Leave blank

1. The line l_1 has equation

$$10x - 2y + 7 = 0$$

(a) Find the gradient of l_1

(1)

The line l_2 is parallel to the line l_1 and passes through the point $\left(-\frac{1}{3}, \frac{4}{3}\right)$.

(b) Find the equation of l_2 in the form $y = mx + c$, where m and c are constants.

(3)

a) $10x + 7 = 2y$

$y = 5x + 3.5 \Rightarrow m = 5$

b) $y - y_1 = m(x - x_1)$

$y - \frac{4}{3} = 5\left[x - \left(-\frac{1}{3}\right)\right]$

$y = 5x + 3$

Leave blank

2. $$f(x) = x^4 - x^3 + 3x^2 + ax + b$$

where a and b are constants.

When $f(x)$ is divided by $(x - 1)$ the remainder is 4

When $f(x)$ is divided by $(x + 2)$ the remainder is 22

Find the value of a and the value of b.

(5)

$f(1) = (1)^4 - (1)^3 + 3(1)^2 + a(1) + b = 4$

$1 - 1 + 3 + a + b = 4$

$a + b = 1 \quad (1)$

$f(-2) = (-2)^4 - (-2)^3 + 3(-2)^2 + a(-2) + b = 22$

$16 + 8 + 12 - 2a + b = 22$

$b - 2a = -14 \quad (2)$

$(1) - (2)$: $a - (-2a) = 1 - (-14)$

$3a = 15$

$a = 5$

(1): $5 + b = 1$

$b = -4$

Leave blank

3. Given that

$$y = \frac{1}{27}x^3$$

express each of the following in the form kx^n where k and n are constants.

(a) $y^{\frac{1}{3}}$

(1)

(b) $3y^{-1}$

(1)

(c) $\sqrt{(27y)}$

(1)

a) $y^{1/3} = \left(\frac{1}{27}x^3\right)^{1/3} = \frac{1}{3}x^1$

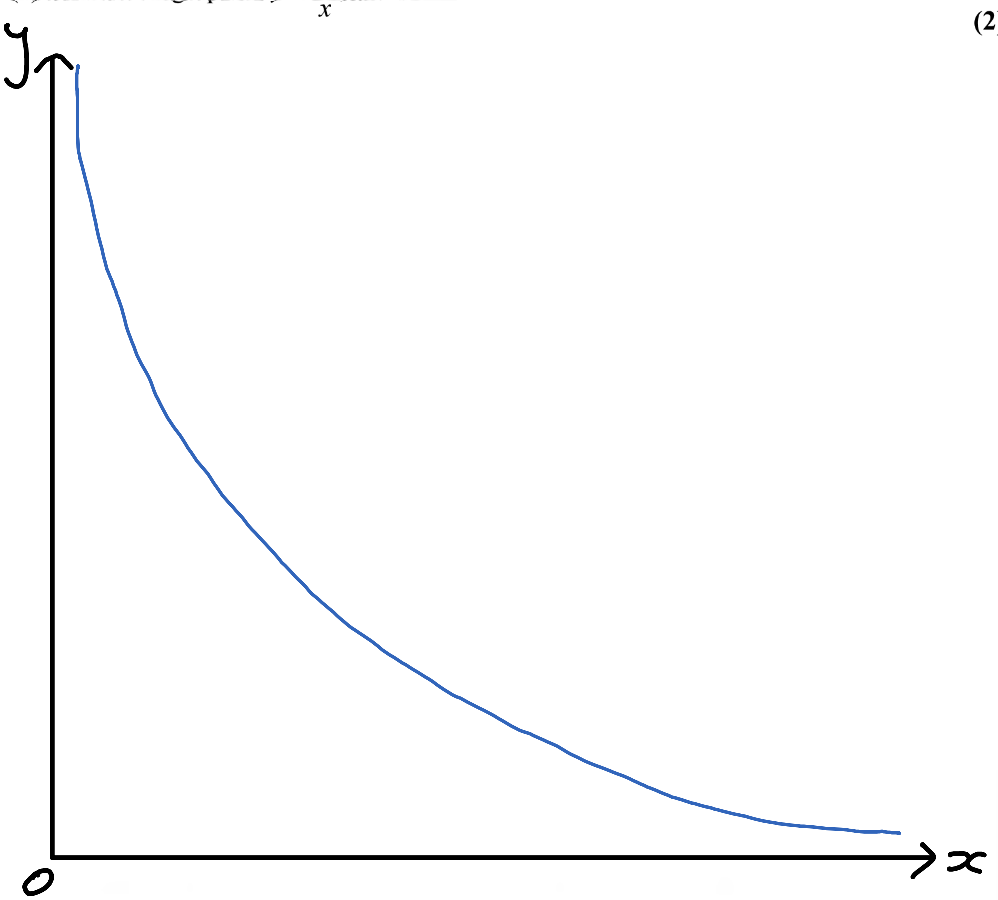
b) $3y^{-1} = 3\left(\frac{1}{27}x^3\right)^{-1} = 3(27)x^{-3} = 81x^{-3}$

