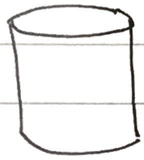
M3 January 2018 (IAL) (MA)

Q1)

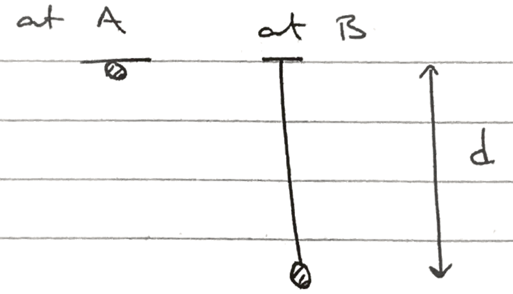
	Shape	Mass (vol.)	$\uparrow^{+}$ Displacement of c.o.m from O
(+)		$\frac{1}{3}\pi(r)^2(4h) = \boxed{\frac{4\pi r^2 h}{3}}$	$\frac{4h}{4} = \boxed{h}$
(+)		$\pi(r)^2(3h) = \boxed{3\pi r^2 h}$	$\boxed{-\frac{3h}{2}}$
(=)		$\boxed{\frac{13}{3}\pi r^2 h}$	$\boxed{\bar{y}}$

taking moments about O . . .

$$\frac{4}{3}(h) + 3\left(-\frac{3h}{2}\right) = \frac{13}{3}(\bar{y})$$

$$\frac{\frac{4h}{3} - \frac{9h}{2}}{\frac{13}{3}} = \bar{y} = \boxed{\frac{19h}{26}}$$

Q2)



at A : $GPE = 0.9gd$

$KE = EPE = 0$

at B : $GPE = 0$

$KE = 0$

$EPE = \frac{\lambda x^2}{2l} = \frac{29.4(d-1.2)^2}{2.4}$

C.O.E : $0.9gd = \frac{49}{4}(d-1.2)^2$

$\frac{18}{245}gd = d^2 - 2.4d + 1.44$

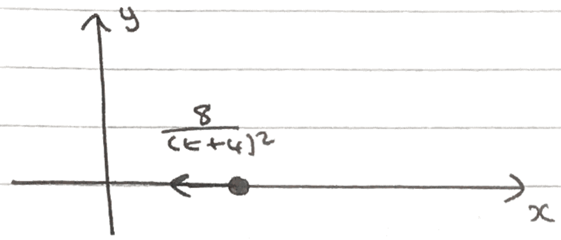
$d^2 - 3.12d + 1.44 = 0$

By Quadratic Formula : $d = 2.56$

$d = 0.56$

$d > 1.2$ so $d = \boxed{2.56 = AB}$

Q3a)



F=ma :
$$\frac{-8}{(t+4)^2} = 0.4a$$

$$\frac{-8}{(t+4)^2} = 0.4\frac{dv}{dt}$$

$$\frac{-20}{(t+4)^2} = \frac{dv}{dt}$$

$$\int (1)\,dv = -20\int (t+4)^{-2}\,dt$$

$$v = -20\left[-(t+4)^{-1}\right] + c$$

$$v = \frac{20}{t+4} + c$$

t=0, v=10 : $10 = \frac{20}{4} + c$

$$c = 10 - 5 = 5$$

$$\boxed{v = \frac{20}{t+4} + 5}$$

b) $v = \frac{20}{t+4} + 5$

$x = \int (v)\,dt = \int \left[\frac{20}{t+4} + 5\right] dt$

$x = 20\ln|t+4| + 5t + c$

$t=0,\ x=0$: $0 = 20\ln 4 + c$

$c = -20\ln 4$

$x = 20\ln|t+4| + 5t - 20\ln 4$

$x = 20\ln\left|\frac{t+4}{4}\right| + 5t$

at $v=6\ldots$ $6 = \frac{20}{t+4} + 5$

$1 = \frac{20}{t+4}$

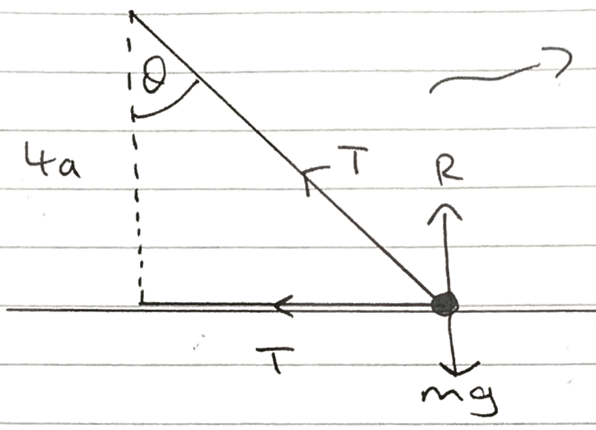
$t+4 = 20 \quad \therefore t = 16$ at $v=6$

