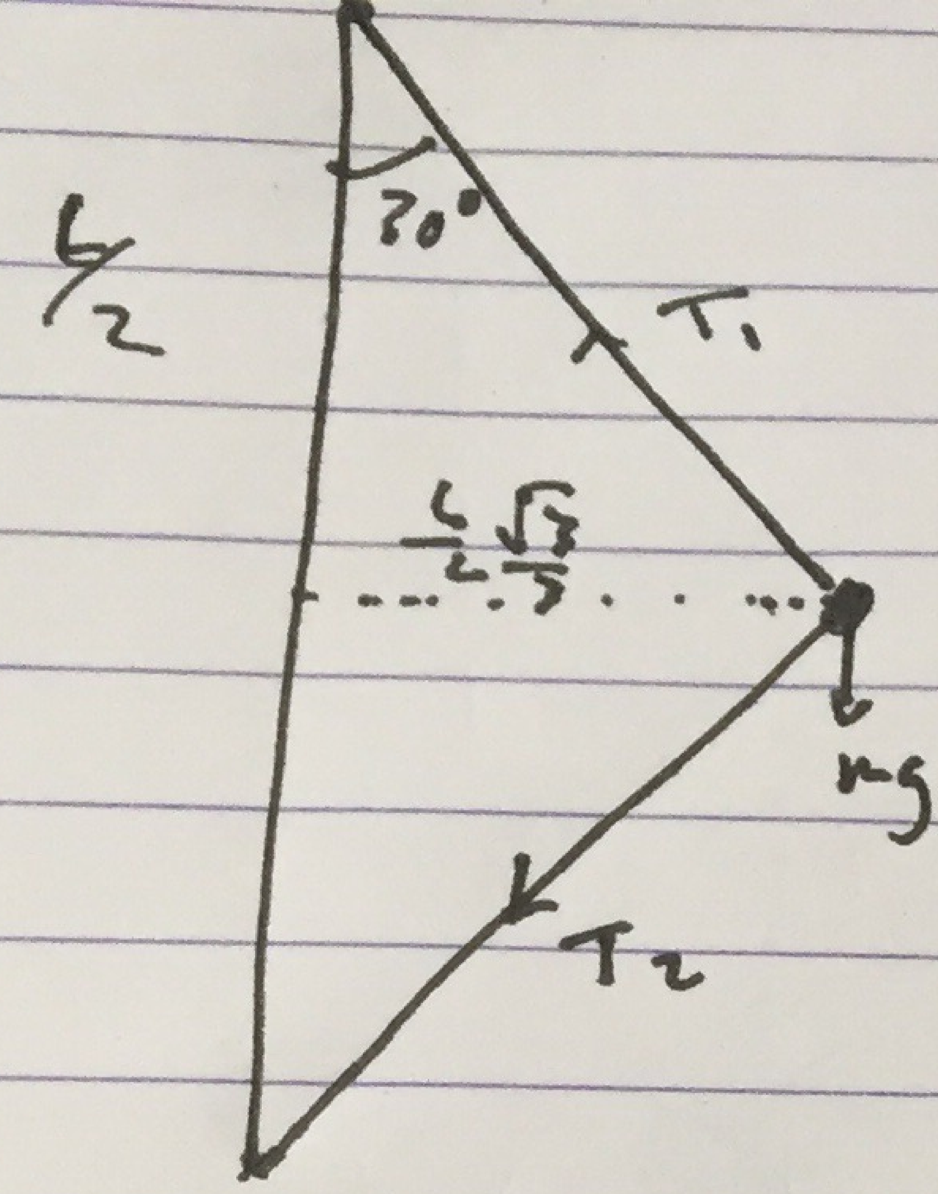


$\frac{h}{4}(6-2) = h = \frac{1}{3}r$

$\tan\theta = \frac{\frac{h}{4}(k-2)}{r} = \frac{\frac{1}{3}r}{r} = \frac{1}{3}$

$\theta = 18.4°$ (3sf)

