

C12 October 2017 (MA)

Q1a) $8x + 2y - 15 = 0$

$$2y = 15 - 8x$$

$$\div 2 : y = \frac{15}{2} - 4x$$

so $\boxed{m = -4}$ = gradient

b) $m_{l_2} = -4$ (parallel) $(-\frac{3}{4}, 16)$

$$\therefore y - 16 = -4(x + \frac{3}{4})$$

$$y = -4x - 3 + 16$$

$$\boxed{y = -4x + 13}$$

Q2a) $f(x+2)$ indicates a translation of +2 in the negative x -direction

so $\boxed{(0, 3)}$

b) $-f(x) \rightarrow$ multiply all y by -1

so $\boxed{(2, -3)}$

c) $y = \frac{1}{2} f(x) \rightarrow$ multiply all y by $\frac{1}{2}$

so $\boxed{(2, \frac{3}{2})}$

d) $y = f(x) - 4 \rightarrow$ translation of +4 in the negative y-dir.

$$\boxed{(2, -1)}$$

$$\text{Q3a)} \quad \frac{x^3 + 4}{2x^2} = \frac{x^3}{2x^2} + \frac{4}{2x^2} = \frac{x}{2} + \frac{2}{x^2}$$

$$= \boxed{\frac{1}{2}x + 2x^{-2}}$$

$$\text{b)} \quad \int \left[\frac{x^3 + 4}{2x^2} \right] dx = \int \left(\frac{1}{2}x + 2x^{-2} \right) dx$$

$$= \frac{x^2}{4} + \frac{2x^{-1}}{-1} + c = \boxed{\frac{1}{4}x^2 - \frac{2}{x} + c}$$

$$\text{Q4a)} \quad \text{Area } \Delta = \frac{1}{2} ab \sin C = \frac{1}{2} (3x)(x) \sin 60$$

$$\Rightarrow \frac{3x^2}{2} \sin 60 = 24\sqrt{3}$$

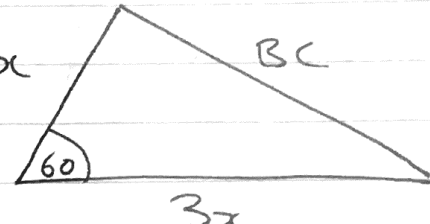
$$\Rightarrow \frac{3x^2}{2} \times \frac{\sqrt{3}}{2} = 24\sqrt{3}$$

$$\Rightarrow x^2 \left(\frac{3\sqrt{3}}{4} \right) = 24\sqrt{3}$$

$$\Rightarrow x^2 = \frac{24\sqrt{3}}{\frac{3\sqrt{3}}{4}} = 32 //$$

$$\text{So } x = \sqrt{32} = \boxed{4\sqrt{2}}$$

b) cosine rule

$$BC^2 = x^2 + (3x)^2 - 2(x)(3x)\cos 60$$


$$BC^2 = 10x^2 - 6x^2\left(\frac{1}{2}\right)$$

$$BC^2 = 7x^2 = 7(4\sqrt{2})^2 = 224 //$$

$$\text{so } BC = \sqrt{224} = \boxed{4\sqrt{14}}$$

(Q5a) $y = 27x^{\frac{3}{2}} - 2x^2$

$$\frac{dy}{dx} = \boxed{\frac{27}{2}x^{-\frac{1}{2}} - 4x}$$

b) $\frac{dy}{dx} = 0$ at P

$$\frac{27}{2}x^{-\frac{1}{2}} - 4x = 0$$

$$4x = \frac{27}{2}x^{-\frac{1}{2}}$$

$$\therefore 4x^{\frac{3}{2}} = \frac{27}{2}$$

$$\text{so } x^{\frac{3}{2}} = \frac{27}{8}$$

$$x = \sqrt[3/2]{\frac{27}{8}} = \frac{9}{4} //$$

$$\text{at } x = \frac{9}{4}, y = 27\left(\frac{9}{4}\right)^{\frac{3}{2}} - 2\left(\frac{9}{4}\right)^2 = \frac{243}{8} //$$

