Your boy Kprime2

C1 ~~May~~ May 17

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1. Find

$$\int\left(2x^5 - \frac{1}{4x^3} - 5\right)dx$$

giving each term in its simplest form.

(4)

1. $\int 2x^5 - \frac{1}{4x^3} - 5 \, dx$

$= \int 2x^5 - \frac{1}{4}x^{-3} - 5 \, dx$

$= \frac{1}{3}x^6 + \frac{1}{8}x^{-2} - 5x + C$

$= \frac{1}{3}x^6 + \frac{1}{8x^2} - 5x + C$

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2. Given

$$y = \sqrt{x} + \frac{4}{\sqrt{x}} + 4, \qquad x > 0$$

find the value of $\frac{dy}{dx}$ when $x = 8$, writing your answer in the form $a\sqrt{2}$, where a is a rational number.

(5)

$$2.\quad y = \sqrt{x} + \frac{4}{\sqrt{x}} + 4 = x^{\frac{1}{2}} + 4x^{-\frac{1}{2}} + 4$$

$$\frac{\partial y}{\partial x} = \frac{1}{2}x^{-\frac{1}{2}} - 2x^{-\frac{3}{2}}$$

$$= \frac{1}{2\sqrt{x}} - \frac{2}{(\sqrt{x})^3}$$

$$\sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$$

$$\therefore \left(\frac{\partial y}{\partial x}\right)_{x=8} = \frac{1}{2\sqrt{8}} - \frac{2}{(\sqrt{8})^3}$$

$$= \frac{1}{4\sqrt{2}} - \frac{2}{(2\sqrt{2})^3}$$

$$= \frac{1}{4\sqrt{2}} - \frac{2}{8\sqrt{8}}$$

$$= \frac{1}{4\sqrt{2}} - \frac{1}{8\sqrt{2}}$$

$$= \frac{2}{8\sqrt{2}} - \frac{1}{8\sqrt{2}}$$

$$= \frac{1}{8\sqrt{2}} = \frac{1}{16}\sqrt{2}$$

3. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 1$$
$$a_{n+1} = \frac{k(a_n + 1)}{a_n}, \qquad n \geqslant 1$$

where k is a positive constant.

(a) Write down expressions for a_2 and a_3 in terms of k, giving your answers in their simplest form.

(3)

Given that $\sum_{r=1}^{3} a_r = 10$

(b) find an exact value for k.

(3)

3(a). $\cancel{a_2 = k(}$ $a_2 = \frac{k(1+1)}{1} = 2k$

$a_3 = \frac{k(2k+1)}{2k} = \frac{2k+1}{2} = k + \frac{1}{2}$

$a_2 = 2k$

$a_3 = k + \frac{1}{2}$

(b) $\sum_{r=1}^{3} a_r = a_1 + a_2 + a_3 = 1 + 2k + k + \frac{1}{2}$

$= 3k + \frac{3}{2} = 10$

$\Rightarrow 3k = \frac{17}{2} \Rightarrow k = \frac{17}{6}$

4. A company, which is making 140 bicycles each week, plans to increase its production. The number of bicycles produced is to be increased by d each week, starting from 140 in week 1, to $140 + d$ in week 2, to $140 + 2d$ in week 3 and so on, until the company is producing 206 in week 12.

(a) Find the value of d.

(2)

After week 12 the company plans to continue making 206 bicycles each week.

(b) Find the total number of bicycles that would be made in the first 52 weeks starting from and including week 1.

(5)

4(a). $a = 140$

$$u_{12} = 140 + 11d = 206$$

$$\Rightarrow d = 6$$

(b)

$$\text{Total} = S_{12} + (206 \times 40)$$

$$= \frac{1}{2}(12)(140 + 206) + (206 \times 40)$$

$$= 2076 + 8240 = 10316$$

5. $$f(x) = x^2 - 8x + 19$$

(a) Express f(x) in the form $(x + a)^2 + b$, where a and b are constants.

(2)

The curve C with equation $y = f(x)$ crosses the y-axis at the point P and has a minimum point at the point Q.

