GradeMax

1. Find

$$\int\left(2x^4 - \frac{4}{\sqrt{x}} + 3\right)\mathrm{d}x$$

giving each term in its simplest form. $4x^{-\frac{1}{2}}$

(4)

$\frac{2x^5}{5} - \frac{4x^{\frac{1}{2}}}{\frac{1}{2}} + 3x + c = \frac{2}{5}x^5 - 8x^{\frac{1}{2}} + 3x + c$

2. Express 9^{3x+1} in the form 3^y, giving y in the form $ax + b$, where a and b are constants.

(2)

$(3^2)^{(3x+1)} = 3^{6x+2}$ $\therefore$ $y = 6x + 2$

3. (a) Simplify

$$\sqrt{50} - \sqrt{18}$$

giving your answer in the form $a\sqrt{2}$, where a is an integer.

(2)

(b) Hence, or otherwise, simplify

$$\frac{12\sqrt{3}}{\sqrt{50} - \sqrt{18}}$$

giving your answer in the form $b\sqrt{c}$, where b and c are integers and $b \neq 1$

(3)

a) $\sqrt{25}\sqrt{2} - \sqrt{9}\sqrt{2} = 5\sqrt{2} - 3\sqrt{2} = 2\sqrt{2}$

b) $\dfrac{12\sqrt{3}}{2\sqrt{2}} = \dfrac{6\sqrt{3}\sqrt{2}}{\sqrt{2}\sqrt{2}} = 3\sqrt{6}$

4.

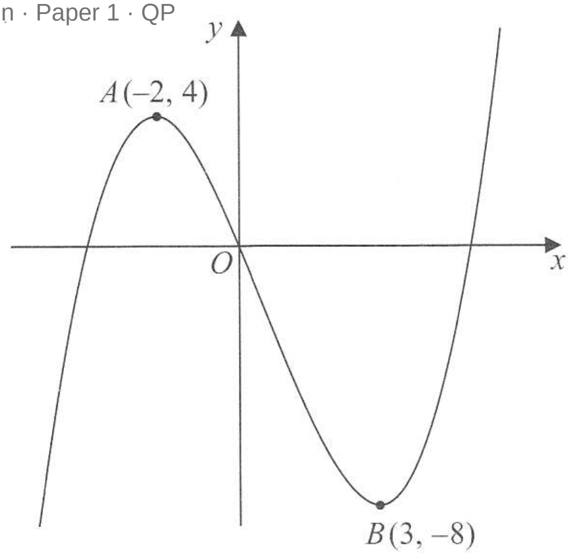


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$. The curve has a maximum point A at $(-2, 4)$ and a minimum point B at $(3, -8)$ and passes through the origin O.

On separate diagrams, sketch the curve with equation

(a) $y = 3\mathrm{f}(x)$,

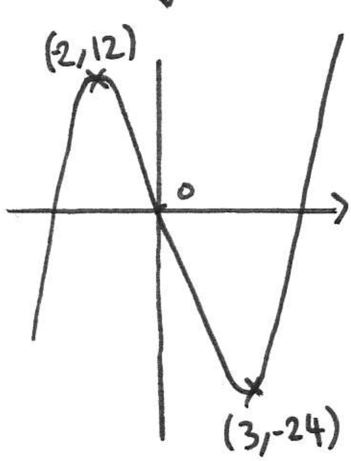
(2)

(b) $y = \mathrm{f}(x) - 4$

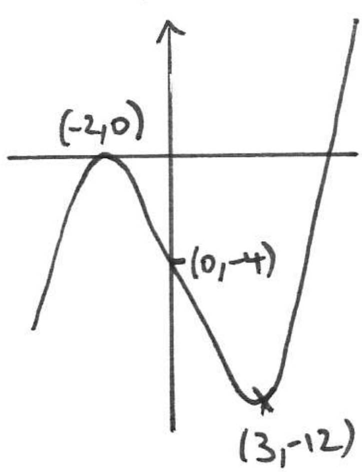
(3)

On each diagram, show clearly the coordinates of the maximum and the minimum points and the coordinates of the point where the curve crosses the y-axis.

a) $3f(x)$ $\updownarrow \times 3$



b) $f(x)-4$ $\downarrow 4$



5. Solve the simultaneous equations

$$y + 4x + 1 \quad = 0$$

$$y^2 + 5x^2 + 2x = 0$$

(6)

$y = -4x - 1 \quad \Rightarrow \quad y^2 = (-4x-1)^2 = 16x^2 + 8x + 1$

$(16x^2 + 8x + 1) + 5x^2 + 2x = 0 \quad \Rightarrow \quad 21x^2 + 10x + 1 = 0$

$(7x+1)(3x+1) = 0 \quad \Rightarrow \quad x = -\frac{1}{7}, \; -\frac{1}{3}$

$y = \frac{4}{7} - 1 = -\frac{3}{7} \quad y = \frac{4}{3} - 1 = \frac{1}{3}$

$\left(-\frac{1}{7}, -\frac{3}{7}\right) \; ; \left(-\frac{1}{3}, \frac{1}{3}\right)$

6. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 4,$$

$$a_{n+1} = 5 - ka_n, \quad n \geqslant 1$$

where k is a constant.