so $P\left(\frac{9}{4}, \frac{243}{8}\right)$

(Q6a)

| | | | |
|--------|--------|--------|---------------------------------------------------------------|
| year 1 | year 2 | year 3 | |
| 600 | 680 | 760 | $\left(\begin{array}{l} a = 600 \\ d = 80 \end{array}\right)$ |

$$n^{\text{th}} \text{ term} = a + (n-1)d$$

$$N^{\text{th}} \text{ term} = 600 + (N-1) \times 80 = 1000$$

$$80(N-1) = 400$$

$$N-1 = 5$$

$$\therefore \boxed{N = 6}$$

b) we require sum of the first 15 terms.

$$S_n = \frac{n}{2} (2a + (n-1)d)$$

$$n = 15 : S_{15} = \frac{15}{2} (2(600) + (14)(80))$$

$$= \boxed{\pounds 17400}$$

c)

| | | | |
|--------|--------|--------|------------------------------------------------------------|
| year 1 | year 2 | year 3 | |
| A | 2A | 3A | $\left(\begin{array}{l} a = A \\ d = A \end{array}\right)$ |

Sum of first 15 terms is equal for Saima and Lin.

$$S_{15} = \frac{15}{2} (2A + (15-1)A) = 17400 //$$

$$\text{So } \frac{15}{2} (2A + 14A) = 17400$$

$$120A = 17400$$

$$A = \frac{17400}{120} = \boxed{145}$$

$$(Q7a) \quad g(x) = 2x^3 + ax^2 - 18x - 8$$

$(x+2)$ is a factor of $g(x)$ so $g(-2) = 0$.

$$g(-2) = 2(-2)^3 + a(-2)^2 - 18(-2) - 8 = 0$$

$$-16 + 4a + 36 - 8 = 0$$

$$4a = -12 \quad \therefore \boxed{a = -3}$$

b)

$$\begin{array}{r} 2x^2 - 7x - 4 \\ x+2 \overline{) 2x^3 - 3x^2 - 18x - 8} \\ \underline{2x^3 + 4x^2} \\ 0 - 7x^2 - 18x \\ \underline{-7x^2 - 14x} \\ 0 - 4x - 8 \\ \underline{-4x - 8} \\ 0 \\ 0 \end{array}$$

$$\therefore g(x) = (x+2)(2x^2 - 7x - 4)$$

$$\text{but } 2x^2 - 7x - 4 = (2x+1)(x-4)$$

$$\text{So } g(x) = \boxed{(x+2)(2x+1)(x-4)}$$

c) $2\sin^3\theta - 3\sin^2\theta - 18\sin\theta - 8 = 0$

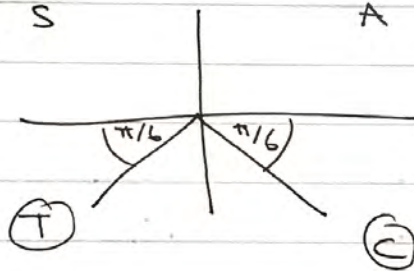
but notice this is exactly the same as $[g(x) = 0]$ if we let $x = \sin\theta$.

so let $x = \sin\theta$:

$$2x^3 - 3x^2 - 18x - 8 = 0 \quad (=g(x))$$

$$\therefore (x+2)(2x+1)(x-4) = 0 //$$

| | | |
|--------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------|
| $x+2 = 0$ $x = -2$ $\sin\theta = -2$ <u>NO SOLUTIONS</u> $-1 \leq \sin\theta \leq 1$ | $2x+1 = 0$ $x = -\frac{1}{2}$ $\sin\theta = -\frac{1}{2}$ $\theta = \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$ | $x-4 = 0$ $x = 4$ $\sin\theta = 4$ <u>NO SOLUTIONS</u> $-1 \leq \sin\theta \leq 1$ |
|--------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------|