(b) Sketch the graph of C showing the coordinates of point P and the coordinates of point Q.

(3)

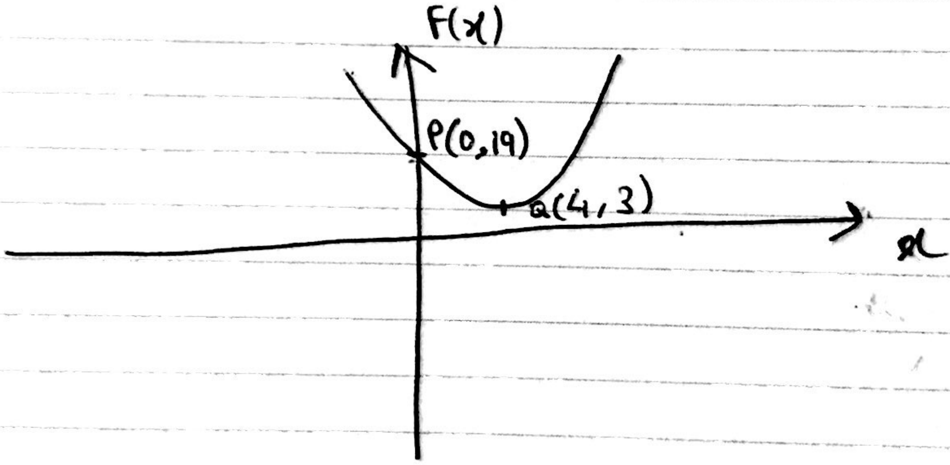
(c) Find the distance PQ, writing your answer as a simplified surd.

(3)

5(a). $f(x) = x^2 - 8x + 19 = (x-4)^2 - 4^2 + 19$

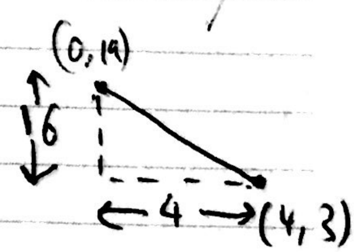
$= (x-4)^2 + 3$

b)



$f(0) = 19$

(c)



distance $PQ = \sqrt{4^2 + 16^2}$

$= \sqrt{16 + 16^2}$

$= \sqrt{16(1+16)}$ $\cancel{= \sqrt{4 \times 17}}$

$= \sqrt{16}\sqrt{17}$ $\cancel{= 2\sqrt{17}}$

$= 4\sqrt{17}$

6. (a) Given $y = 2^x$, show that

$$2^{2x+1} - 17(2^x) + 8 = 0$$

can be written in the form

$$2y^2 - 17y + 8 = 0$$

(2)

(b) Hence solve

$$2^{2x+1} - 17(2^x) + 8 = 0$$

(4)

6(a)

$y = 2^x \Rightarrow 2^{2x+1} - 17(2^x) + 8$

$= (2^{2x})(2) - 17(2^x) + 8$

$= 2(2^x)^2 - 17(2^x) + 8$

$= 2y^2 - 17y + 8 = 0$

(b) $2y^2 - 17y + 8 = 0$

$2y^2 - 17y + 8 = 2y^2 - 16y - y + 8$

$= 2y(y-8) - (y-8)$

$= (2y-1)(y-8) = 0$

$\Rightarrow y = \frac{1}{2}$ $\quad y = 2^x \Rightarrow 2^x = \frac{1}{2} = 2^{-1} \Rightarrow x = -1$

$y = 8$ $\quad 2^x = 8 = 2^3 \Rightarrow x = 3$

P 4 8 7 6 0 A 0 1 2 2 8

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7. The curve C has equation $y = f(x)$, $x > 0$, where

$$f'(x) = 30 + \frac{6 - 5x^2}{\sqrt{x}}$$

Given that the point $P(4, -8)$ lies on C,

(a) find the equation of the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

(b) Find $f(x)$, giving each term in its simplest form.

