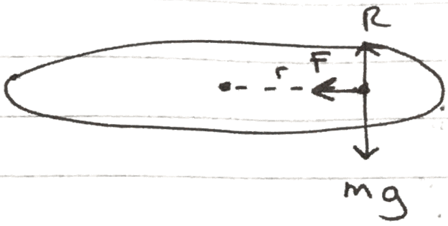
# M3 June 2018 (MA)

Q1)




$\overleftarrow{+}$ N2L (P) : $F = ma$

$F = mr\omega^2$

but $F \leq \mu R$ since P doesn't slip...

$\therefore F \leq \mu mg$ (since $R = mg$)

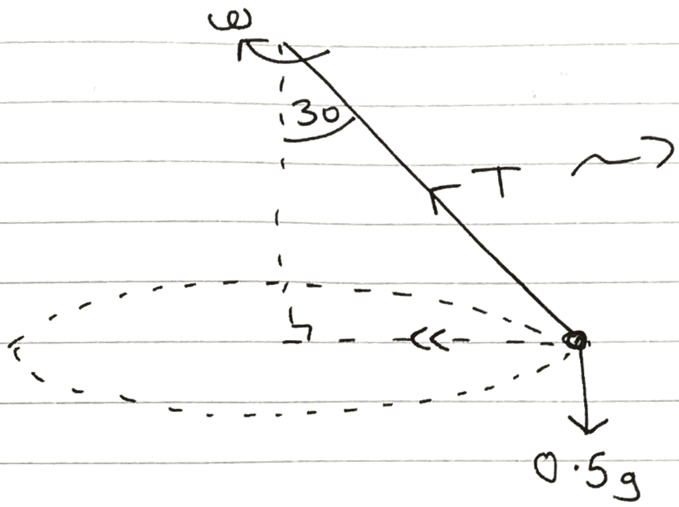
$\Rightarrow \cancel{m} r\omega^2 \leq \mu \cancel{m} g$

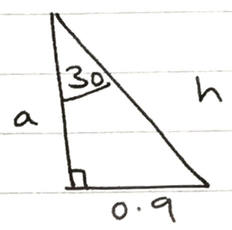
$\omega^2 \leq \frac{\mu g}{r}$

hence $\omega \leq \sqrt{\frac{\mu g}{r}}$

Q2)

a)




$$\tan 30 = \frac{0.9}{a}$$

$$a = \frac{0.9}{\tan 30} = \frac{9\sqrt{3}}{10}$$

$$\therefore h = \sqrt{\left(\frac{9\sqrt{3}}{10}\right)^2 + (0.9)^2}$$

$$h = 1.8$$

//

$$R(\updownarrow): \quad T\cos 30 = 0.5g$$

$$T = \frac{\lambda x}{l} = \lambda\left(\frac{1.8 - 1.2}{1.2}\right)$$

$$\therefore \lambda\left(\frac{1}{2}\right)\cos 30 = 0.5(9.8)$$

$$\lambda = \frac{0.5 \times 9.8}{\frac{1}{2} \times \cos 30} = \frac{98\sqrt{3}}{15} = \boxed{11.3}$$

b) $\overset{+\leftarrow}{\text{N2L (particle)}}$ : $T\sin 30 = mr\omega^2$

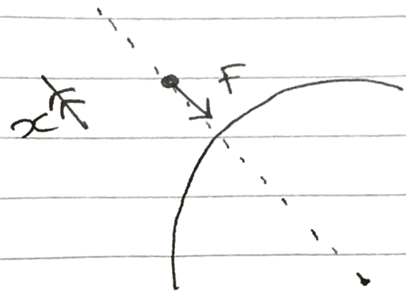
$$\frac{T}{2} = 0.5(0.9)\omega^2$$

$$\frac{T}{0.9} = \omega^2 = \frac{\lambda x}{0.9l} = \frac{(11.3)(0.6)}{0.9(1.2)}$$

$$= 6.287\ldots$$

so $\omega = \sqrt{6.287\ldots} = \boxed{2.51}$

Q3.) $-F = ma$




$-\frac{\cancel{m}gR^2}{x^2} = \cancel{m}v\frac{dv}{dx}$

$\int (v)\,dv = -gR^2 \int (x^{-2})\,dx$

$\frac{v^2}{2} = -gR^2\left[-\frac{1}{x}\right] + c$

$\frac{v^2}{2} = \frac{gR^2}{x} + c$

$v^2 = \frac{2gR^2}{x} + c'$

$x = 3R,\ v = \sqrt{\frac{gR}{3}}$ : $\frac{gR}{3} = \frac{2gR^{\cancel{2}}}{3\cancel{R}} + c'$