c) $\sqrt{27y} = \left(27 \cdot \frac{1}{27}x^3\right)^{1/2} = (x^3)^{1/2} = x^{3/2}$

Leave blank

4. (a) Sketch the graph of $y = \frac{1}{x}, x > 0$

(2)



The table below shows corresponding values of x and y for $y = \frac{1}{x}$, with the values for y rounded to 3 decimal places where necessary.

x	1	1.5	2	2.5	3
y	1	0.667	0.5	0.4	0.333

(b) Use the trapezium rule with all the values of y from the table to find an approximate value, to 2 decimal places, for $\int_1^3 \frac{1}{x}\,dx$

(4)

$$\int_1^3 \frac{1}{x}\,dx \approx \frac{1}{2}(0.5)\left[1 + 0.333 + 2(0.667 + 0.5 + 0.4)\right]$$

$$= 1.12 \ (2dp)$$

Leave blank

5. (i) Find, giving your answer to 3 significant figures, the value of y for which

$$3^y = 12$$

(2)

(ii) Solve, giving an exact answer, the equation

$$\log_2(x+3) - \log_2(2x+4) = 4$$

(You should show each step in your working.)

(4)

i) $3^y = 12$

$y = \log_3 12 = 2.26$ (3sf)

ii) $\log_2(x+3) - \log_2(2x+4) = 4$

$\log_2\left(\frac{x+3}{2x+4}\right) = 4$

$\frac{x+3}{2x+4} = 2^4$

$32x + 64 = x + 3$

$31x = -61$

$x = -\frac{61}{31}$

Leave blank

6. (a) Find the first 3 terms in ascending powers of x of the binomial expansion of

$$(2 + ax)^6$$

where a is a non-zero constant. Give each term in its simplest form.

(4)

Given that, in the expansion, the coefficient of x is equal to the coefficient of x^2

(b) find the value of a.

(2)

a) $(2+ax)^6 = 2^6 + {}^6C_1 2^5 ax + {}^6C_2 2^4 (ax)^2 + \ldots$

$= 64 + 192ax + 240a^2x^2$

b) $192a = 240a^2$

$a = \frac{192}{240} = \frac{4}{5}$

Leave blank

7.

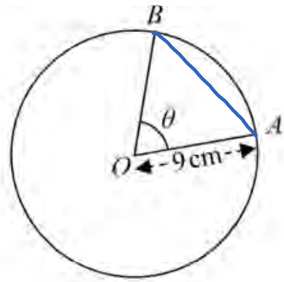


Figure 1

Figure 1 shows a circle with centre O and radius 9 cm. The points A and B lie on the circumference of this circle. The minor sector OAB has perimeter 30 cm and the angle between the radii OA and OB of this sector is θ radians.

