

C12 October 2016 (MA)

$$\text{Q1) } \int f(x) dx = \int (3x^2 + x - 4x^{-\frac{1}{2}} + 6x^{-3}) dx$$

$$= x^3 + \frac{x^2}{2} - \frac{4x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{6x^{-2}}{-2} + c$$

$$= \boxed{x^3 + \frac{x^2}{2} - 8x^{\frac{1}{2}} - 3x^{-2} + c}$$

$$\text{Q2a) } 7^{2x} = 14$$

$$\log(7^{2x}) = \log(14)$$

$$2x \log 7 = \log 14$$

$$2x = \frac{\log 14}{\log 7} \quad \therefore x = \frac{1}{2} \times \frac{\log 14}{\log 7} \approx \boxed{0.678}$$

$$\text{b) } \log_5(3x+1) = -2$$

$$5^{-2} = 3x+1$$

$$\frac{5^{-2} - 1}{3} = x = \frac{\frac{1}{25} - 1}{3} = \boxed{\frac{-8}{25}}$$

$$\begin{aligned}
 \text{Q3:)} \quad \sqrt{45} - \frac{20}{\sqrt{5}} + \sqrt{6} \cdot \sqrt{30} &= \sqrt{45} - \frac{20\sqrt{5}}{5} + \sqrt{6 \times 30} \\
 &= \sqrt{9} \times \sqrt{5} - (\sqrt{5} \times 4) + \sqrt{180} \\
 &= 3\sqrt{5} - 4\sqrt{5} + \sqrt{5} \times \sqrt{36} \\
 &= 3\sqrt{5} - 4\sqrt{5} + 6\sqrt{5} = (6 + 3 - 4)\sqrt{5} \\
 &= \boxed{5\sqrt{5}}
 \end{aligned}$$

$$\begin{aligned}
 \text{ii)} \quad \frac{17\sqrt{2}}{\sqrt{2}+6} \times \frac{(\sqrt{2}-6)}{(\sqrt{2}-6)} \\
 &= \frac{17\sqrt{2}(\sqrt{2}-6)}{(\sqrt{2}+6)(\sqrt{2}-6)} = \frac{17(2) - 6(17)\sqrt{2}}{2 - 6\sqrt{2} + 6\sqrt{2} - 36} \\
 &= \frac{34 - 102\sqrt{2}}{-34} = \boxed{-1 + 3\sqrt{2}}
 \end{aligned}$$

$$\begin{aligned}
 \text{Q4 ai)} \quad 2x+1 &= 0 \\
 2x &= -1 \\
 x &= -\frac{1}{2} \quad \text{so } f\left(-\frac{1}{2}\right) = 6\left(-\frac{1}{2}\right)^3 - 7\left(-\frac{1}{2}\right)^2 - 43\left(-\frac{1}{2}\right) + 30 \\
 &= \boxed{49}
 \end{aligned}$$

$$\begin{aligned}
 \text{ii)} \quad x-3 &= 0 \\
 x &= 3 \quad \text{so } f(3) = 6(3)^3 - 7(3)^2 - 43(3) + 30 \\
 &= -30 + 30 = \boxed{0}
 \end{aligned}$$

b)  $f(3) = 0$  so  $(x-3)$  is a factor.

$$\begin{array}{r}
 6x^2 + 11x - 10 \\
 x-3 \overline{) 6x^3 - 7x^2 - 43x + 30} \\
 \underline{6x^3 - 18x^2} \phantom{+ 30} \\
 0 + 11x^2 - 43x \phantom{+ 30} \\
 \underline{11x^2 - 33x} \phantom{+ 30} \\
 0 - 10x + 30 \\
 \underline{-10x + 30} \\
 0 \phantom{+ 30} \\
 \hline
 \hline
 \end{array}$$

$$\therefore f(x) = (x-3)(6x^2 + 11x - 10)$$

$$\text{but } 6x^2 + 11x - 10 = (2x+5)(3x-2)$$

$$\text{so } f(x) = \boxed{(x-3)(2x+5)(3x-2)}$$

$$\text{Q5a) } \left(3 - \frac{ax}{2}\right)^5 \approx (3)^5 + \binom{5}{1}(3)^4 \left(-\frac{ax}{2}\right)^1$$