(5)

7. @ P, gradient $= f'(4) = 30 + \dfrac{6-5(4)^2}{\sqrt{4}}$

$= 30 + \dfrac{6-80}{2} = 30 - \dfrac{74}{2} = 30 - 37$

$= -7 \Rightarrow m = -7$

@ P $y = -8 \Rightarrow$ $\cancel{-8 = -7(4) + c}$

$\Rightarrow -8 = -7(4) + c$

$\therefore 28 - 8 = c$

$\Rightarrow c = 20$

$\therefore y = -7x + 20$

Question 7 continued

(b) $f'(x) = 30 + 6x^{-\frac{1}{2}} - 5\dfrac{x^2}{x^{1/2}}$

$= 30 + 6x^{-1/2} - 5x^{3/2}$

$\therefore f(x) = \int f'(x)\,\partial x = 30x + 12x^{\frac{1}{2}} - 2x^{5/2} + C$

@ P, $f(4) = 30(4) + 12\sqrt{4} - 2(4)^{5/2} + C$

$= 120 + 24 - 2(\sqrt{4})^5 + C$

$= 144 - 2(32) + C$

$= 80 + C = -8$

$\Rightarrow C = -88$

$\therefore f(x) = 30x + 12\sqrt{x} - 2(\sqrt{x})^5 - 88$

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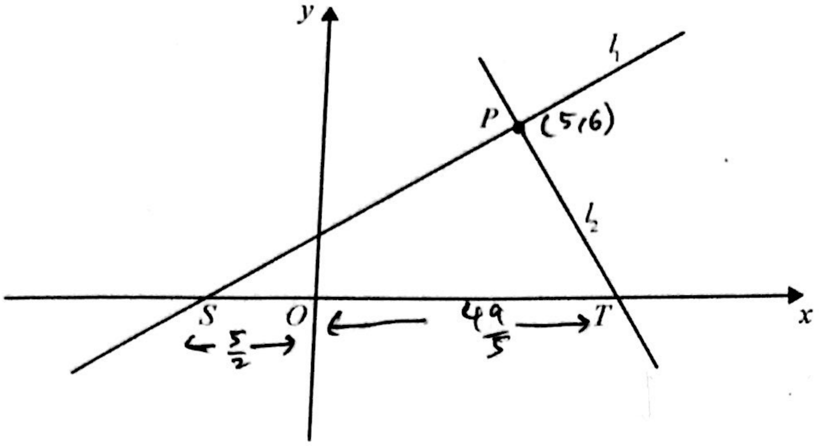
8.



Not to scale

Figure 1

The straight line l_1, shown in Figure 1, has equation $5y = 4x + 10$

The point P with x coordinate 5 lies on l_1

The straight line l_2 is perpendicular to l_1 and passes through P.

(a) Find an equation for l_2, writing your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(4)

The lines l_1 and l_2 cut the x-axis at the points S and T respectively, as shown in Figure 1.

(b) Calculate the area of triangle SPT.

(4)

8 (a).

@P, $x=5 \Rightarrow 5y = 4(5)+10 = 30$

$\Rightarrow y = \frac{30}{5} = 6$

l_1: $5y = 4x+10 \Rightarrow y = \frac{4}{5}x + 2 \Rightarrow$ gradient $= \frac{4}{5}$