Find

(a) the length of the arc AB, **(1)**

(b) the value of θ, **(2)**

(c) the area of the minor sector OAB, **(2)**

(d) the area of triangle OAB, giving your answer to 3 significant figures. **(2)**

a) $30 = 9 + 9 + \overset{\frown}{AB}$

$\overset{\frown}{AB} = 12\text{ cm}$

b) $12 = r\theta$

$\theta = \frac{12}{9} = \frac{4}{3}$

c) $A = \frac{1}{2}r^2\theta$

$= \frac{1}{2}(9)^2\frac{4}{3}$

$= 54\text{ cm}^2$

d) $A' = \frac{1}{2}r^2\sin\theta$

$= \frac{1}{2}(9)^2\sin\left(\frac{4}{3}\right)$

$= 39.4\text{ cm}^2$ (3sf)

Leave blank

8. A 25-year programme for building new houses began in Core Town in the year 1986 and finished in the year 2010.

The number of houses built each year form an arithmetic sequence. Given that 238 houses were built in the year 2000 and 108 were built in the year 2010, find

(a) the number of houses built in 1986, the first year of the building programme.

(5)

(b) the total number of houses built in the 25 years of the programme.

(2)

a) $u_{15} = 238$ $u_{25} = 108$

$a + 14d = 238$ (1) $a + 24d = 108$ (2)

(2)-(1): $10d = -130$

$d = -13$

(1): $a + 14(-13) = 238$

$a = 420$

b) $S_{25} = \frac{25}{2}(420 + 108) = 6600$

Leave blank

9. The equation $x^2 + (6k + 4)x + 3 = 0$, where k is a constant, has no real roots.

(a) Show that k satisfies the inequality

$$9k^2 + 12k + 1 < 0$$

(3)

(b) Find the range of possible values for k, giving your boundaries as fully simplified surds.

(4)

a) $\Delta < 0$

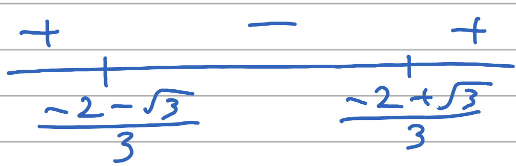
$(6k+4)^2 - 4(3) < 0$

$36k^2 + 48k + 16 - 12 < 0$

$9k^2 + 12k + 1 < 0$

b) $k = \dfrac{-12 \pm \sqrt{12^2 - 4(9)(1)}}{2(9)} = \dfrac{-12 \pm 6\sqrt{3}}{18}$

$= \dfrac{-2 \pm \sqrt{3}}{3}$



$\Rightarrow \dfrac{-2-\sqrt{3}}{3} < k < \dfrac{-2+\sqrt{3}}{3}$

Leave blank

10. A sequence is defined by

$$u_1 = 4$$

$$u_{n+1} = \frac{2u_n}{3}, \qquad n \geqslant 1$$

(a) Find the exact values of u_2, u_3 and u_4 **(2)**

(b) Find the value of u_{20}, giving your answer to 3 significant figures. **(2)**

(c) Evaluate

$$12 - \sum_{i=1}^{16} u_i$$

giving your answer to 3 significant figures. **(3)**

(d) Explain why $\sum_{i=1}^{N} u_i < 12$ for all positive integer values of N. **(1)**

a) $u_2 = \frac{2(4)}{3} = \frac{8}{3}$, $u_3 = \frac{2}{3} \times \frac{8}{3} = \frac{16}{9}$

$u_4 = \frac{2}{3} \times \frac{16}{9} = \frac{32}{27}$

b) G.P. with $a = 4$, $r = \frac{2}{3}$

$u_{20} = 4\left(\frac{2}{3}\right)^{19} = 0.00180$ (3sf)

c) $12 - \sum_{i=1}^{16} u_i = 12 - S_{16} = 12 - \frac{4\left[1 - \left(\frac{2}{3}\right)^{16}\right]}{1 - \left(\frac{2}{3}\right)}$