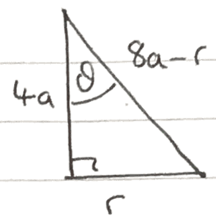
so at $v=6\ldots$ $x = 20\ln\left|\frac{20}{4}\right| + 80$

$x = 20\ln 5 + 80$

$b = 20$
$a = 80$

(Q4)





$r^2 + (4a)^2 = (8a - r)^2$

$r^2 + 16a^2 = (8a - r)^2$

$\cancel{r^2} + 16a^2 = 64a^2 - 16ar + \cancel{r^2}$

$48a^2 = 16ar$

$\therefore r = \frac{48a}{16}$

$= 3a$ //

so $\cos\theta = \frac{4}{5}$

$\sin\theta = \frac{3}{5}$

$R(\updownarrow)$: $T\cos\theta + R = mg$

$R = mg - \frac{4}{5}T$ //

$\overset{+}{\leftarrow}$ N2L(P) : $T + T\sin\theta = m(3a)\omega^2$

$T(1 + \sin\theta) = 3ma\omega^2$

$T = \frac{3ma\omega^2}{1 + \frac{3}{5}} = \frac{15}{8}ma\omega^2$ //

$\therefore R = mg - \frac{4}{5}\left(\frac{15}{8}\right)ma\omega^2$

$R = mg - \frac{3}{2}ma\omega^2$

but $R \geq 0$ for this circular motion to continue.

$\therefore \cancel{m}g - \frac{3}{2}\cancel{m}a\omega^2 \geq 0$

$g \geq \frac{3}{2}a\omega^2$

$\omega^2 \leq \frac{2g}{3a}$ //

So $\omega \leq \sqrt{\frac{2g}{3a}}$

now $T = \frac{2\pi}{\omega}$ $\longrightarrow$ $\omega = \frac{2\pi}{T}$

$$\omega \leq \sqrt{\frac{2g}{3a}}$$

so $\frac{2\pi}{S} \leq \sqrt{\frac{2g}{3a}}$

take reciprocal from each side . . .
so sign will flip :

$$\frac{S}{2\pi} \geq \sqrt{\frac{3a}{2g}}$$

$$S \geq 2\pi\sqrt{\frac{3a}{2g}}$$

$$S \geq \pi\sqrt{\frac{4 \times 3a}{2g}}$$

$$S \geq \pi\sqrt{\frac{6a}{g}}$$

Q5a) $V = \pi\int_0^{\pi/2} y^2\,dx = \pi\int_0^{\pi/2}[\sin^2 x]\,dx$

$\cos 2x = 1 - 2\sin^2 x$

$\therefore \sin^2 x = \dfrac{1-\cos 2x}{2}$

$V = \frac{1}{2}\pi\int_0^{\pi/2}[1-\cos 2x]\,dx$

$= \frac{\pi}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^{\pi/2}$

$= \frac{\pi}{2}\left[\frac{\pi}{2} - \cancel{\frac{1}{2}\sin\pi}^{\,0}\right] = \boxed{\frac{\pi^2}{4}}$

b) $M\bar{x} = \pi\int_a^b y^2 x\,dx = \pi\int_0^{\pi/2}[x\sin^2 x]\,dx$

By Parts : $\dfrac{dv}{dx} = \sin^2 x$ from a

$v = \frac{1}{2}x - \frac{1}{4}\sin 2x$

$u = x$

$u' = 1$

$\Rightarrow M\bar{x} = \pi\left[\left[\frac{1}{2}x^2 - \frac{1}{4}x\sin 2x\right]_0^{\frac{\pi}{2}} - \int_0^{\pi/2}\left[\frac{1}{2}x - \frac{1}{4}\sin 2x\right]dx\right]$