$$+ \binom{5}{2}(3)^3 \left(-\frac{ax}{2}\right)^2$$

$$+ \binom{5}{3}(3)^2 \left(-\frac{ax}{2}\right)^3$$

$$\approx \boxed{
 \begin{array}{l}
 243 - \frac{405ax}{2} + \frac{135a^2x^2}{2} \\
 - \frac{45a^3x^3}{4}
 \end{array}
 }$$

$$b) \text{ coeff. of } x \text{ is } -\frac{405a}{2}$$

$$\text{coeff. of } x^3 \text{ is } -\frac{45}{4}a^3$$

$$\text{they are equal so } -\frac{405}{2}a = -\frac{45}{4}a^3$$

$$\Rightarrow -\frac{405}{2} = -\frac{45}{4}a^2$$

$$\Rightarrow a^2 = \frac{\frac{405}{2}}{\frac{45}{4}} = 18 //$$

$$\therefore a = \sqrt{18} = \boxed{3\sqrt{2}} \quad (a > 0).$$

$$\begin{aligned} \text{Q6 a) } u_2 &= \frac{2}{3}(36) = \boxed{24} \\ u_3 &= \frac{2}{3}(24) = \boxed{16} \\ u_4 &= \frac{2}{3}(16) = \boxed{\frac{32}{3}} \end{aligned} \left. \vphantom{\begin{aligned} u_2 \\ u_3 \\ u_4 \end{aligned}} \right\} \text{Geometric series!}$$

$$\begin{aligned} a &= 36 \\ r &= \frac{2}{3} \end{aligned}$$

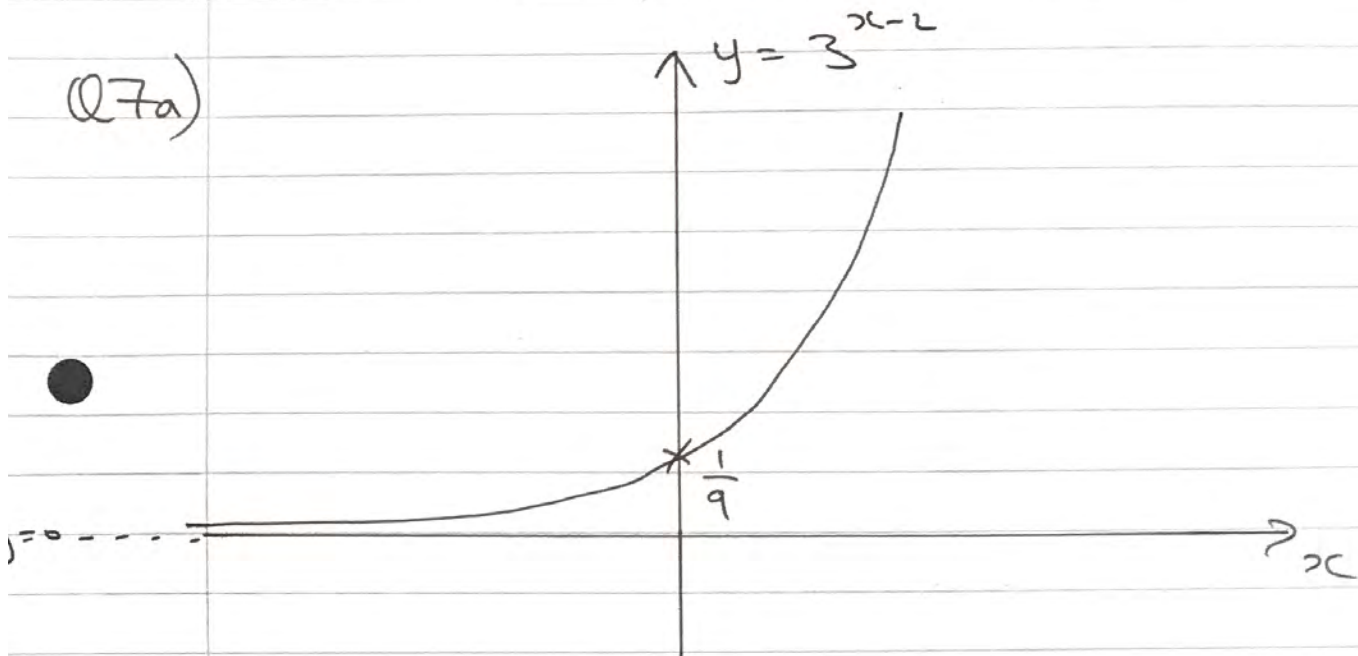
$$b) \boxed{\frac{2}{3}}$$

$$c) u_{11} = ar^{11-1} = 36 \times \left(\frac{2}{3}\right)^{10} = \boxed{0.6243}$$

$$d) S_6 = \frac{a(1-r^6)}{1-r} = \frac{36(1 - (\frac{2}{3})^6)}{1 - \frac{2}{3}} = \boxed{\frac{2660}{27}}$$

$$e) S_{\infty} = \frac{a}{1-r} = \frac{36}{1-\frac{2}{3}} = \boxed{108}$$

Q7a)