$= 0.0183$ (3sf)

d) $S_\infty = \frac{a}{1-r} = \frac{4}{1-\left(\frac{2}{3}\right)} = 12$

Leave blank

11. The curve C has equation $y = \mathrm{f}(x)$, $x > 0$, where

$$\mathrm{f}'(x) = 3\sqrt{x} - \frac{9}{\sqrt{x}} + 2$$

Given that the point P (9, 14) lies on C,

(a) find f(x), simplifying your answer,

(6)

(b) find an equation of the normal to C at the point P, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(5)

a) $f'(x) = 3x^{1/2} - 9x^{-1/2} + 2$

$f(x) = \frac{2}{3} \cdot 3x^{3/2} - 2(9)x^{1/2} + 2x + C$

$f(9) = 2(9)^{3/2} - 18(9)^{1/2} + 2(9) + C = 14$

$54 - 54 + 18 + C = 14$

$C = -4$

$\Rightarrow f(x) = 2x^{3/2} - 18x^{1/2} + 2x - 4$

b) $f'(9) = 3(9)^{1/2} - 9(9)^{-1/2} + 2 = 9 - 3 + 2$

$= 8$

$\Rightarrow m_N = -\frac{1}{8}$

$y - y_1 = m(x - x_1)$

$y - 14 = -\frac{1}{8}(x - 9)$

$8y - 112 = -x + 9$

$x + 8y - 121 = 0$

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12.

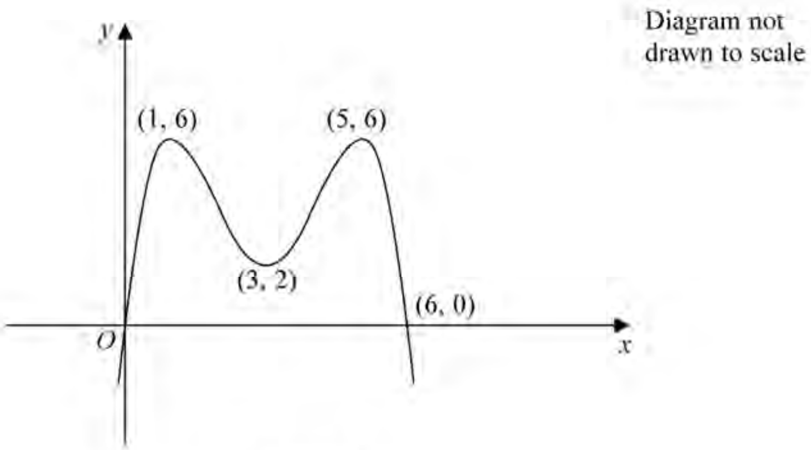


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$.

The curve crosses the x-axis at the origin and at the point (6, 0). The curve has maximum points at (1, 6) and (5, 6) and has a minimum point at (3, 2).

On **separate** diagrams sketch the curve with equation

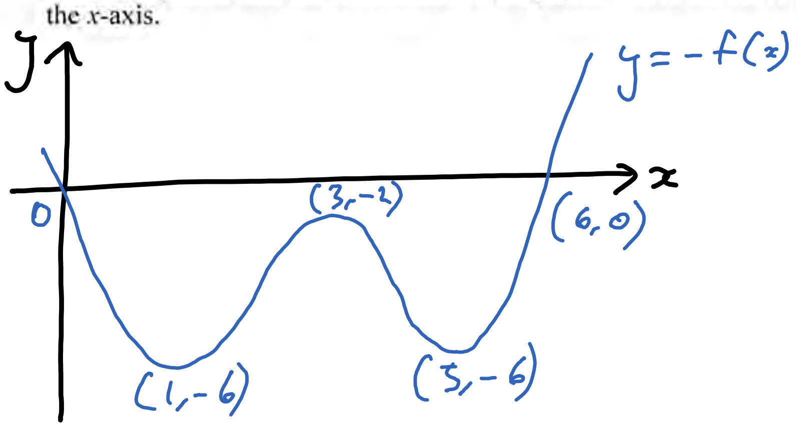
(a) $y = -\mathrm{f}(x)$ **(3)**

(b) $y = \mathrm{f}\left(\frac{1}{2}x\right)$ **(3)**

(c) $y = \mathrm{f}(x + 4)$ **(3)**

On each diagram show clearly the coordinates of the maximum and minimum points, and the coordinates of the points where the curve crosses the x-axis.