$$\theta = \pi + \frac{\pi}{6}, \quad 2\pi - \frac{\pi}{6}$$

$\theta = \frac{7\pi}{6}, \quad \frac{11\pi}{6}$

$$\bullet \text{ (Q8a)} \quad L = \boxed{r\theta = 6}$$

$$\frac{1}{2} r^2 \theta = \text{Area OAB} = 20$$

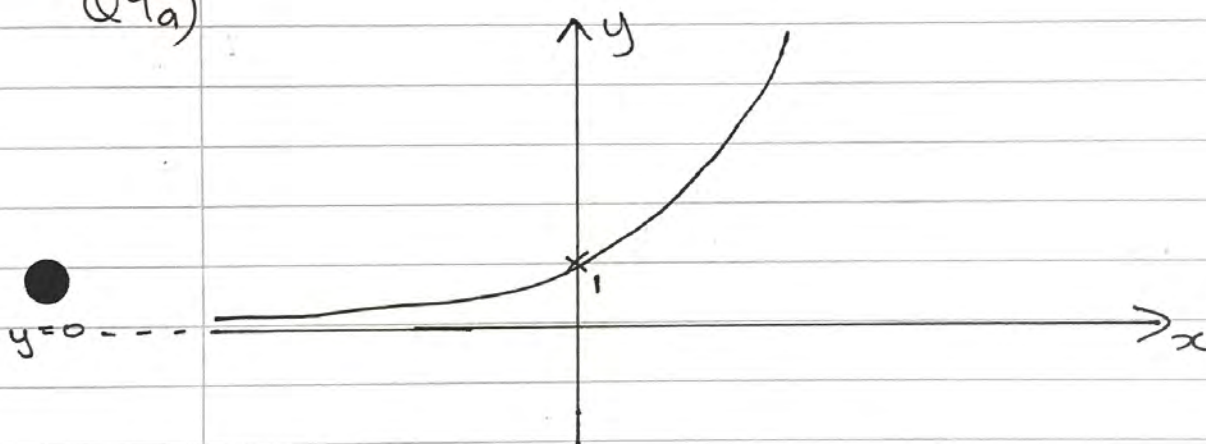
$$20 = \frac{1}{2} r^2 \theta \rightarrow \boxed{40 = r^2 \theta}$$

$$\bullet \text{ b) } r\theta = 6 \rightarrow 40 = 6 \times r$$

$$r = \frac{40}{6} = \boxed{\frac{20}{3}}$$

$$\theta = \frac{6}{r} = \frac{6}{\frac{20}{3}} = \boxed{\frac{9}{10}}$$

(Q9a)



$$\bullet \text{ b) } h = \frac{b-a}{n} = \frac{4 - -4}{4} = 2$$

$$\text{Area} \approx \frac{1}{2} \times 2 \left[0.0625 + 16 + 2(0.25 + 1 + 4) \right]$$

$$\approx \boxed{26.56}$$

$$c_i) 2^{x+2} = 2^x (2^2) = 2^2 \times 2^x //$$

$$\text{so } \int_{-4}^4 2^{x+2} dx = 2^2 \times 26.56 \approx \boxed{106}$$

or $\frac{425}{16}$ (exact)

$$ii) \int_{-4}^4 (3 + 2^x) dx = \int_{-4}^4 (3) dx + \int_{-4}^4 (2^x) dx$$

$$= \left[3x \right]_{-4}^4 + 26.56$$

$$= [3 \times 4] + [3 \times 4] + 26.56 \approx \boxed{50.6}$$

or $\frac{809}{16}$ (exact)

Q10a) A(7, -3) and D(10, 5)
C(p, q)

$$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

since D is the midpoint...

$$\hookrightarrow \left(\frac{7+p}{2}, \frac{-3+q}{2} \right) = (10, 5)$$

$$\frac{7+p}{2} = 10 \quad \therefore p = 20 - 7 = \boxed{13}$$

$$\frac{-3+q}{2} = 5 \quad \therefore q = 10 + 3 = \boxed{13}$$

$$b) m_{AC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{13 - -3}{13 - 7} = \frac{16}{6} = \frac{8}{3}$$

but l is perp to AC so $m_l = -\frac{3}{8} //$
 $(-\frac{3}{8} \times \frac{8}{3} = -1) //$

