C12 January 2018 (MA)

Q1 a) $y = \frac{1}{3}x^{\frac{2}{3}} + \frac{1}{2}$

$\frac{dy}{dx} = \frac{2}{3}\left(\frac{1}{3}\right)x^{-\frac{1}{3}} + 0$

$\therefore \boxed{\frac{dy}{dx} = \frac{2}{9}x^{-\frac{1}{3}}}$

b) $\int\left(\frac{1}{3}x^{\frac{2}{3}} + \frac{1}{2}\right)dx = \frac{\frac{1}{3}x^{\frac{5}{3}}}{\frac{5}{3}} + \frac{1}{2}x + c$

$= \boxed{\frac{1}{5}x^{\frac{5}{3}} + \frac{1}{2}x + c}$

Q2 a) $u_2 = 2 - 3(1) = \boxed{-1}$

$u_3 = 2 - 3(-1) = \boxed{5}$

$\Sigma r \qquad \Sigma u_r$

b) $\sum_{r=1}^{4} r - u_r = (1+2+3+4) - (u_1+u_2+u_3+u_4)$

$= (10) - (1 - 1 + 5 + 2 - 3(5))$

$= 10 - -8 = \boxed{18}$

Q3a) $(3x^{\frac{1}{2}})^4 = 3^4 \times x^{\frac{1}{2}\times 4} = \boxed{81x^2}$

b) $\dfrac{2y^7 \times (4y)^{-2}}{3y} = \dfrac{2y^7 \times \frac{1}{16y^2}}{3y}$

$= \dfrac{\frac{2y^7}{16y^2}}{3y} = \dfrac{\frac{1}{8}y^5}{3y} = \dfrac{y^5}{24y} = \boxed{\dfrac{y^4}{24}}$

Q4a) $(p-2)x^2 + (8)x + (p+4) = 0$

no real roots so $b^2-4ac < 0$

$\left.\begin{array}{l} a = p-2 \\ b = 8 \\ c = p+4 \end{array}\right\}$ $b^2 - 4ac < 0$

$(8)^2 - 4(p-2)(p+4) < 0$

$64 - 4(p^2 + 2p - 8) < 0$

$64 - 4p^2 - 8p + 32 < 0$

$-4p^2 - 8p + 96 < 0$

$\underline{\div 4}$: $-p^2 - 2p + 24 < 0$

$\underline{\times -1}$: $p^2 + 2p - 24 > 0$

b) $p^2 + 2p - 24 = 0$

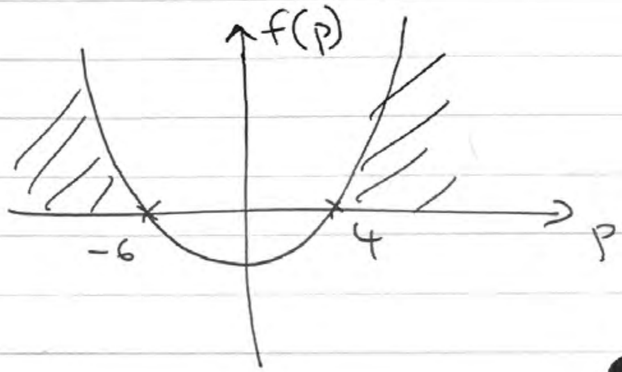
finding critical values ... By Quadratic Formula

$a = 1$
$b = 2$
$c = -24$

$$p = \frac{-2 \pm \sqrt{4 - 4(-24)}}{2}$$

$$p = \frac{-2 \pm \sqrt{100}}{2}$$

So $p = 4$, $p = -6$



we want where $y > 0$.