a)



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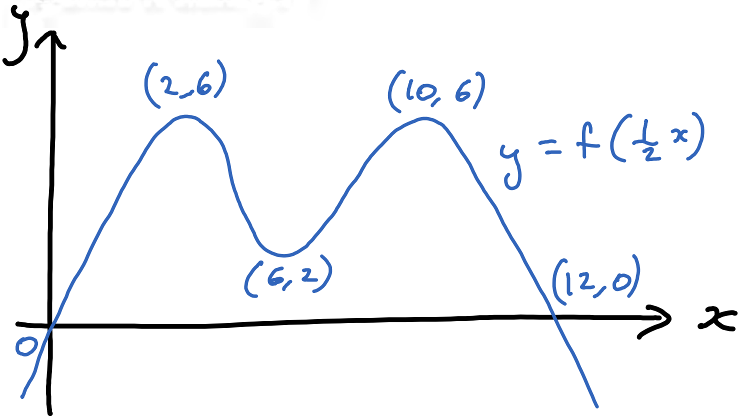
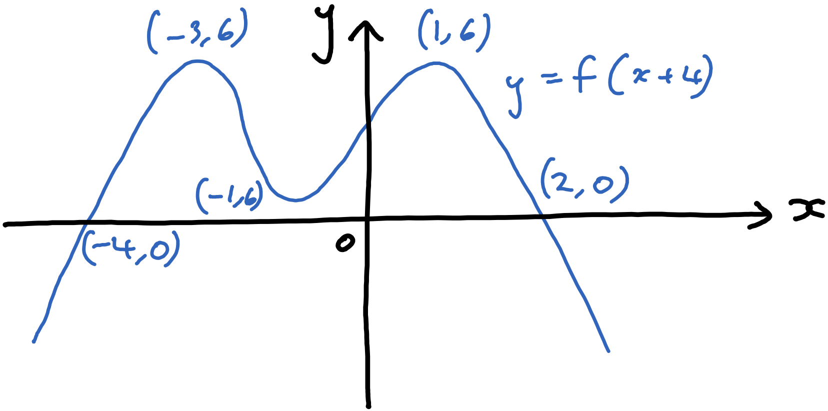
Question 12 continued

b)



c)



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13. (i) Showing each step in your reasoning, prove that

$$(\sin x + \cos x)(1 - \sin x \cos x) \equiv \sin^3 x + \cos^3 x$$

(3)

(ii) Solve, for $0 \leqslant \theta < 360°$,

$$3\sin\theta = \tan\theta$$

giving your answers in degrees to 1 decimal place, as appropriate.

(6)

(Solutions based entirely on graphical or numerical methods are not acceptable.)

i) $\text{LHS} \equiv (\sin x + \cos x)(1 - \sin x \cos x)$

$\equiv \sin x - \sin^2 x \cos x + \cos x - \cos^2 x \sin x$

$\equiv \sin x - (1-\cos^2 x)\cos x + \cos x - (1-\sin^2 x)\sin x$

$\equiv \sin x - \cos x + \cos^3 x + \cos x - \sin x + \sin^3 x$

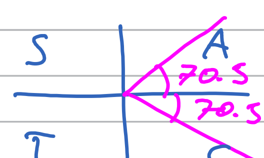
$\equiv \cos^3 x + \sin^3 x$

$\equiv \text{RHS}$

ii) $3\sin\theta = \tan\theta$

$3\sin\theta\cos\theta = \sin\theta$

$\sin\theta(3\cos\theta - 1) = 0$



$\sin\theta = 0$ or $\cos\theta = \frac{1}{3}$

$\theta = 0, 180°$ $\quad\theta = 70.5, 289.5$

Leave blank

14.