$$\Rightarrow y - 5 = -\frac{3}{8}(x - 10)$$

$$\Rightarrow y = -\frac{3}{8}x + \frac{30}{8} + 5$$

$$\Rightarrow y = -\frac{3}{8}x + \frac{35}{4}$$

$$\xrightarrow{\times 8} 8y = -3x + 70$$

$$\rightarrow \boxed{3x + 8y = 70}$$

c) AB is simply the line $x = 7$ (for $-3 \leq y \leq 20$)

$$\text{so sub } x=7 : 3(7) + 8y = 70$$

$$8y = 49$$

$$\boxed{y = \frac{49}{8}}$$

$$\text{so } \boxed{E\left(7, \frac{49}{8}\right) //$$

$$\begin{aligned}
 \text{Q11a)} \quad (3+ax)^5 &\approx (3)^5 + \binom{5}{1}(3)^4(ax)^1 \\
 &\quad + \binom{5}{2}(3)^3(ax)^2 \\
 &\approx \boxed{243 + 405ax + 270a^2x^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{b)} \quad f(x) &= (a-x)(243 + 405ax + 270a^2x^2) \\
 &= 243 + \underline{405a^2x} + 270a^3x^2 \\
 &\quad - \underline{243x} - 405ax^2 - 270a^2x^3
 \end{aligned}$$

x terms are $(+405a^2x)$ and $(-243x)$

coeff. of x is 0 so $405a^2 - 243 = 0$

$$\Rightarrow a^2 = \frac{243}{405} = \frac{3}{5}$$

$$\therefore a = \boxed{\sqrt{\frac{3}{5}}}$$

$$\bullet \text{ (12i)} \quad 3 \sin(\theta + 30) = 2 \cos(\theta + 30)$$

$$\div \underline{\cos(\theta + 30)} : \quad 3 \tan(\theta + 30) = 2$$

$$\tan(\theta + 30) = \frac{2}{3}$$

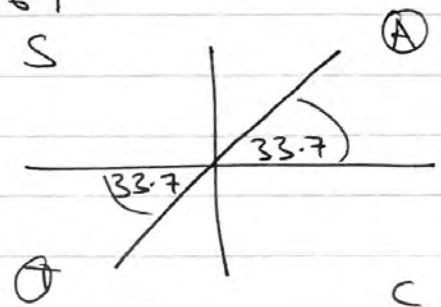
$$\text{so } \theta + 30 = \tan^{-1}\left(\frac{2}{3}\right) = 33.69^\circ \dots$$