So $\boxed{\begin{matrix} p > 4 \\ p < -6 \end{matrix}}$

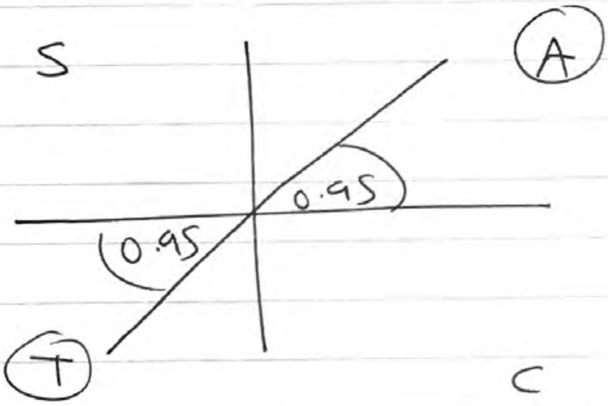
Q5 i) $5\sin 3\theta - 7\cos 3\theta = 0$

$\div \underline{\cos 3\theta}$: $5\tan 3\theta = 7$

$\tan 3\theta = \frac{7}{5}$

$3\theta = \tan^{-1}\left(\frac{7}{5}\right) = 0.9505\ldots$

range : $\underline{0 < 3\theta < \frac{3\pi}{2}}$



$3\theta = 0.9505, \pi + 0.9505$

$3\theta = 0.9505, 4.092$

so $\theta = \boxed{0.317, 1.36}$

ii) $9\cos^2 x + 5\cos x = 3 - 3\cos^2 x$

$12\cos^2 x + 5\cos x - 3 = 0$

$(3\cos x - 1)(4\cos x + 3) = 0$

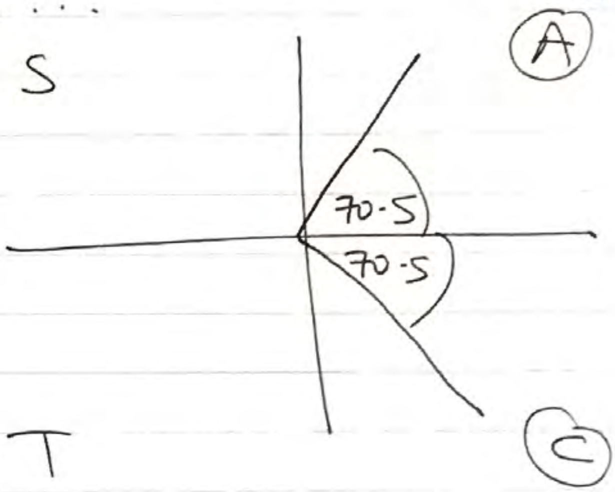
$3\cos x - 1 = 0$
$\cos x = \frac{1}{3}$ //

$4\cos x + 3 = 0$
$\cos x = -\frac{3}{4}$ //

$x = \cos^{-1}\left(\frac{1}{3}\right) = 70.53°...$

$x = 70.5°, 360 - 70.5°$

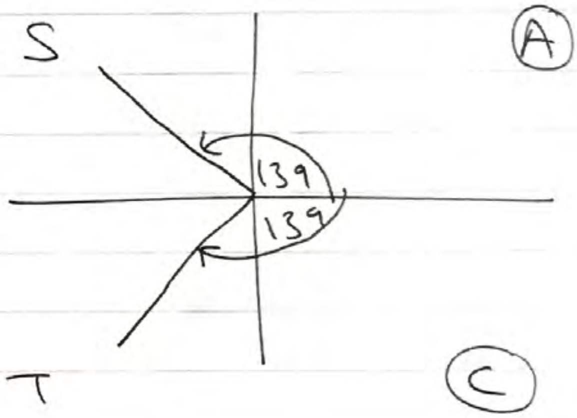
$\boxed{x = 70.5°, 289.5°}$



$x = \cos^{-1}\left(-\frac{3}{4}\right) = 138.6°$

$x = 138.6°, 360 - 138.6°$

$\boxed{x = 138.6°, 221.4°}$



Q6a) $f(x)$ div by $(x+1) \rightarrow$ no remainder

So $f(-1) = 0$.