$c' = \frac{gR}{3} - \frac{2gR}{3} = -\frac{gR}{3}$

$\therefore v^2 = \frac{2gR^2}{x} - \frac{gR}{3}$

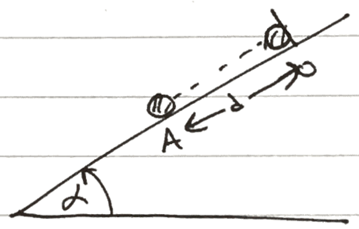
$v = 2\sqrt{\frac{gR}{3}}$ : $\frac{4\cancel{g}R}{3} = -\frac{\cancel{g}R}{3} + \frac{2\cancel{g}R^{\cancel{2}}}{x}$

$$\frac{5}{3} = \frac{2R}{x}$$

$$\frac{5}{3}x = 2R \rightarrow x = \frac{6R}{5}$$

$\therefore$ distance from surface of earth $= \frac{6R}{5} - R = \boxed{\frac{R}{5}}$

Q4)



$\sin\alpha = \frac{3}{5} \rightarrow \cos\alpha = \frac{4}{5}$

$\mu = \frac{1}{4}$

<u>Initially</u>

KE = 0
GPE = $mgd\sin\alpha$
EPE = 0

<u>Finally</u>

KE = 0
GPE = 0
EPE = $\frac{2mg}{2l}(d-l)^2$

W·D due to friction $= \mu R d = \frac{1}{4}(mg\cos\alpha)(d)$

$$= \frac{mgd}{5}$$

C.O.E

$$\Rightarrow mgd\left(\frac{3}{5}\right) = \frac{mg}{l}(d-l)^2 + \frac{mgd}{5}$$

$$\Rightarrow \frac{2mgd}{5} = \frac{mg}{l}(d^2 - 2dl + l^2)$$

$$\frac{2dl}{5} = d^2 - 2dl + l^2$$

$$d^2 - \frac{12dl}{5} + l^2 = 0$$

By Quadratic formula . . .

$a = 1$
$b = -\frac{12l}{5}$
$c = l^2$

$$d = \frac{\frac{12l}{5} \pm \sqrt{\left(\frac{12l}{5}\right)^2 - 4l^2}}{2}$$

$$d = \frac{\frac{12l}{5} \pm \sqrt{\frac{44}{25}l^2}}{2}$$

$$d = \frac{\frac{12}{5}l \pm 1.327l}{2}$$

$d > l$ so $d = l\left(\frac{12}{5} + 1.327\right)$

$\therefore k = \frac{12}{5} + 1.327$

$\simeq \boxed{1.86}$

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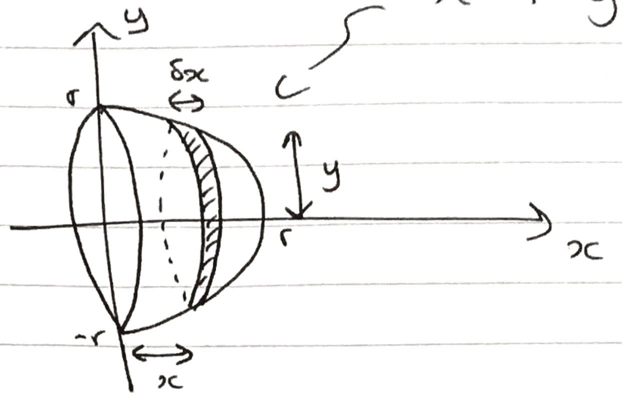
Q5a)

$x^2 + y^2 = r^2 \quad (x \geq 0)$




[splitting up the hemisphere into an infinite number of discs; thickness δx.

$\frac{\text{mass}}{\text{volume}} = \frac{m}{\frac{2}{3}\pi r^3} = \frac{3m}{2\pi r^3}$ for entire hemisphere.

so for one disc, mass $= \frac{3m}{2\pi r^3} \times \pi y^2 \delta x$

$m = \frac{3my^2 \delta x}{2r^3}$ //

recall from M2 that $\bar{x} \sum m_i = \sum m_i x_i$

$\Rightarrow \bar{x}(m) = \sum_{x=0}^{r} \frac{3my^2 \delta x}{2r^3} \times x$

$\lim_{x \to 0} \sum_{0}^{r} \frac{3mxy^2 \delta x}{2r^3} = \frac{3m}{2r^3} \int_0^r (xy^2)\, dx$

$\Rightarrow \frac{3m}{2r^3} \int_0^r x(r^2 - x^2)\, dx = \frac{3m}{2r^3} \int_0^r (r^2x - x^3)\, dx$