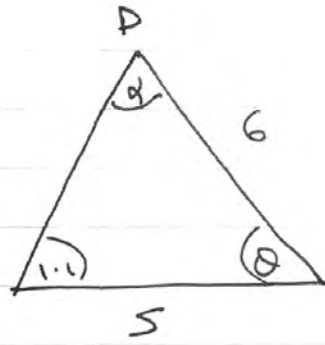
$$b) h = \frac{b-a}{n} = \frac{3-0.5}{5} = \frac{2.5}{5} = \frac{1}{2} //$$

$$\therefore \text{Area} \approx \frac{1}{2} \times \frac{1}{2} [0.192 + 3 + 2(0.333 + 0.577 + 1 + 1.732)]$$

$$\approx \boxed{2.62}$$

Q8a) sine rule

$$\frac{\sin d}{5} = \frac{\sin 1.1}{6}$$



$$\sin d = \frac{5}{6} \sin 1.1$$

$$d = \sin^{-1} \left( \frac{5}{6} \sin 1.1 \right)$$

$$\begin{aligned} \text{so } \theta = \text{angle required} &= \pi - 1.1 - \sin^{-1} \left( \frac{5}{6} \sin 1.1 \right) \\ &= \boxed{1.20} \end{aligned}$$

$$\text{b) } \angle DBC = \pi - 1.20 = 1.94$$

$$\text{so Area BCD} = \frac{1}{2} (6)^2 (1.94) = 34.9$$

$$\text{and Area ABD} = \frac{1}{2} (6)(5) \sin(1.20) = 14.0$$

$$34.9 + 14.0 = \boxed{48.9} \text{ cm}^2$$

● Q9a) Sum of first 10 terms =  $\frac{10}{2} (2a + 9d) = 395$

$$\Rightarrow 5(2a + 9d) = 395$$

$$\Rightarrow 10a + 45d = 395 \quad \sim \textcircled{1}$$

b)  $S_{18} = \frac{18}{2} (2a + 17d) = 927$

$$18a + 153d = 927 \quad \sim \textcircled{2}$$

c)  $\textcircled{2} : a = \frac{927 - 153d}{18}$

$$\hookrightarrow \textcircled{1} : 10 \left( \frac{927 - 153d}{18} \right) + 45d = 395$$

$$515 - 85d + 45d = 395$$

$$-40d = -120$$

$$\therefore d = \frac{120}{40} = \boxed{3}$$

$$\text{so } a = \frac{927 - 153(3)}{18} = \boxed{26}$$

d)  $n^{\text{th}}$  term =  $a + (n-1)d$

●  $20^{\text{th}}$  term =  $26 + 19(3) = \boxed{83}$

$$\text{Q10a)} \quad \frac{8 \sin x}{\cos x} = -3 \cos x$$

$$\times \cos x : 8 \sin x = -3 \cos^2 x$$

$$8 \sin x = -3(1 - \sin^2 x)$$

$$8 \sin x = -3 + 3 \sin^2 x$$

$$\therefore 3 \sin^2 x - 8 \sin x - 3 = 0$$

b) Same eqn as in (a) but  $2\theta$  instead of  $x$ .

$$\Rightarrow 3 \sin^2(2\theta) - 8 \sin(2\theta) - 3 = 0$$

By Quadratic formula...

$$\begin{aligned} a &= 3 \\ b &= -8 \\ c &= -3 \end{aligned}$$

$$\left. \begin{aligned} a &= 3 \\ b &= -8 \\ c &= -3 \end{aligned} \right\} \begin{aligned} & \left[ \sin 2\theta = 3 \right] \text{ or } \left[ \sin 2\theta = -\frac{1}{3} \right] \\ & \quad \uparrow \\ & \text{NO SOLUTIONS} \\ & \quad \sin 2\theta \leq 1 \end{aligned}$$