$f(-1) = -a - 8 - b + 6 = 0$

$a + b = -2$

$f(x)$ div by $(x-2) \rightarrow$ remainder $= -12$

so $f(2) = -12$

$\hookrightarrow f(2) = 8a - 32 + 2b + 6 = -12$

$\therefore \; 8a + 2b = 14$

$\underline{\div 2}: \; 4a + b = 7$

$- \; (a + b = -2)$

$3a + 0 = 9$

$\therefore 3a = 9$ so $\boxed{a = 3}$

$b = -2 - a = -2 - 3 = \boxed{-5}$

b)

$$\begin{array}{r} 3x^2 - 11x + 6 \\ x+1 \,\big)\, \overline{3x^3 - 8x^2 - 5x + 6} \\ 3x^3 + 3x^2 \\ \hline 0 - 11x^2 - 5x \\ -11x^2 - 11x \\ \hline 0 + 6x + 6 \\ 6x + 6 \\ \hline 0 \quad 0 \end{array}$$

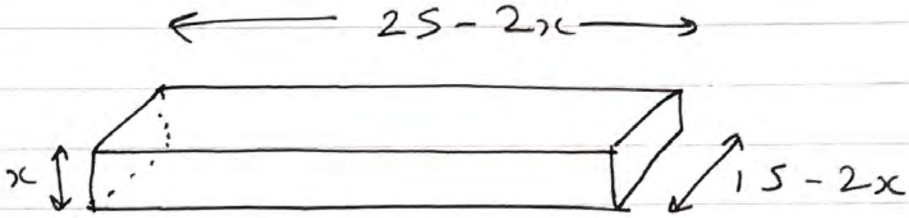
//

$\therefore f(x) = (x+1)(3x^2 - 11x + 6)$

but $(3x^2 - 11x + 6) = (3x-2)(x-3)$ //

$\therefore f(x) = \boxed{(x+1)(3x-2)(x-3)}$

(Q7a)



$$\text{Volume} = (25-2x)\times(15-2x)\times(x)$$

$$V = (375 - 50x - 30x + 4x^2)\times x$$

$$V = (375 - 80x + 4x^2) \times x$$

$$V = 4x^3 - 80x^2 + 375x$$

b) $\dfrac{dV}{dx} = 12x^2 - 160x + 375$

at max value, $\dfrac{dV}{dx} = 0.$

$\Rightarrow \quad 12x^2 - 160x + 375 = 0$

By Quadratic formula...

$$\left.\begin{aligned} a&=12 \\ b&=-160 \\ c&=375 \end{aligned}\right\} \quad x = 3.03,\ 10.3$$

but $x \neq 10.3$ since side length is $15-2x$

so $\boxed{x = 3.03}$

c) $\frac{d^2V}{dx^2} = 24x - 160$

$x = 3.03 \rightarrow \left(\frac{d^2V}{dx^2}\right)_{x=3.03} = 24(3.03) - 160$

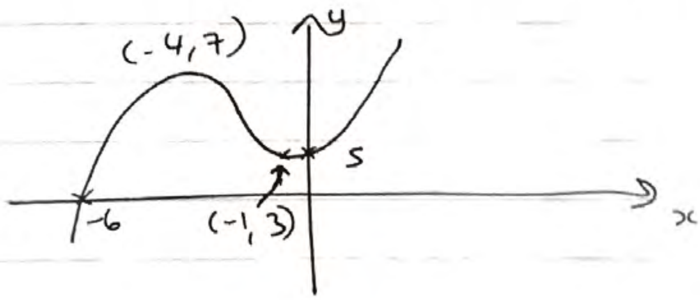
$\simeq -87 < 0$

hence value is a maximum.