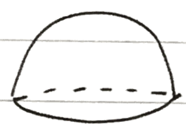
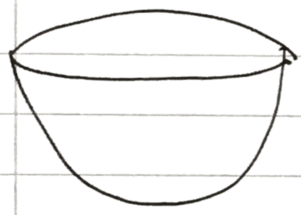
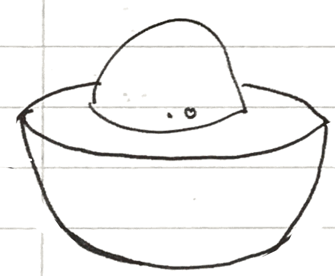
$= \frac{3m}{2r^3} \left[ \frac{r^2x^2}{2} - \frac{x^4}{4} \right]_0^r = \frac{3m}{2r^3} \left[ \frac{r^4}{2} - \frac{r^4}{4} \right] = \frac{3mr^4}{2(4r^3)}$

$= \frac{3mr}{2 \times 4}$ //

so $\bar{x}(m) = \frac{3mr}{8}$

hence $\bar{x} = \frac{3r}{8}$

b)

| Shape | Mass (vol.) | ↓+ Displacement of c.o.m from O |
|---|---|---|
|  | $\frac{2}{3}\pi\left(\frac{a}{2}\right)^3 \times 4$ $= \frac{2\pi 4a^3}{24}$ | $-\frac{3}{8}\left(\frac{a}{2}\right) = -\frac{3a}{16}$ |
|  | $\frac{2}{3}\pi a^3$ | $\frac{3a}{8}$ |
|  | $\pi a^3\left[\frac{2}{3} + \frac{24}{24}\right]$ | $\bar{y}$ |

Moments about a diameter through O

$$\frac{4}{12}\left(-\frac{3a}{16}\right) + \frac{2}{3}\left(\frac{3a}{8}\right) = \left(\frac{2}{3} + \frac{4}{12}\right)\bar{y}$$

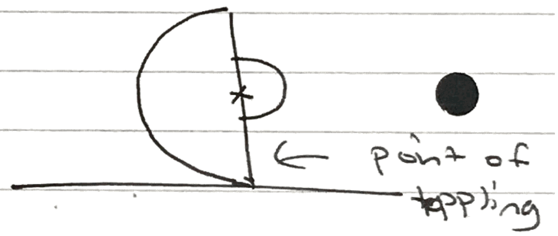
$$-\frac{34a}{192} + \frac{a}{4} = \left(\frac{2}{3} + \frac{4}{12}\right)\bar{y}$$

$$\bar{y} = \frac{a\left(\frac{1}{4} - \frac{3u}{192}\right)}{\left(\frac{2}{3} + \frac{u}{12}\right)} \begin{matrix} \times 192 \\ \times 192 \end{matrix}$$

$$= \frac{a(48 - 3u)}{128 + 16u} \quad //$$

$$\therefore \underline{\text{distance}} = \boxed{\left|\frac{a(48-3u)}{16(u+8)}\right|}$$

c) the information given means that the centre of mass is at O.




so $\bar{y} = 0$

$$a(48 - 3u) = 0$$

$$48 - 3u = 0$$

$$u = \frac{48}{3} = \boxed{16}$$

Q6a)

**Initially**

$KE = \frac{1}{2}mu^2$

$GPE = 0$

**At angle $\theta$ to horizontal**

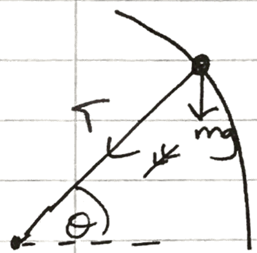
$KE = \frac{1}{2}mv^2$

$GPE = mga\sin\theta$

C.O.E : $\frac{mu^2}{2} = \frac{mv^2}{2} + mga\sin\theta$

$\times 2$ : $u^2 = v^2 + 2ag\sin\theta$

$v^2 = u^2 - 2ag\sin\theta$ //




N2L (↙+) : $T + mg\sin\theta = \frac{mv^2}{r}$

$T = \frac{m}{a}(u^2 - 2ag\sin\theta) - mg\sin\theta$

$T = \frac{mu^2}{a} - 2mg\sin\theta - mg\sin\theta$

$T = \frac{mu^2}{a} - 3mg\sin\theta$

$\therefore T = \frac{m}{a}(u^2 - 3ag\sin\theta)$

b) $T \geq 0$ at top.

min. value of $u$ is when $T = 0$. at top.