(a) Write down expressions for a_2 and a_3 in terms of k.
(2)

Find

(b) $\sum_{r=1}^{3}(1 + a_r)$ in terms of k, giving your answer in its simplest form,
(3)

(c) $\sum_{r=1}^{100}(a_{r+1} + ka_r)$
(1)

a) $a_1 = 4 \quad a_2 = 5 - 4k \quad a_3 = 5 - k(5-4k)$

$= 5 - 5k + 4k^2$

b) $(1+4) + (1+5-4k) + (1+5-5k+4k^2)$

$= 17 - 9k + 4k^2$

c) $= (a_2 + ka_1) = 5 - \cancel{4k} + \cancel{4k}$

$+ (a_3 + ka_2) = 5 - \cancel{5k} + \cancel{4k^2} + \cancel{5k} - \cancel{4k^2}$

$+ (a_4 + ka_3)$

$+$

$\vdots$

$+ (a_{101} + ka_{100})$

$= 5 \times 100 = 500$

7. Given that

$$y = 3x^2 + 6x^{\frac{1}{3}} + \frac{2x^3 - 7}{3\sqrt{x}}, \quad x > 0$$

find $\frac{\mathrm{d}y}{\mathrm{d}x}$. Give each term in your answer in its simplified form.

(6)

$$y' = 6x + 2x^{-\frac{2}{3}} + \frac{5}{3}x^{\frac{3}{2}} + \frac{7}{6}x^{-\frac{3}{2}}$$

$$\frac{2x^3}{3x^{\frac{1}{2}}} - \frac{7}{3\sqrt{x}}$$

$$= \frac{2}{3}x^{\frac{5}{2}} - \frac{7}{3}x^{-\frac{1}{2}}$$

8. The straight line with equation $y = 3x - 7$ does not cross or touch the curve with equation $y = 2px^2 - 6px + 4p$, where p is a constant.

(a) Show that $4p^2 - 20p + 9 < 0$

(4)

(b) Hence find the set of possible values of p.

(4)

$3x - 7 = 2px^2 - 6px + 4p$

$\Rightarrow 2px^2 - 6px - 3x + 4p + 7 = 2px^2 - (6p+3)x + (4p+7) = 0$

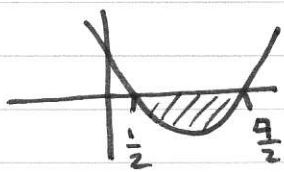
If line does not cross or touch curve $\Rightarrow b^2 - 4ac < 0$

$(6p+3)^2 - 4(2p)(4p+7) < 0$

$(36p^2 + 36p + 9) - 32p^2 - 56p < 0 \Rightarrow 4p^2 - 20p + 9 < 0$

b) $(2p-9)(2p-1) < 0$

$p = \frac{9}{2} \quad p = \frac{1}{2}$



$p > \frac{1}{2}$ and $p < \frac{9}{2}$

(or) $\frac{1}{2} < p < \frac{9}{2}$

9 On John's 10th birthday he received the first of an annual birthday gift of money from his uncle. This first gift was £60 and on each subsequent birthday the gift was £15 more than the year before. The amounts of these gifts form an arithmetic sequence.

(a) Show that, immediately after his 12th birthday, the total of these gifts was £225

(1)

(b) Find the amount that John received from his uncle as a birthday gift on his 18th birthday.

(2)

(c) Find the total of these birthday gifts that John had received from his uncle up to and including his 21st birthday.

(3)

When John had received n of these birthday gifts, the total money that he had received from these gifts was £3375

(d) Show that $n^2 + 7n = 25 \times 18$

(3)

(e) Find the value of n, when he had received £3375 in total, and so determine John's age at this time.

(2)