d) $V = 4(3.03)^3 - 80(3.03)^2 + 375(3.03)$

$\simeq \boxed{513}\ cm^3$

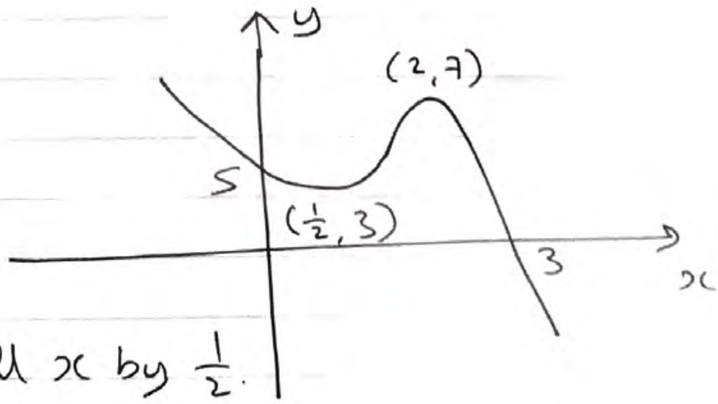
Q8a) $f(-x) \rightarrow$ multiply all x by -1



b) $f(2x)$

$\downarrow$

multiply all x by $\frac{1}{2}$.



Q9a) $a \quad ar \quad ar^2$

$20 \quad 20(0.9) \quad 20(0.9)^2$

5^{th} term $= ar^4 = 20 \times (0.9)^4$

$= \boxed{13.122}$

b) $S_8 = \dfrac{20(1-0.9^8)}{1-0.9} = \boxed{113.9}$

c) $S_\infty = \dfrac{a}{1-r} = \dfrac{20}{1-0.9} = 200$

$S_\infty - S_N = 200 - \dfrac{20(1-0.9^N)}{0.1}$

$S_\infty - S_N = 200 - 200(1-0.9^N)$

$= 200 - 200 + 200(0.9^N)$

$= 200(0.9^N)$ //

hence $200(0.9^N) < 0.04$

$\underline{\div 200}$: $0.9^N < \dfrac{0.04}{200}$

$\therefore \quad 0.9^N < 0.0002$ //

d) $\log(0.9^N) < \log(0.0002)$

$N\log(0.9) < \log(0.0002)$

$N > \dfrac{\log 0.0002}{\log 0.9}$

÷ by a −ve value flips the signs

$N > 80.84\ldots$

So $\boxed{N_{min} = 81}$

Q10i) $3\log_8(2) + \log_8(7-x) = 2 + \log_8(x)$

$\log_8(2^3) + \log_8(7-x) - \log_8(x) = 2$

$\cancel{\log_8(8)}^{1} + \log_8\left(\dfrac{7-x}{x}\right) = 2$

$1 + \log_8\left(\dfrac{7-x}{x}\right) = 2$

$\therefore \log_8\left(\dfrac{7-x}{x}\right) = 1$

$\therefore 8^1 = \dfrac{7-x}{x}$

So $8x = 7 - x$

$9x = 7$: $\boxed{x = \frac{7}{9}}$

ii) $3^{2y} + 3^{y+1} = 10$

$3^{2y} + (3^y)(3) - 10 = 0$

let $3^y = u$,

then $u^2 + 3u - 10 = 0$

$(u+5)(u-2) = 0$

$u = -5$	$u = 2$
$3^y = -5$	$3^y = 2$
$y\log 3 = \log(-5)$	$\log(3^y) = \log(2)$
↙	$y\log 3 = \log 2$
no valid solutions....	$y = \boxed{\frac{\log 2}{\log 3}}$ //
($a^x > 0$ for all x)	