Q5



$r = \frac{L}{2} \cdot \frac{\sqrt{3}}{3} = \frac{L\sqrt{3}}{6}$

R(↑) $\quad T_1 \cos 30 - T_2 \cos 30 = mg$

$$T_1 - T_2 = \frac{2}{\sqrt{3}} mg \qquad ①$$

R(←) $\quad T_1 \sin 30 + T_2 \sin 30 = m\left(\frac{L\sqrt{3}}{6}\right)\omega^2$

$$T_1 + T_2 = \frac{mL\sqrt{3}}{3}\omega^2 \qquad ②$$

i) Adding ①+② => $2T_1 = \frac{2}{\sqrt{3}} mg + \frac{mL\sqrt{3}}{3}\omega^2$

$$T_1 = \frac{mg}{\sqrt{3}} + \frac{mL\sqrt{3}}{6}\omega^2$$

$$T_1 = \frac{m\sqrt{3}}{6}\left(L\omega^2 + 2g\right)$$

ii) Subtracting ②−① >> $T_1 = \frac{mL\sqrt{3}\,\omega^2}{6} - \frac{mg}{\sqrt{3}}$

$$= \frac{m\sqrt{3}}{6}\left(L\omega^2 - 2g\right)$$

b) $T_1 > 0 \qquad L\omega^2 - 2g > 0 \qquad \omega^2 > \frac{2g}{L}$

$\omega > \sqrt{\frac{2g}{L}} \qquad \frac{2\pi}{T} > \sqrt{\frac{2g}{L}} \qquad 2\pi > T\sqrt{\frac{2g}{L}}$

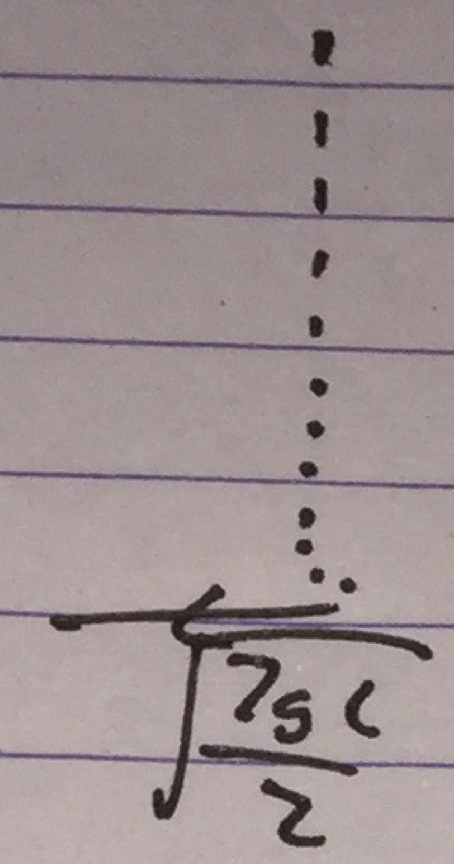
$T < \frac{2\pi}{\sqrt{\frac{2g}{L}}} = 2\pi\sqrt{\frac{L}{2g}}$ 2 into the sqrt

$$= \pi\sqrt{\frac{4L}{2g}} = \pi\sqrt{\frac{2L}{g}}$$

Q8 Mass: $\underline{\underline{2m}}$

$v \uparrow$

a)