$$\sin 2\theta = -\frac{1}{3}$$

$$\sin^{-1}\left(-\frac{1}{3}\right) = 2\theta = -19.47^\circ //$$

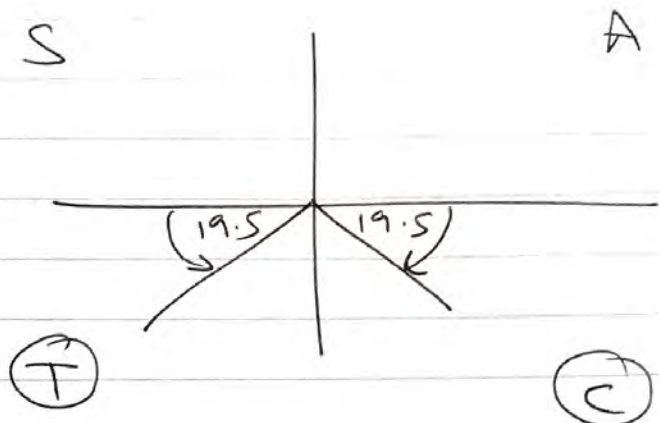
range :  $0 \leq 2\theta < 720^\circ$

$$2\theta = 180 + 19.5,$$

$$360 - 19.5,$$

$$540 + 19.5,$$

$$720 - 19.5,$$



$$\text{so } 2\theta = 199.47^\circ, 340.53^\circ, 559.47^\circ, 700.53^\circ$$

$$\theta = 99.7^\circ, 170.3^\circ, 279.7^\circ, 350.3^\circ$$

Q11a)  $5x^2 + 6 = k(13x^2 - 12x)$

$$5x^2 - 13kx^2 + 6 = -12kx$$

$$(5 - 13k)x^2 + (12k)x + 6 = 0$$

But this eqn has two distinct real roots,

$$\text{so } \underline{b^2 - 4ac > 0}$$

$$\left. \begin{array}{l} a = 5 - 13k \\ b = 12k \\ c = 6 \end{array} \right\} \begin{array}{l} b^2 - 4ac = (12k)^2 - 4(5 - 13k)(6) \\ = 144k^2 - 24(5) + 24(13k) \end{array}$$

$$\therefore b^2 - 4ac = 144k^2 + 312k - 120 //$$

$$b^2 - 4ac > 0 \text{ hence } 144k^2 + 312k - 120 > 0 //$$

$$\div 24: \underline{6k^2 + 13k - 5 > 0}$$

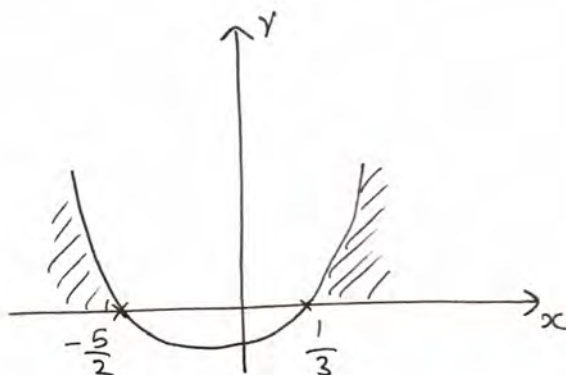
$$b) \quad 6u^2 + 13u - 5 > 0$$

finding critical values... (6u<sup>2</sup> + 13u - 5 = 0) <sup>Solve</sup>

$$\left. \begin{array}{l} a=6 \\ b=13 \\ c=-5 \end{array} \right\} \text{ By Quadratic Formula,}$$

$$u = \frac{1}{3}, \quad u = -\frac{5}{2}$$

So  $y = 6u^2 + 13u - 5$



$6u^2 + 13u - 5 > 0$  represented by the regions where  $y > 0$ .

∴

$$\begin{array}{l} u > \frac{1}{3} \\ u < -\frac{5}{2} \end{array}$$

$$\bullet \text{ (Q12a)} \quad y=0 : \quad 0 = \frac{x^3 - 9x^2 - 81x}{27}$$

$$\Rightarrow x^3 - 9x^2 - 81x = 0$$

$$\Rightarrow x(x^2 - 9x - 81) = 0$$

$$x \neq 0 \text{ so } x^2 - 9x - 81 = 0 \\ \text{(at A \& B)}$$

$$\bullet \quad \text{By Quadratic Formula... } x = \frac{9 \pm 9\sqrt{5}}{2}$$

$$\left( \begin{array}{l} a=1 \\ b=-9 \\ c=-81 \end{array} \right) \quad \text{so } \boxed{\begin{array}{l} A \left( \frac{9-9\sqrt{5}}{2}, 0 \right) \\ B \left( \frac{9+9\sqrt{5}}{2}, 0 \right) \end{array}}$$

$$\bullet \text{ b) } y = \frac{1}{27} (x^3 - 9x^2 - 81x)$$

at C/D,  $\frac{dy}{dx} = 0$  as they are turning points.