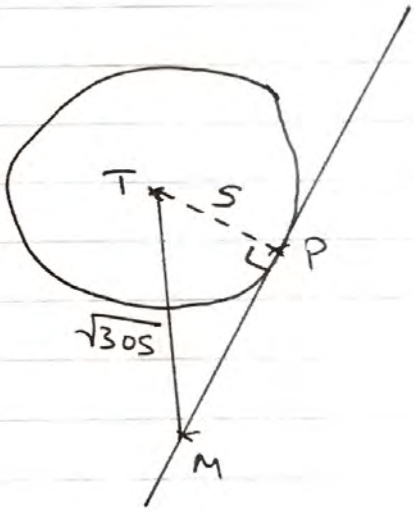
Q11a) /ii $x^2 - 8x + y^2 - 10y + 16 = 0$

$(x-4)^2 - \cancel{16} + (y-5)^2 - 25 + \cancel{16} = 0$

$\therefore \boxed{(x-4)^2 + (y-5)^2 = 25}$

so $\boxed{\begin{array}{l} T(4,5) \\ r = 5 \end{array}}$

b) $MT = \sqrt{(4-20)^2 + (5-12)^2} = \boxed{\sqrt{305}}$



c) $PM = \sqrt{(305) - (5)^2} = 2\sqrt{70}$ //

$\therefore \Delta PMT$ area $= \frac{1}{2}(2\sqrt{70})(5)$

$= \boxed{5\sqrt{70}}$

Q12a) $\underline{x = -1}$: $-1 + 3y - 11 = 0$

$3y = 12 \quad \therefore y = \boxed{4 = p}$

$\underline{y = 2}$: $x + 6 - 11 = 0$

$x = 5 \quad \therefore \boxed{q = 5}$

b) $(-1, 4)$ $(5, 2)$

$AB = \sqrt{(5--1)^2 + (2-4)^2} = \boxed{2\sqrt{10}}$

c) midpoint : $\left(\frac{-1+5}{2}, \frac{4+2}{2} \right)$

$\Rightarrow (2, 3)$ //

$m_{AB} = \frac{2-4}{5--1} = \frac{-2}{6} = -\frac{1}{3}$ //

so $m_{L_2} = 3$ // $\quad (3 \times -\frac{1}{3} = -1)$

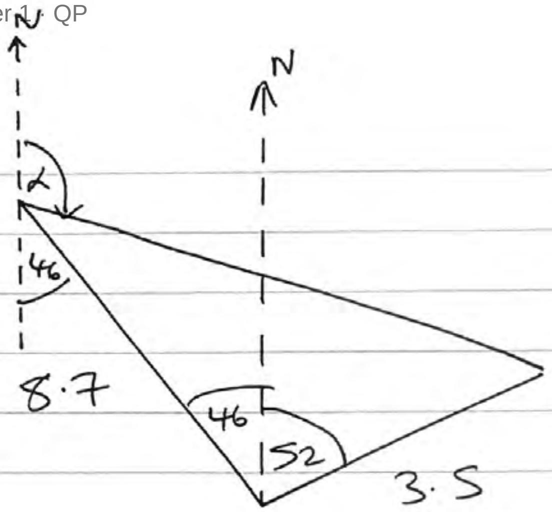
since L_2 is pep to L_1.

$\Rightarrow y - 3 = 3(x - 2)$

$\Rightarrow y = 3x - 6 + 3$

$\Rightarrow \boxed{y = 3x - 3}$

Q13a)



$360 - 314 = 46$

$\angle APB = 46 + 52 = \boxed{98^\circ}$

b) <u>cosine rule</u>

$AB^2 = (8.7)^2 + (3.5)^2 - 2(8.7)(3.5)\cos 98$
$\simeq 96.42\ldots$

so $AB = \boxed{9.8\text{ km}}$

c) sine rule to find $\angle PAB$.

$$\frac{9.8}{\sin 98} = \frac{3.5}{\sin \angle PAB}$$

$$\therefore \sin \angle PAB = \frac{3.5}{9.8} \sin 98$$

$$\therefore \angle PAB = \sin^{-1}\left(\frac{3.5}{9.8}\sin 98\right) \simeq 20.66^\circ$$