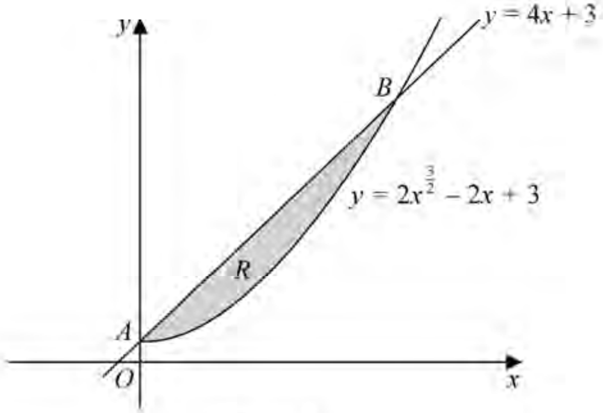


Figure 3

The finite region R, which is shown shaded in Figure 3, is bounded by the straight line l with equation $y = 4x + 3$ and the curve C with equation $y = 2x^{\frac{3}{2}} - 2x + 3,\ x \geqslant 0$

The line l meets the curve C at the point A on the y-axis and l meets C again at the point B, as shown in Figure 3.

(a) Use algebra to find the coordinates of A and B.

(4)

(b) Use integration to find the area of the shaded region R.

(6)

a) $4x+3 = 2x^{3/2} - 2x + 3$ When $x = 0$, $y = 4(0)+3 = 3$

$2x^{3/2} - 6x = 0$ When $x = 9$ $y = 4(9)+3 = 39$

$x(x^{1/2} - 3) = 0$

$x = 0$ or $x^{1/2} = 3$ $\therefore A(0,3)$ and $B(9,39)$

$x = 9$

b) $R = \int_0^9 [4x+3-(2x^{3/2}-2x+3)]\,dx$

$= \int_0^9 (6x - 2x^{3/2})\,dx = \left[3x^2 - \frac{4}{5}x^{5/2}\right]_0^9$

$= 3(9)^2 - \frac{4}{5}(9)^{5/2} = \frac{243}{5}$

Leave blank

15.

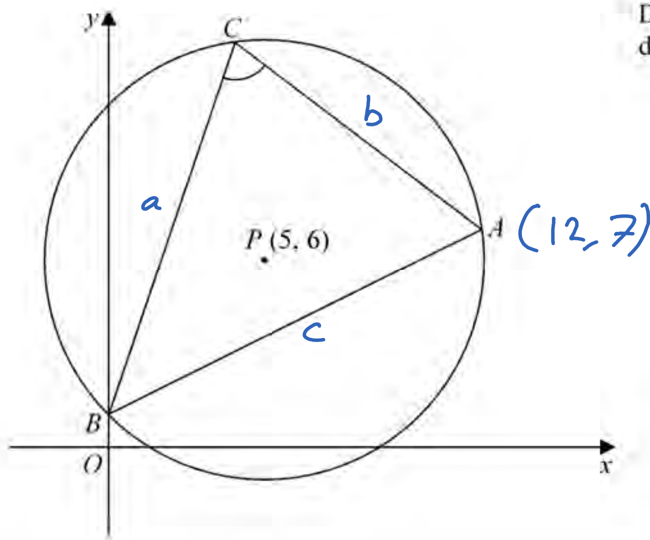


Figure 4

The circle shown in Figure 4 has centre P (5, 6) and passes through the point A (12, 7).

Find

(a) the exact radius of the circle, **(2)**

(b) an equation of the circle, **(3)**

(c) an equation of the tangent to the circle at the point A. **(4)**

The circle also passes through the points B (0, 1) and C (4, 13).