$$\frac{dy}{dx} = \frac{1}{27} (3x^2 - 18x - 81) = 0$$

$$3x^2 - 18x - 81 = 0$$

$$(3x - 27)(x + 3) = 0$$

$$(0 = 3x - 27) \rightarrow x = 9 \quad (x + 3 = 0) \rightarrow x = -3$$

so C has  $x$  coordinate  $-3$ .

$$f(-3) = \frac{(-3)^3 - 9(-3)^2 - 81(-3)}{27}$$

$$= 5 //$$
$$\therefore \boxed{C(-3, 5)}$$

and D has  $x$  coordinate 9

$$f(9) = \frac{(9)^3 - 9(9)^2 - 81(9)}{27}$$

$$= -27 //$$

$$\therefore \boxed{D(9, -27)}$$

c) D is transformed to the  $y$ -axis.  
ie 9 units to the left.

$$\text{So } \boxed{a=9}$$

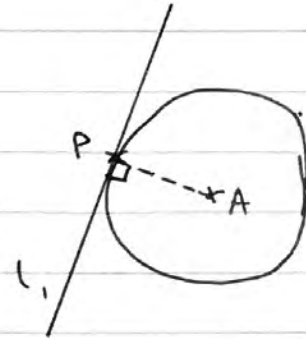
● (Q13a) distance  $AP = \sqrt{(1-8)^2 + (-3+2)^2} = \sqrt{50} //$

$$\therefore r = \sqrt{50}$$

$$\text{So } \boxed{(x-1)^2 + (y+3)^2 = 50}$$

b)  $l_1$  is perpendicular to  $AP$ .

$$m_{AP} = \frac{-2 - -3}{8 - 1} = \frac{1}{7} //$$



$$\text{So } m_{l_1} = -7 // \quad (-7 \times \frac{1}{7} = -1)$$

$$\therefore y - -2 = -7(x - 8)$$

$$y = -7x + 56 - 2$$

$$\boxed{y = -7x + 54}$$

c) sub  $y = x + 6$  into eqn for  $C$ .

$$(x-1)^2 + (x+6+3)^2 = 50$$

$$(x-1)^2 + (x+9)^2 = 50$$

$$x^2 - 2x + 1 + x^2 + 18x + 81 = 50$$

$$2x^2 + 16x + 82 - 50 = 0$$

$$2x^2 + 16x + 32 = 0$$

$$x^2 + 8x + 16 = 0$$

$$(x + 4)^2 = 0$$

$$x = -4 //$$

$$\text{at } x = -4, y = (-4) + 6 = 2 //$$

$$\text{so } \boxed{Q(-4, 2)}$$

Q14g)  $y = -x^2 + 6x - 8$

$$\frac{dy}{dx} = -2x + 6$$

$$\text{at } P, m = -2(5) + 6 = 6 - 10 = -4 //$$

$$\text{so at normal to } P, m = \frac{1}{4} // \left(\frac{1}{4}x - 4 = -1\right)$$