Bearing $= \alpha = 180 - 46 - 20.66$
$= \boxed{113^\circ}$

Q14a) $8 - x = 14 + 3x - 2x^2$

$2x^2 - 4x - 6 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3 \qquad\qquad x = -1$

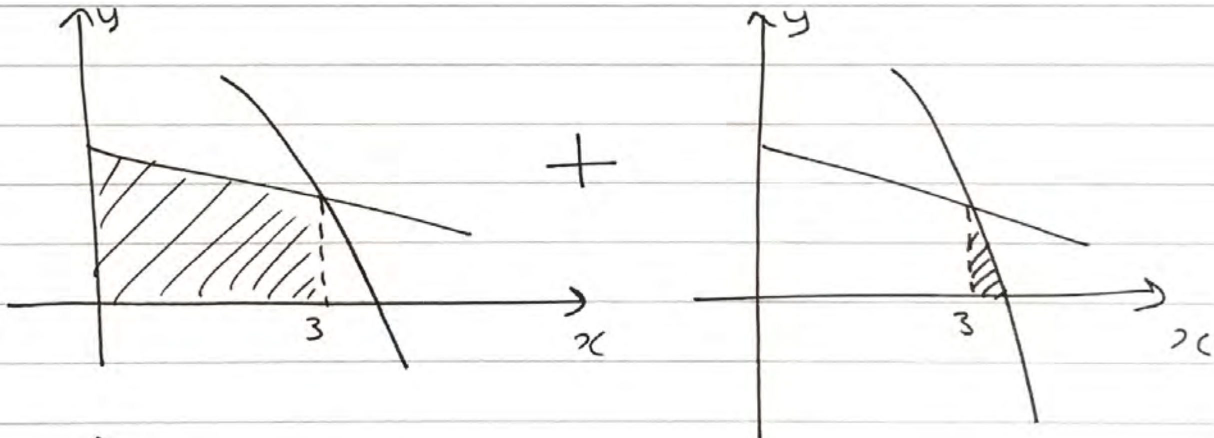
$\downarrow \qquad\qquad \downarrow$

$y = 8 - 3 = 5 \qquad\qquad y = 8 - -1 = 9$

so $\boxed{B(3,5)}$ so $\boxed{A(-1,9)}$

b)

R =



$14 + 3x - 2x^2 = 0 \longrightarrow x = 3.5 \quad , \quad x = \frac{2}{7}$

Trapezium Area 1 $= \frac{(8+5)(3)}{2} = \frac{39}{2}$ //

Area 2 $= \int_{3}^{3.5} \left[14 + 3x - 2x^2\right] dx$

$= \left[14x + \frac{3}{2}x^2 - \frac{2x^3}{3}\right]_{3}^{3.5}$

$= \left[\frac{931}{24}\right] - \left[\frac{75}{2}\right] = \frac{31}{24}$ //

$\therefore R = \frac{39}{2} + \frac{31}{24} = \boxed{\frac{499}{24}}$

Q15a) $(1+kx)^n \simeq 1 + nkx + \frac{n(n-1)}{2}k^2x^2$

$= 1 + 36x + 126kx^2$

so... $36 = nk$ and $\frac{n(n-1)k^2}{2} = 126k$

$\therefore \frac{n(n-1)k}{2} = 126$

hence $nk(n-1) = 252$
(×2)

b) $nk(n-1) = 252$

but $nk = 36$

$\therefore 36(n-1) = 252$

$n - 1 = \frac{252}{36}$

$n - 1 = 7 \quad \therefore \boxed{n = 8}$

and $k = \frac{36}{n} = \frac{36}{8} = \boxed{\frac{9}{2}}$

c) x^3 term $= \frac{n(n-1)(n-2)}{6}(k^3x^3)$

so coefficient $= \frac{8(8-1)(8-2)}{6} \times \left(\frac{9}{2}\right)^3 = \boxed{5103}$