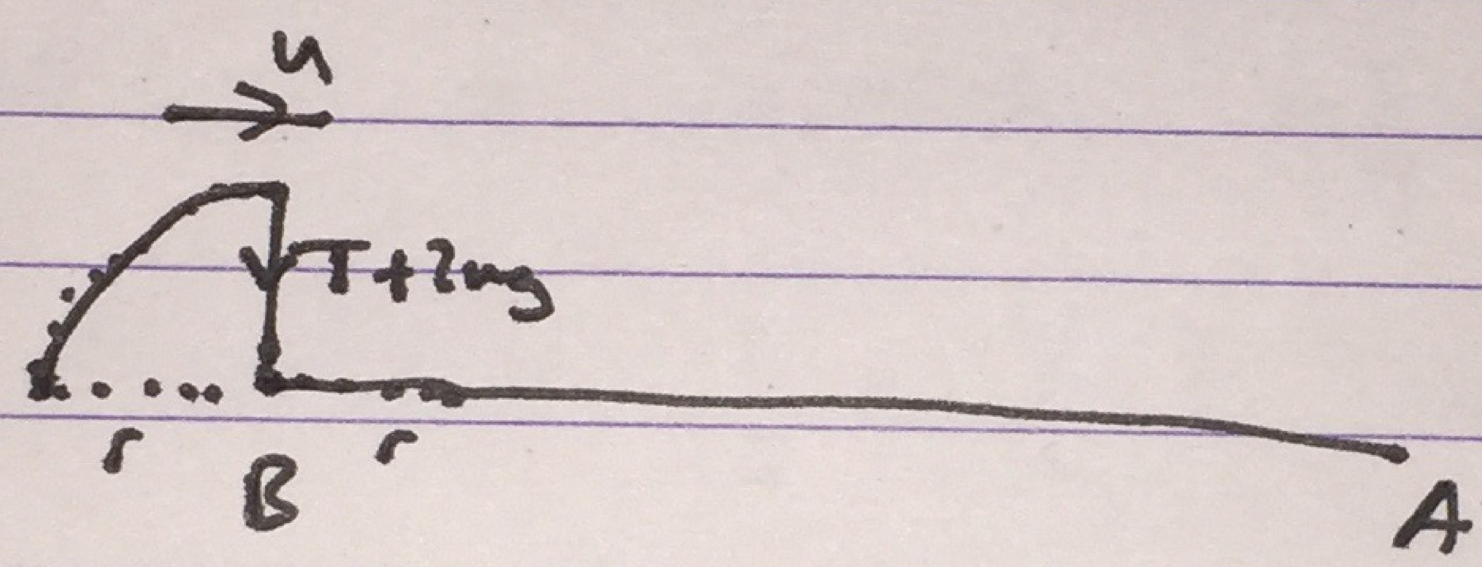


$2mgl + \frac{1}{2}(2m)v^2 = \frac{1}{2}(2m)\left(\frac{7gl}{2}\right)$

$gl + \frac{v^2}{2} = \frac{7gl}{4} \Rightarrow \frac{v^2}{2} = \frac{3gl}{4}$ $\quad v^2 = \frac{3gl}{2}$

$\underline{\underline{v = \sqrt{\frac{3gl}{2}}}}$

b)



$\underline{\underline{AB = l - r}}$

At top $T \geq 0$

$T + 2mg = \frac{2m(u^2)}{r}$

By conservation of energy $\frac{1}{2}(2m)u^2 + 2mgr = \frac{1}{2}(2m)\left(\frac{3gl}{2}\right)$

$\frac{u^2}{2} = \frac{3gl}{4} - gr \quad \underline{\underline{u^2 = \frac{3gl}{2} - 2gr}}$

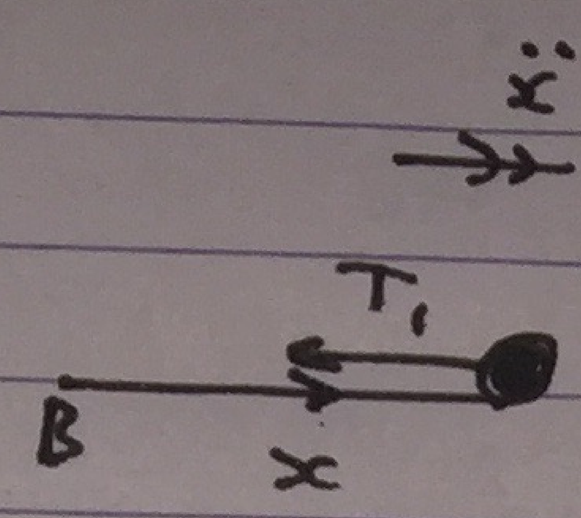
So $T + 2mg = \frac{2m\left(\frac{3gl}{2} - 2gr\right)}{r} = \frac{3mgl - 4mgr}{r} = \frac{3mgl}{r} - 4mg$

$T = \frac{3mgl}{r} - 6mg \geq 0 \Rightarrow \frac{3l}{r} \geq 6 \quad r \leq l/2$

So $-r \geq -l/2$ Adding l

$\underline{\underline{-l/2 + l = l/2 \leq -r + l = AB}}$

Q7



a) i) $R(\rightarrow)$ $\quad m\ddot{x} = -\dfrac{15x}{1.2} = -\dfrac{25}{2}x$

$\ddot{x} = -25x \quad \therefore$ SHM $\quad \omega^2 = 25$

$\omega = 5$

ii) $T = 2\pi/\omega = \dfrac{2\pi}{5}$

b) $v = a\omega = 5(0.8) = 4\,ms^{-1}$.