$$y - 3 = \frac{1}{4}(x - 5)$$

$$y + 3 = \frac{1}{4}x - \frac{5}{4}$$

$$y = \frac{1}{4}x - \frac{17}{4}$$

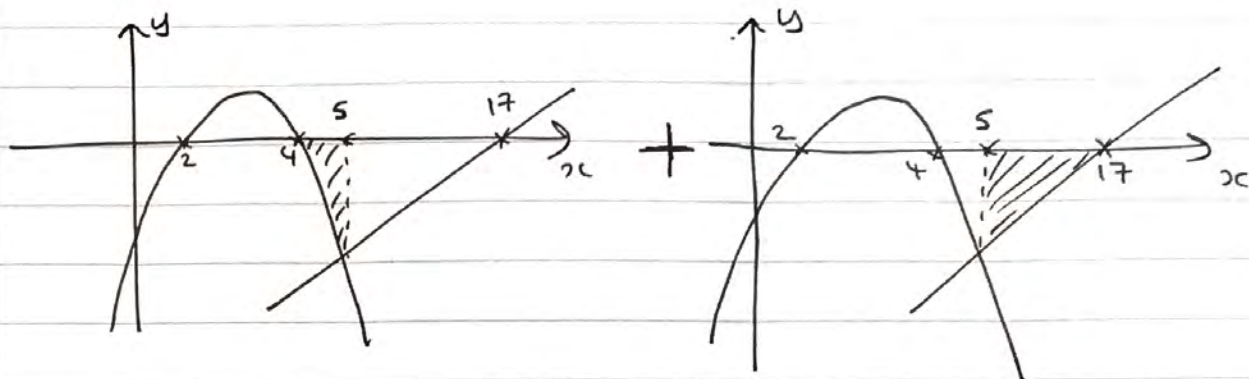
$$\times 4 : 4y = x - 17$$

$$\boxed{x - 4y - 17 = 0}$$

$$\text{at } y=0, \underline{x=4}, \underline{x=2}$$

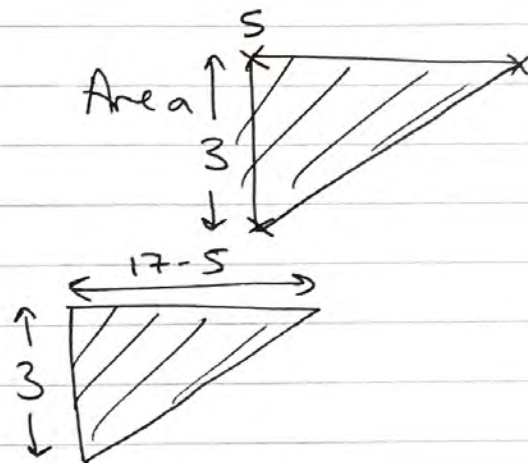
b)

$R =$



$$R = \int_4^5 (-x^2 + 6x - 8) dx + \text{Area}$$

$$= \left[ -\frac{x^3}{3} + 3x^2 - 8x \right]_4^5 +$$



$$= \left[ -\frac{20}{3} \right] - \left[ -\frac{16}{3} \right] + \frac{1}{2} (3)(12)$$

$$= \left| -\frac{4}{3} \right| + \frac{36}{2} = 18 + \frac{4}{3}$$

$$= \boxed{\frac{58}{3}}$$

$$\begin{aligned} \text{(Q15a) Surface Area} &= \frac{\pi r^2}{2} + \frac{\pi r^2}{2} + \frac{1}{2}(2\pi r h) \\ &\quad + h(2r) \\ &= \pi r^2 + \pi r h + 2hr // \end{aligned}$$


$$\therefore 200 = \pi r^2 + \pi r h + 2hr$$

the Volume equation we need to prove does not include  $h$ , so make  $h$  the subject here:

$$200 = \pi r^2 + h(\pi r + 2r)$$

$$h = \frac{200 - \pi r^2}{r(\pi + 2)} //$$

$$\text{and } V = \frac{\pi r^2 h}{2} = \frac{\pi r^2}{2} \left( \frac{200 - \pi r^2}{r(\pi + 2)} \right)$$

$$\Rightarrow V = \frac{\pi r^2 (200 - \pi r^2)}{2r(\pi + 2)} = \frac{\pi r (200 - \pi r^2)}{4 + 2\pi}$$


just a constant.

$$b) \quad V = \left( \frac{\pi}{4+2\pi} \right) \left[ 200r - \pi r^3 \right]$$

$$\frac{dV}{dr} = \frac{\pi}{4+2\pi} \left[ 200 - 3\pi r^2 \right] = 0$$

(at max value,  $\frac{dV}{dr} = 0$ )

$$\Rightarrow 200 - 3\pi r^2 = 0$$

$$\Rightarrow 200 = 3\pi r^2$$

$$\Rightarrow r^2 = \frac{200}{3\pi}$$

$$\Rightarrow r = \sqrt{\frac{200}{3\pi}} \quad (= 4.6)$$

$$\begin{aligned} \text{So } V_{\max} &= \frac{\pi}{4+2\pi} \left[ 200 \sqrt{\frac{200}{3\pi}} - \pi \left( \sqrt{\frac{200}{3\pi}} \right)^3 \right] \\ &= \boxed{188} \text{ cm}^3 \end{aligned}$$

$$c) \quad \frac{d^2V}{dr^2} = \frac{\pi}{4+2\pi} \left[ -6\pi r \right] < 0$$

for all  $r$  //

hence value of  $V$  found is a maximum.