$$\text{range } \dots \quad \underline{30^\circ < \theta + 30 \leq 390^\circ}$$

$$\theta + 30^\circ = 33.69^\circ, 180 + 33.69^\circ$$

$$\theta + 30 = 33.69^\circ, 213.69^\circ$$

$$\boxed{\theta = 3.69^\circ, 183.69^\circ}$$



$$\text{ii a)} \quad \frac{\cos^2 x + 2 \sin^2 x}{1 - \sin^2 x} = 5$$

$$\frac{\cos^2 x + 2 \sin^2 x}{\cos^2 x} = 5$$

$$1 + 2 \tan^2 x = 5$$

$$2 \tan^2 x = 4$$

$$\therefore \boxed{\tan^2 x = 2}$$

$$(n=2)$$

$$0 < x < 2\pi$$

b) we need to solve: $\tan^2 x = 2$

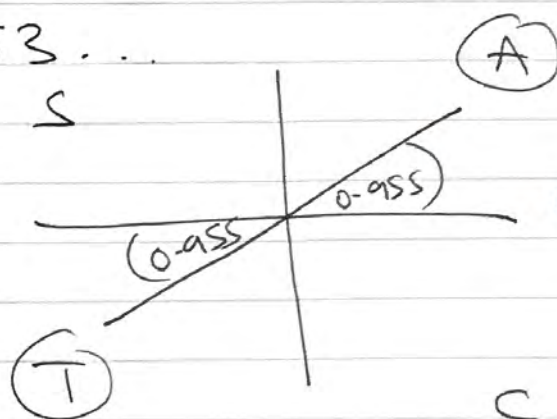
$$\tan x = \pm\sqrt{2}$$

$$\tan x = \sqrt{2}$$

$$x = \tan^{-1}(\sqrt{2}) = 0.9553 \dots$$

$$x = 0.955^\circ, 4.10^\circ, \dots$$

$$x = 0.955^\circ, 4.097^\circ$$



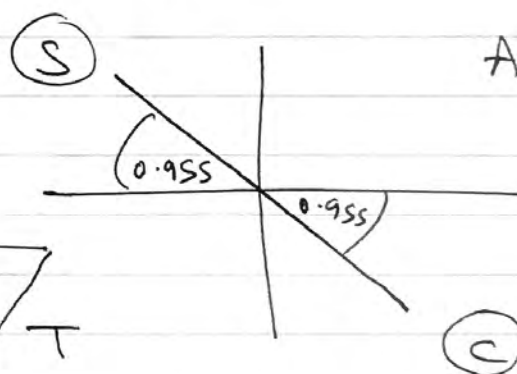
$$\tan x = -\sqrt{2}$$

$$x = \tan^{-1}(-\sqrt{2}) = -0.9553 \dots$$

$$x = \pi - 0.9553$$

$$2\pi - 0.9553$$

$$x = 2.186^\circ, 5.328^\circ$$



1

● (Q13a) $(x-3)^2 + (y+4)^2 = 30$

centre $(3, -4)$

ii) radius $\sqrt{30}$

b) since P lies inside the circle we know that the distance from the centre to P will be less than the radius.

so $\sqrt{(6-3)^2 + (4+k)^2} < \sqrt{30}$

square both sides ...

$$(6-3)^2 + (k+4)^2 < 30$$

$$9 + k^2 + 16 + 8k < 30$$

$$k^2 + 8k + 25 - 30 < 0$$

$$k^2 + 8k - 5 < 0$$

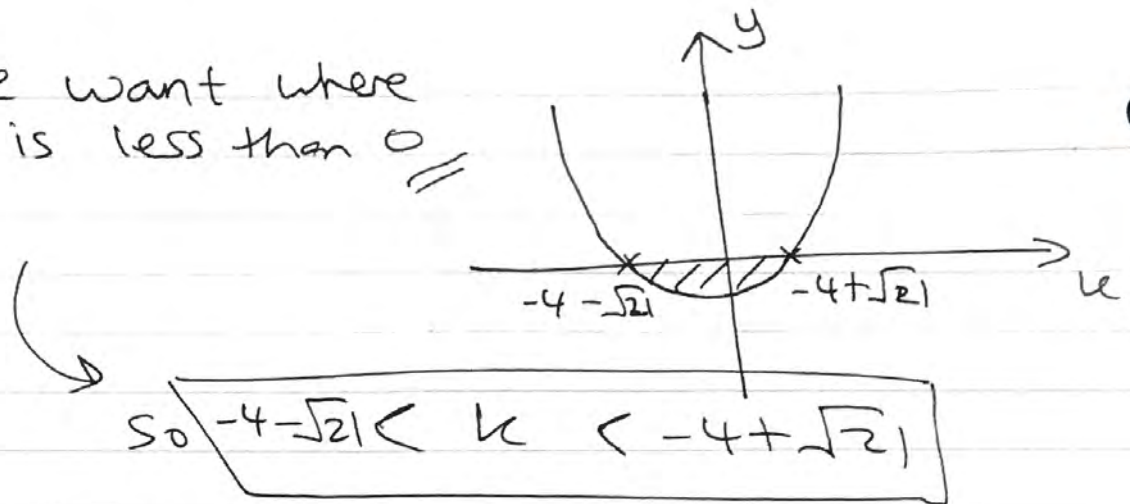
c) we want the values of k for which

$$k^2 + 8k - 5 < 0$$

critical values: $k^2 + 8k - 5 = 0$

By Quadratic formula, $k = -4 \pm \sqrt{21}$

we want where
 y is less than 0



(Q14a)