$= \pi\left[\frac{\pi^2}{8}\right] - \pi\left[\frac{x^2}{4} + \frac{1}{8}\cos 2x\right]_0^{\pi/2}$

$$= \frac{\pi^3}{8} - \pi\left[\frac{\pi^2}{16} - \frac{1}{8}\right] + \pi\left[\frac{1}{8}\right]$$

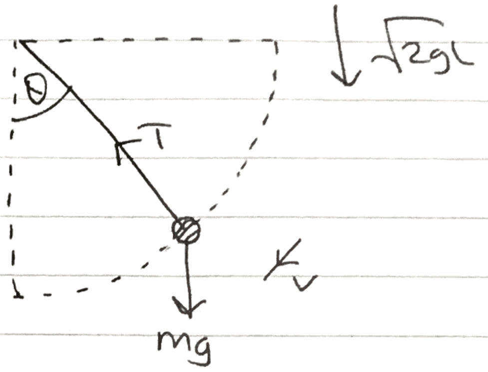
$$= \frac{\pi^3}{8} - \frac{\pi^3}{16} + \frac{\pi}{8} + \frac{\pi}{8} = \frac{\pi^3}{16} + \frac{\pi}{4} = M\bar{x}$$

$$\frac{M\bar{x}}{M} = \bar{x} = \frac{\cancel{\pi}\left(\frac{\pi^2}{16} + \frac{1}{4}\right)}{\frac{\pi \cancel{2}}{4}}$$

$$= \frac{\frac{\pi^2}{16} + \frac{1}{4}}{\frac{\pi}{4}} = \boxed{\frac{\pi^2 + 4}{4\pi}}$$

x16 to top and bottom

(Q6a)



at A : $KE = \frac{1}{2}m(2gl) = mgl$

$GPE = mgl$

at angle θ to d.v : $KE = \frac{1}{2}mv^2$

$GPE = mgl(1-\cos\theta)$

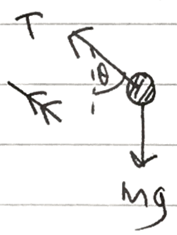
C.O.E : $mgl + mgl = \frac{1}{2}mv^2 + mgl(1-\cos\theta)$

$2mgl = \frac{1}{2}mv^2 + mgl - mgl\cos\theta$

$\times 2$: $2mgl + 2mgl\cos\theta = mv^2$

$v^2 = 2gl(1+\cos\theta)$ //

N2L $\nwarrow^{+}$ (P) : $T - mg\cos\theta = \frac{mv^2}{l}$



$T = mg\cos\theta + \frac{m}{l}(2gl)(1+\cos\theta)$

$T = mg\cos\theta + 2mg + 2mg\cos\theta$ //

hence $T = mg(3\cos\theta + 2)$

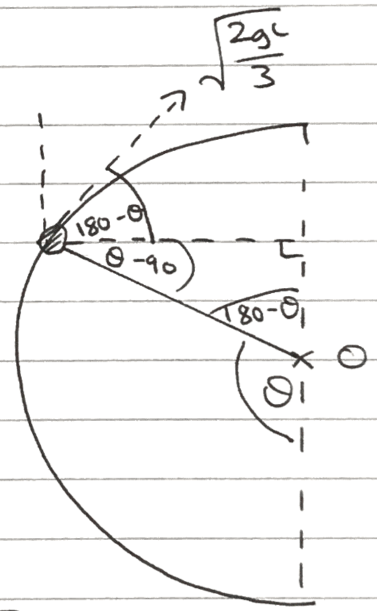
b) $\underline{T=0}$: $3\cos\theta + 2 = 0$

$\cos\theta = -\frac{2}{3}$

from (a), $v^2 = 2gl(1+\cos\theta)$

$v^2 = 2gl\left(1-\frac{2}{3}\right) = \frac{2gl}{3}$

hence $\boxed{v = \sqrt{\frac{2gl}{3}}}$



c) vertical speed at B = $\uparrow u$

$\uparrow u = \sqrt{\frac{2gl}{3}} \sin(180-\theta)$

$\sin\theta = \sqrt{1-\cos^2\theta}$

$\sin\theta = \sqrt{1-\left(-\frac{2}{3}\right)^2} = \frac{\sqrt{5}}{3}$

$\therefore \sin(180-\theta) = \sin(\theta) = \frac{\sqrt{5}}{3}$ //

so $\uparrow u = \frac{\sqrt{5}}{3} \times \sqrt{\frac{2gl}{3}} = \frac{1}{3}\sqrt{\frac{10gl}{3}}$ //