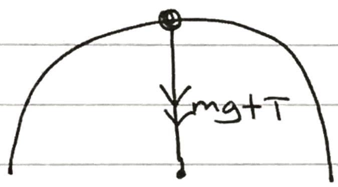
$\Rightarrow \quad \frac{m}{a}(u^2 - 3ag\sin\theta) = 0$

$\Rightarrow \quad u^2 - 3ag\sin\theta = 0$

at top, $\theta = 90°$ so $u^2 = 3ag$ //

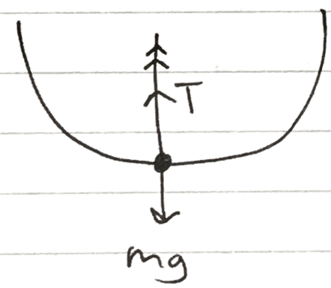
hence $u_{min} = \boxed{\sqrt{3ag}}$

c)




Tension is minimum at top.

$\underline{\theta = 90°}$: $T = S = \frac{m}{a}(u^2 - 3ag)$ // ~ ①




$\underline{\theta = 270°}$: (At bottom tension is max)

$T = 4S = \frac{m}{a}(u^2 + 3ag)$

$S = \frac{m}{4a}(u^2 + 3ag)$ // ~ ②

$\underline{① = ②}$: $\frac{m}{a}(u^2 - 3ag) = \frac{m}{4a}(u^2 + 3ag)$

$u^2 - 3ag = \frac{u^2}{4} + \frac{3ag}{4}$

so $\frac{3u^2}{4} = ag\left(3 + \frac{3}{4}\right)$

$u^2 = \frac{4ag}{3}\left(\frac{15}{4}\right)$

$u^2 = 5ag$ hence $\boxed{u = \sqrt{5ag}}$

(Q7a)

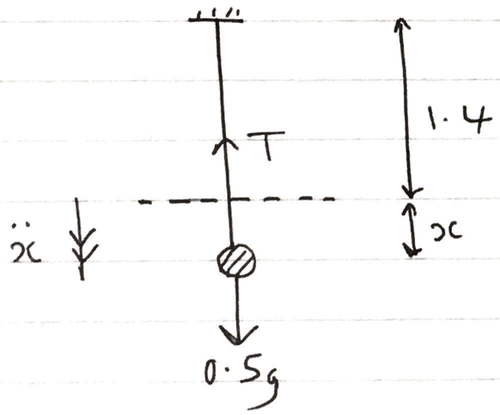

$T = mg$

$\frac{29.4(1.4 - l)}{l} = 4.9$

$41.16 - 29.4l = 4.9l$

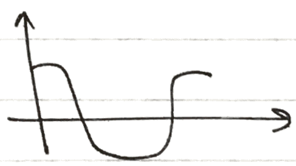
$l = \frac{41.16}{29.4 + 4.9} = \boxed{1.2}$

b)




$x$ = distance from equilibrium (B) here.

P starts at an endpoint so $x = a\cos\omega t$ applies



so $x$ is max at $t = 0$ which means $x$ increases in the direction AB. So $\ddot{x}$ is positive in this direction too.

N2L (P) ↓+ : $0.5g - T = 0.5\ddot{x}$

$g - 2T = \ddot{x}$

$g - 2\left(\frac{29.4}{1.2}\right)(x+0.2) = \ddot{x}$

$9.8 - 49(x) - 49(0.2) = \ddot{x}$

$9.8 - 9.8 - 49x = \ddot{x}$

hence $\ddot{x} = -49x$ so P moves with S.H.M.

c) $x = -0.2$ : $-0.2 = 0.4\cos(7t)$

$-\frac{1}{2} = \cos 7t$ $(\omega = \sqrt{49} = 7)$

$t = \frac{1}{7}\cos^{-1}\left(-\frac{1}{2}\right)$ = time from C to slack point

$\dot{x} = \left|-(0.4)(7)\sin\left(\cos^{-1}\left(-\frac{1}{2}\right)\right)\right| \simeq$ $\boxed{2.4\text{ ms}^{-1}}$

alt : $V^2 = \omega^2(a^2 - x^2)$

$= 49(0.4^2 - 0.2)^2 =$ $\boxed{2.4\text{ ms}^{-1}}$

d) when string is slack, P moves under gravity.

$+\uparrow$
$s =$
$u = 2.4$
$v = 0$
$a = -g$
$t = t$

$v = u + at$
$0 = 2.4 - gt$

$t = \frac{2.4}{g}$ = time from slack to D
= time from D to slack

but time from C to slack point is equal to the time from slack point to D.

$= \frac{1}{7}\cos^{-1}\left(-\frac{1}{2}\right)$ (from c)

So total time $= \frac{1}{7}\cos^{-1}\left(-\frac{1}{2}\right) + \frac{2.4}{g}$

$\simeq$ **0.547s**