| year 1 | year 2 | year 3 |
|--------|----------------------|------------------------|
| 8000 | $8000 \times (0.85)$ | $8000 \times (0.85)^2$ |

$$\begin{pmatrix} a = 8000 \\ r = 0.85 \end{pmatrix}$$

$$\begin{aligned} \text{so } 6^{\text{th}} \text{ year} &\rightarrow ar^{n-1} = 8000(0.85)^5 \\ &= 3549.6 \\ &\approx \boxed{3550} \end{aligned}$$

b) $\boxed{r < 1}$ as $n \rightarrow \infty$, $S_n \rightarrow \text{constant}$.
 (ie S_∞)

$$c) S_\infty = \frac{a}{1-r} = \frac{8000}{1-0.85} \approx \boxed{53333}$$

$$d) S_n = \frac{a(1-r^n)}{1-r} = 40000$$

$$\frac{8000(1-0.85^N)}{0.15} = 40000$$

$$8000(1 - 0.85^N) = 6000$$

$$1 - 0.85^N = \frac{3}{4}$$

$$0.85^N = \frac{1}{4}$$

$$\log(0.85^N) = \log\left(\frac{1}{4}\right)$$

$$N \log 0.85 = \log \frac{1}{4}$$

$$N = \frac{\log \frac{1}{4}}{\log 0.85} = 8.53 \dots$$

So $\boxed{N=9}$ (in the 8th year, total mass < 40000)
so $N=9$

(Q15ai) $\underline{x=0}$: $y = \frac{(-3)^2(4)}{2} = \boxed{18}$

ii) $\underline{y=0}$: $\frac{(x-3)^2(x+4)}{2} = 0$

$\boxed{x=3}$ ($x > 0$)

b) $\underline{y=18}$: $18 = \frac{(x-3)^2(x+4)}{2}$

$36 = (x-3)^2(x+4)$

$$(x^2 - 6x + 9)(x + 4) = 36$$

$$x^3 - 2x^2 - 15x + 36 = 36$$

$$x^3 - 2x^2 - 15x = 0$$

$$x(x^2 - 2x - 15) = 0$$

$$x > 0 \quad \therefore \boxed{x^2 - 2x - 15 = 0}$$

$$c) \quad (x - 5)(x + 3) = 0$$

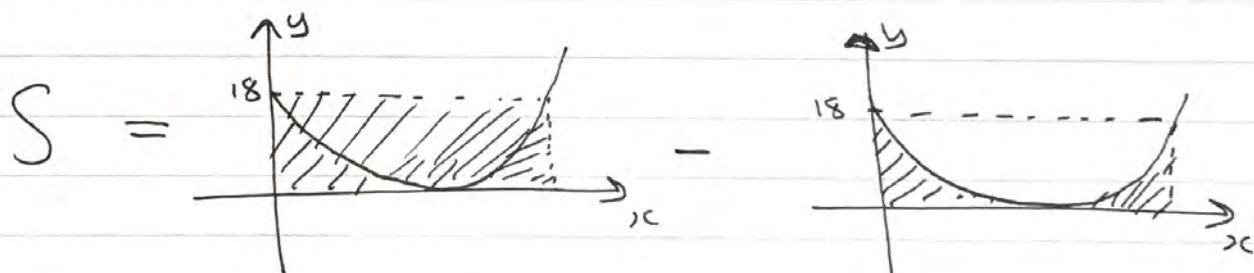
$$x = 5$$

$$x = -3$$

$$x > 0 \quad \text{so } \boxed{x = 5}$$

$$\text{hence } \boxed{(5, 18)}$$

d)



$$S =$$

$$- \int_0^5 f(x) dx$$

$$\therefore S = (18 \times 5) - \frac{1}{2} \int_0^5 [x^3 - 2x^2 - 15x + 36] dx$$

$$= 90 - \frac{1}{2} \left[\frac{x^4}{4} - \frac{2x^3}{3} - \frac{15x^2}{2} + 36x \right]_0^5$$

$$= 90 - \frac{1}{2} \left[\frac{785}{12} \right] = \boxed{\frac{1375}{24}}$$