$\uparrow +$

$s = h$
$u = \frac{1}{3}\sqrt{\frac{10gl}{3}}$
$v = 0$
$a = -g$
$t =$

$v^2 = u^2 + 2as$

$0^2 = \frac{1}{9}\left(\frac{10gl}{3}\right) - 2gh$

$\frac{10gl}{27} = 2gh$

$\therefore h = \frac{5l}{27}$ //

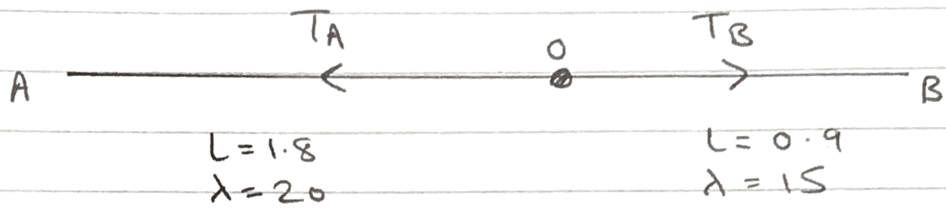
so height above O $= \frac{5l}{27} + l\cos(180 - \theta)$

$\sin(180 - \theta) = \frac{\sqrt{5}}{3} \quad \therefore \cos\theta = \sqrt{1 - \left(\frac{\sqrt{5}}{3}\right)^2} = \frac{2}{3}$

so height above O $= \frac{5l}{27} + \frac{2l}{3}$

$= \boxed{\frac{23l}{27}}$

(Q7a)



$$T_A = T_B$$

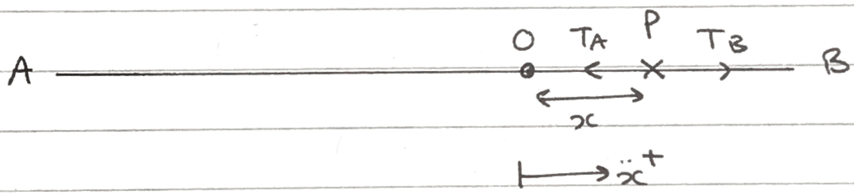
$$\frac{20}{1.8}(AO - 1.8) = \frac{15}{0.9}(4.2 - AO - 0.9)$$

$$\frac{100}{9}(AO) - 20 = 70 - 15 - \frac{50}{3}(AO)$$

$$\left(\frac{100}{9} + \frac{50}{3}\right)AO = 75 = \frac{250}{9}AO$$

$$\therefore AO = \frac{75}{\frac{250}{9}} = \boxed{2.7\text{m}}$$

b)



$\underline{N2L(P)}$: $T_B - T_A = m\ddot{x}$

$$\frac{15}{0.9}(0.6 - x) - \frac{20}{1.8}(0.9 + x) = m\ddot{x}$$

$$\Rightarrow 10 - \frac{50}{3}x - 10 - \frac{100}{9}x = m\ddot{x}$$

$$\Rightarrow -\frac{250}{9m} = \ddot{x}$$ hence P moves with S.H.M centre O.

c i) $AC = 2.9 \therefore a = 0.2m$

$\omega = \sqrt{\frac{250}{9 \times 10}} = \frac{5}{3}$

$J = 10(v-u)$

$V_{max} = a\omega = \frac{5}{3} \times 0.2 = \frac{1}{3}$ = speed right after impulse.

so $J = 10\left(\frac{1}{3} - 0\right) = \boxed{\frac{10}{3}}$

$x = a\sin\omega t$ as P starts at the centre of oscillation.

$x = 0.2\sin\left(\frac{5t}{3}\right)$

when total distance travelled = 0.5m, $x = -0.1$

$-0.1 = 0.2\sin\left(\frac{5t}{3}\right)$

$-\frac{1}{2} = \sin\left(\frac{5t}{3}\right)$

$t = \frac{3}{5}\sin^{-1}\left(-\frac{1}{2}\right) = \boxed{2.2s}$