a) 10^{th} $u_1 = 60$ $\quad S_3 = 60 + 75 + 90 = £225$

11^{th} $u_2 = 75$

12^{th} $u_3 = 90$ $\quad a = 60 \quad d = 15$

$\rightarrow -9$

b) $18^{th} = u_9 = a + 8d = 60 + 8 \times 15 = £180$

c) $21^{st} = u_{12} = a + 11d = 60 + 11 \times 15 = £225$

$S_{12} = \frac{n}{2}[a + L] = 6[60 + 225) = 6 \times 285 = £1710$

d) $\frac{n}{2}[2a + (n-1)d] = 3375 \Rightarrow n[120 + (n-1) \times 15] = 6750$

$\Rightarrow 120n + 15n^2 - 15n = 6750 \Rightarrow 15n^2 + 105n = 6750$

$\div 15 \Rightarrow n^2 + 7n = 450 \quad \therefore n^2 + 7n = 25 \times 18$ #

c) $n^2 + 7n - 25 \times 18 = 0 \Rightarrow (n + 25)(n - 18) = 0$

$\therefore n = \cancel{-25} \quad \underline{n = 18}$ $+9$ he was 27

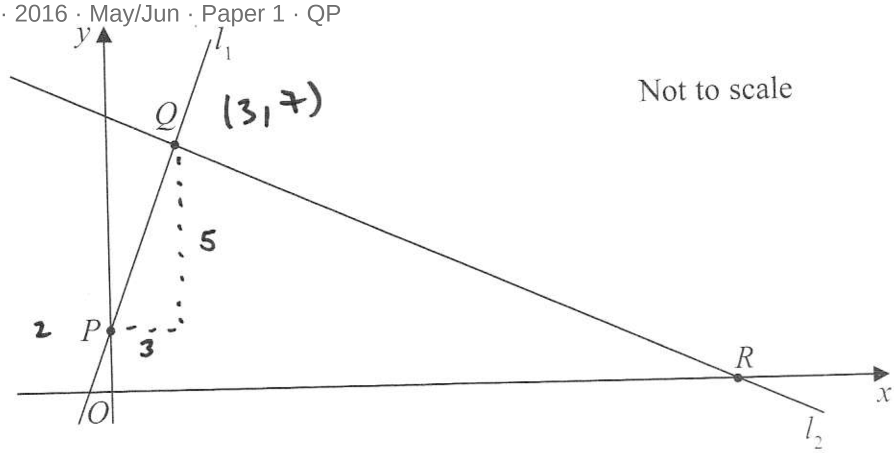


Figure 2

The points P (0, 2) and Q (3, 7) lie on the line l_1, as shown in Figure 2.

The line l_2 is perpendicular to l_1, passes through Q and crosses the x-axis at the point R, as shown in Figure 2.

Find

(a) an equation for l_2, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers,
(5)

(b) the exact coordinates of R,
(2)

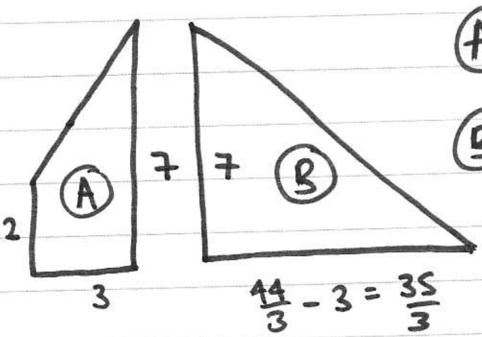
(c) the exact area of the quadrilateral $ORQP$, where O is the origin.
(5)

a) $m_{l_1} = \frac{5}{3} \therefore m_{l_2} = -\frac{3}{5}$ $\quad y - 7 = -\frac{3}{5}(x-3)$

$\Rightarrow 5y - 35 = -3x + 9 \quad \Rightarrow 3x + 5y - 44 = 0$

b) $y = 0 \Rightarrow 3x = 44 \quad \therefore x = \frac{44}{3} \quad R\left(\frac{44}{3}, 0\right)$

c)



$A = \frac{(2+7)}{2} \times 3 = \frac{27}{2} = \frac{81}{6}$

$B = \frac{35}{6} \times 7 = \frac{245}{6}$

$\therefore \text{Area} = \frac{326}{6} = \frac{163}{3}$

11. The curve C has equation $y = 2x^3 + kx^2 + 5x + 6$, where k is a constant.

(a) Find $\frac{dy}{dx}$ **(2)**

The point P, where $x = -2$, lies on C.

The tangent to C at the point P is parallel to the line with equation $2y - 17x - 1 = 0$

Find

(b) the value of k, **(4)**

(c) the value of the y coordinate of P, **(2)**

(d) the equation of the tangent to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(2)**

a) $y' = 6x^2 + 2kx + 5$

$2y = 17x + 1$

$y = \frac{17}{2}x + \frac{1}{2}$

b) $\therefore M_t = \frac{17}{2}$ as parallel when $x = -2$.

$6x^2 + 2kx + 5 = \frac{17}{2} \Rightarrow 12x^2 + 4kx + 10 = 17$

$x = -2 \Rightarrow 48 - 8k = 7 \Rightarrow 8k = 41 \quad \therefore k = \frac{41}{8}$

c) $y = 2(-2)^3 + \frac{41}{8}(-2)^2 + 5(-2) + 6$

$y = -16 + \frac{41}{2} - 10 + 6 = \frac{41}{2} - 20 = \frac{41}{2} - \frac{40}{2} = \frac{1}{2} \quad P(-2, \frac{1}{2})$

d) $y - \frac{1}{2} = \frac{17}{2}(x+2)$

$2y - 1 = 17x + 34$ $\therefore$ $17x - 2y + 35 = 0$