$\therefore$ for l_2 gradient $= -\frac{1}{4/5} = -\frac{5}{4}$

let l_2 be $y = mx + k$

For l_2: @P, $y = 6 = -\frac{5}{4}(5) + k$

$= -\frac{25}{4} + k$

$\therefore k = 6 + \frac{25}{4} = \frac{24}{4} + \frac{25}{4} = \frac{49}{4}$

$\therefore l_2$ has eqn: $y = \frac{-5}{4}x + \frac{49}{4}$

$\therefore 4y = -5x + 49$

$\therefore 5x + 4y - 49 = 0$

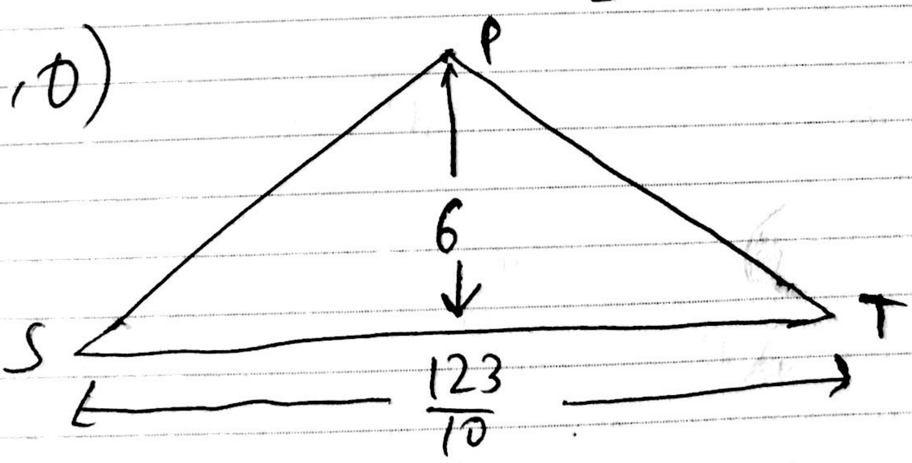
(b) Consider S:

l_1: $y = 0 \Rightarrow 5y = 0 = 4x + 10$
$\Rightarrow x_S = -\frac{5}{2}$

$\therefore S\left(-\frac{5}{2}, 0\right)$

Consider T: $y = 0 \Rightarrow 5x_T + 0 - 49 = 0$
$\therefore x_T = \frac{49}{5}$

$\therefore T\left(\frac{49}{5}, 0\right)$



$\frac{5}{2} + \frac{49}{5}$

$= \frac{25}{10} + \frac{98}{10}$

$= \frac{123}{10}$

$\therefore$ Area $= \frac{1}{2} \times \frac{123}{10} \times 6 = 3 \times \frac{123}{10}$

$= \frac{369}{10}$

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9. (a) On separate axes sketch the graphs of

(i) $y = -3x + c$, where c is a positive constant,

(ii) $y = \dfrac{1}{x} + 5$

On each sketch show the coordinates of any point at which the graph crosses the y-axis and the equation of any horizontal asymptote.

(4)

Given that $y = -3x + c$, where c is a positive constant, meets the curve $y = \dfrac{1}{x} + 5$ at two distinct points,

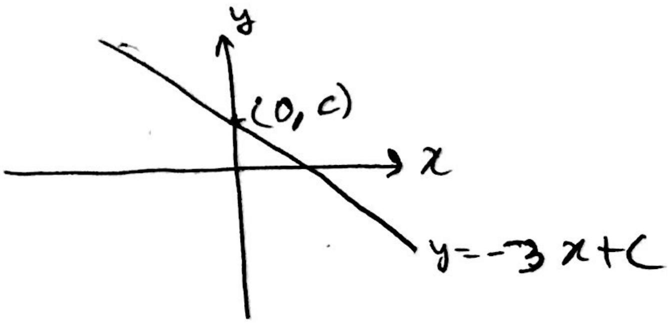
(b) show that $(5 - c)^2 > 12$

(3)

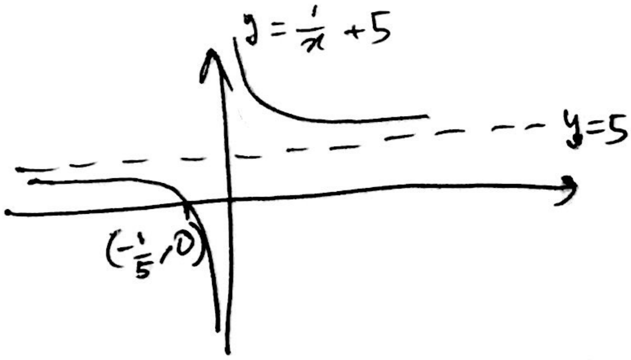
(c) Hence find the range of possible values for c.