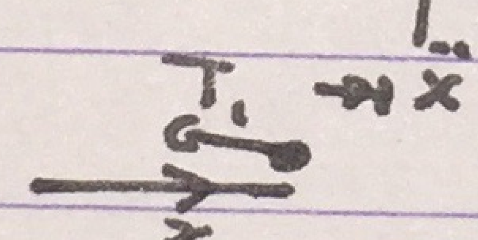
c) $DB = 0.6m$

So $\quad x = 0.8\cos(5t)$

D behind B $\quad x = -0.6 \quad -\dfrac{3}{4} = \cos(5t)$

$t = \dfrac{1}{5}\cos^{-1}\left(-\dfrac{3}{4}\right) = 0.484s$ (3sf)

d) $m = 0.8kg \quad T_1$ is unchanged.

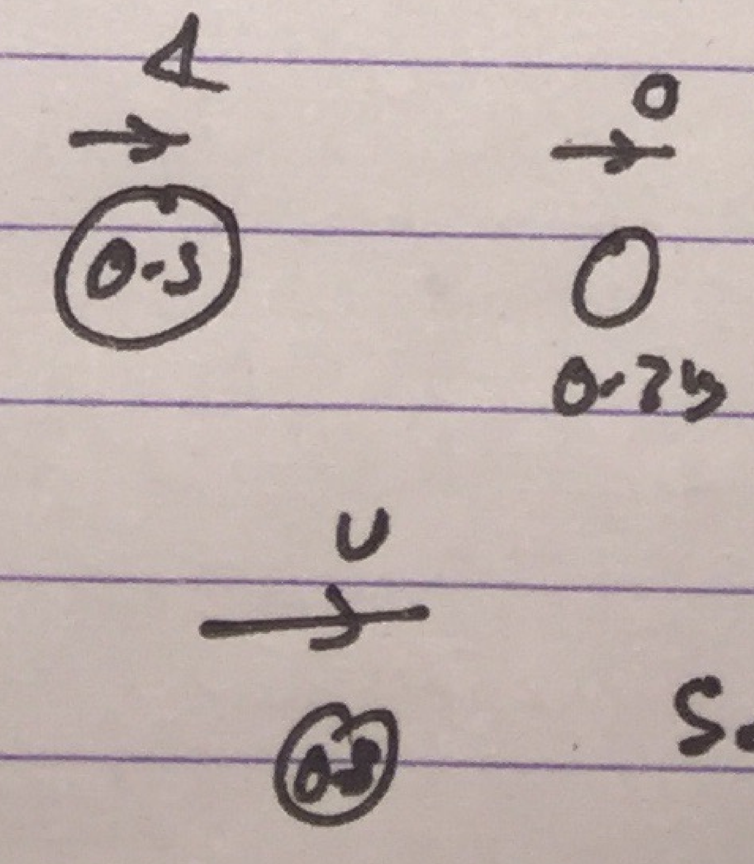


$0.8\ddot{x} = \dfrac{-15}{1.2}x = -\dfrac{25}{2}x$

$\ddot{x} = -\dfrac{125}{8}x \quad \therefore$ SHM. $\quad \omega^2 = \dfrac{125}{8}$

$\omega = \sqrt{\dfrac{125}{8}}$

e) COLM:



$0.8v = 0.5 \times 4$

$v = 2.5\,ms^{-1}$

So $\quad 2.5 = \sqrt{\dfrac{125}{8}}\,a$

$a = \dfrac{\sqrt{10}}{5}m = 0.632m$ (3sf)