(d) Use the cosine rule on triangle ABC to find the size of the angle BCA, giving your answer in degrees to 3 significant figures. **(5)**

a) $r^2 = (12-5)^2 + (7-6)^2 = 50$

$r = 5\sqrt{2}$

b) $(x-5)^2 + (y-6)^2 = 50$

Leave blank

Question 15 continued

c) $m_{PA} = \frac{7-6}{12-5} = \frac{1}{7} \Rightarrow m_T = -7$

$y - y_1 = m(x - x_1)$

$y - 7 = -7(x - 12)$

$y = 91 - 7x$

d) $a^2 = (4-0)^2 + (13-1)^2 = 160$

$b^2 = (12-4)^2 + (13-7)^2 = 100$

$c^2 = (12-0)^2 + (7-1)^2 = 180$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$= \frac{160 + 100 - 180}{2 \times 4\sqrt{10} \times 10}$

$= \frac{1}{\sqrt{10}}$

$C = 71.6^\circ$ (3sf)

Leave blank

16. [*In this question you may assume the formula for the area of a circle and the following formulae:*

a ***sphere*** *of radius r has volume* $V = \frac{4}{3}\pi r^3$ *and surface area* $S = 4\pi r^2$

a ***cylinder*** *of radius r and height h has volume* $V = \pi r^2 h$ *and curved surface area* $S = 2\pi rh$]

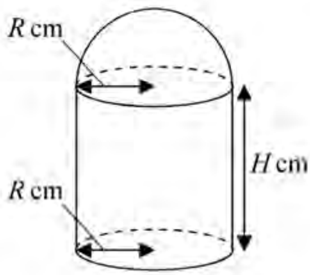


Figure 5

Figure 5 shows the model for a building. The model is made up of three parts. The roof is modelled by the curved surface of a hemisphere of radius R cm. The walls are modelled by the curved surface of a circular cylinder of radius R cm and height H cm. The floor is modelled by a circular disc of radius R cm. The model is made of material of negligible thickness, and the walls are perpendicular to the base.

It is given that the volume of the model is 800π cm^3 and that $0 < R < 10.6$

(a) Show that

$$H = \frac{800}{R^2} - \frac{2}{3}R$$

(2)

(b) Show that the surface area, A cm^2, of the model is given by

$$A = \frac{5\pi R^2}{3} + \frac{1600\pi}{R}$$

(3)

(c) Use calculus to find the value of R, to 3 significant figures, for which A is a minimum.

(5)

(d) Prove that this value of R gives a minimum value for A.

(2)

(e) Find, to 3 significant figures, the value of H which corresponds to this value for R.

(1)

Leave blank

Question 16 continued

a) $V = \pi R^2 H + \frac{1}{2} \times \frac{4}{3}\pi R^3 = 800\pi$

$R^2 H = 800 - \frac{2R^3}{3}$

$H = \frac{800}{R^2} - \frac{2R}{3}$

b) $A = \pi R^2 + 2\pi R H + \frac{1}{2} \times 4\pi R^2$

$= 3\pi R^2 + 2\pi R\left(\frac{800}{R^2} - \frac{2R}{3}\right)$

$= 3\pi R^2 + \frac{1600\pi}{R} - \frac{4\pi R^2}{3}$

$= \frac{5\pi R^2}{3} + \frac{1600\pi}{R}$

c) $\frac{dA}{dR} = \frac{10\pi R}{3} - \frac{1600\pi}{R^2} = 0$

$R^3 - 480 = 0$

$R = 7.83 \text{ cm } (3sf)$

d) $\frac{d^2A}{dR^2} = \frac{10\pi}{3} + \frac{3200\pi}{R^3}$

When $R = 7.83$, $\frac{d^2A}{dR^2} = \frac{10\pi}{3} + \frac{3200\pi}{480} > 0$

e) $H = \frac{800}{(7.83)^2} - \frac{2}{3}(7.83) = 7.83 \text{ cm } (3sf)$