(4)

9(a)(i)



(ii)



$y=0 \Rightarrow 0 = \frac{1}{x} + 5$

$\Rightarrow x = -\frac{1}{5}$

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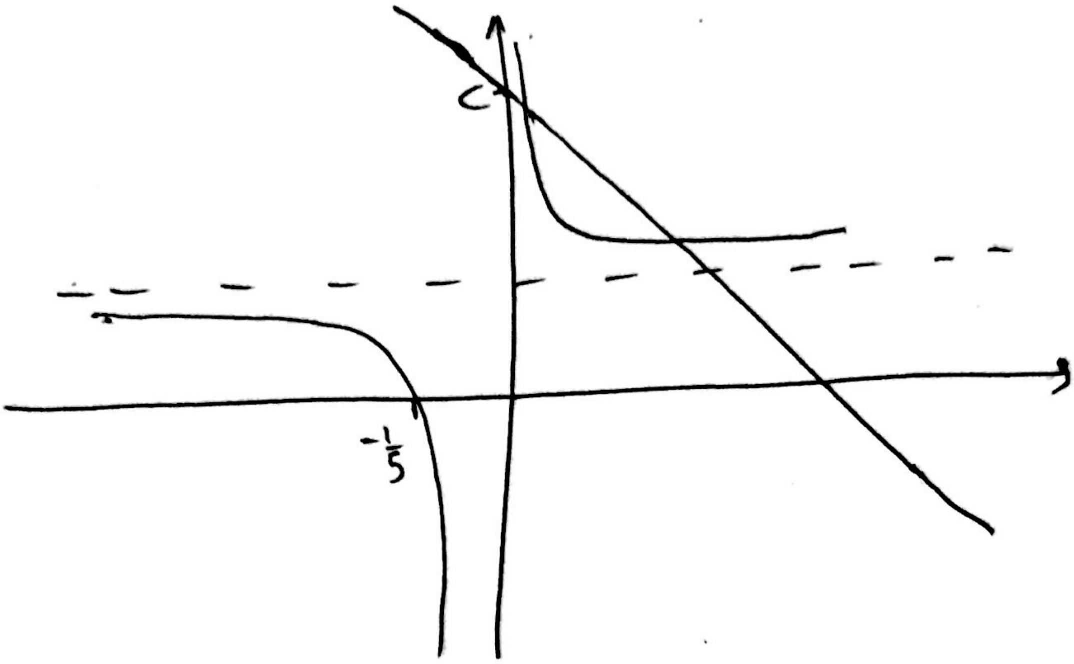
GradeMax

Question 9 continued

(b)



$y = -3x + c$ & $y = \frac{1}{x} + 5$ meet

$\therefore \quad -3x + c = \frac{1}{x} + 5 \Rightarrow \frac{1}{x} + 3x + 5 - c = 0$

We're told they meet at two distinct points.

$\therefore$ The discriminant of eqn: $\left(\frac{1}{x} + 3x + 5 - c = 0\right)$

is > 0

$\frac{1}{x} + 3x + 5 - c = 0$

$(\times x)$

$\Rightarrow 1 + 3x^2 + (5-c)x = 0$

$3x^2 + (5-c)x + 1 = 0$

discriminant $= b^2 - 4ac = (5-c)^2 - 4(3)(1) > 0$

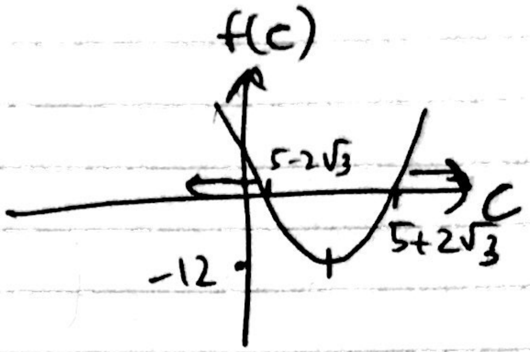
$\therefore (5-c)^2 - 12 > 0 \Rightarrow (5-c)^2 > 12$

Question 9 continued

c) $(5-c)^2 > 12$

$(5-c)^2 - 12 > 0$

let $f(c) = (5-c)^2 - 12$



critical points are

$c = 5 - 2\sqrt{3}$

$c = 5 + 2\sqrt{3}$

$(5-c)^2 - 12 = 0$

$(5-c)^2 = 12$

$(5-c) = \pm 2\sqrt{3}$

$\therefore \quad c > 5 + 2\sqrt{3}$

C is a positive constant.

$0 < c < 5 - 2\sqrt{3}$

$c > 5 + 2\sqrt{3}$

10.

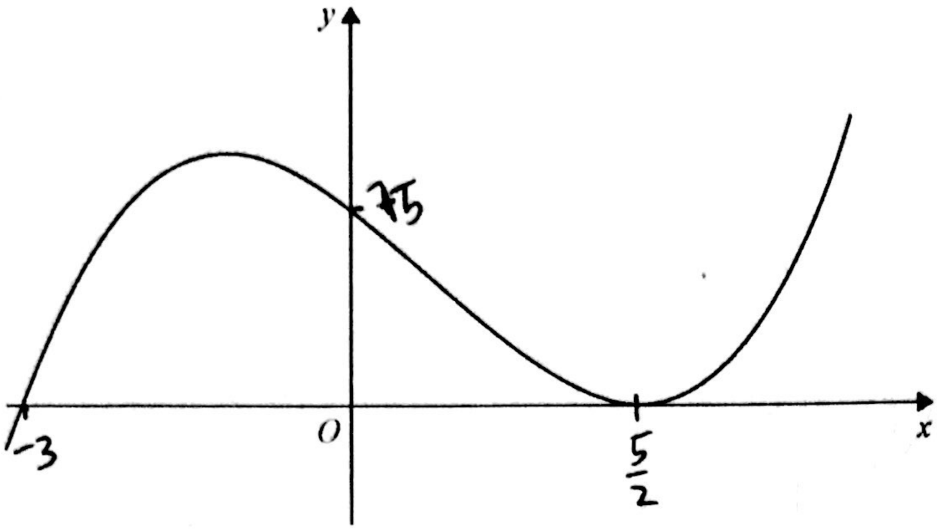


Figure 2

Figure 2 shows a sketch of part of the curve $y = f(x)$, $x \in \mathbb{R}$, where

$$f(x) = (2x - 5)^2(x + 3)$$

(a) Given that

(i) the curve with equation $y = f(x) - k$, $x \in \mathbb{R}$, passes through the origin, find the value of the constant k,

(ii) the curve with equation $y = f(x + c)$, $x \in \mathbb{R}$, has a minimum point at the origin, find the value of the constant c.

(3)

(b) Show that $f'(x) = 12x^2 - 16x - 35$

(3)

Points A and B are distinct points that lie on the curve $y = f(x)$.

The gradient of the curve at A is equal to the gradient of the curve at B.

Given that point A has x coordinate 3

(c) find the x coordinate of point B.

(5)

10(a)(i) $f(0) = 25 \times 3 = 75$ graph goes down 75 units to land on origin

$\therefore k = 75$

(ii) $c = \frac{5}{2}$

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(b) $f(x) = (2x-5)^2(x+3)$

$= (2x-5)(2x-5)(x+3)$

$= (4x^2 - 20x + 25)(x+3)$

$= 4x^3 - 8x^2 - 35x + 75 \cancel{+75}$

$\cancel{\therefore f'(x) = \frac{\partial}{\partial x}(4x^3 - 8x^2 - 35x + 25x} + 75)$

$\cancel{= 12x^2 - 16x - 3}$

$f'(x) = \frac{\partial}{\partial x}(4x^3 - 8x^2 - 35x + 75)$

$= (12x^2 - 16x - 35)$

(c). $f'(3) = 12(3)^2 - 16(3) - 35 = 25$

25 is gradient at A and B

$\therefore 12x^2 - 16x - 35 = 25$

$\Rightarrow 12x^2 - 16x - 60 = 0$

$x = 3$ is a solution

$\therefore (x-3)(12x+20) = 0$

$\Rightarrow x_B = -\frac{5}{3}$

P 4 8 7 6 0 A 0 2 5 2 8