C1 June 2018 (MA)

Q1i) $\sqrt{48} - \frac{6}{\sqrt{3}} = \sqrt{16\times 3} - \frac{6\sqrt{3}}{3} = 4\sqrt{3} - 2\sqrt{3}$

$= \boxed{2\sqrt{3}}$

ii) $3^{6x-3} = 81$

$3^{6x-3} = 3^4$

$\therefore \quad 6x - 3 = 4$

$6x = 7$

$\boxed{x = \frac{7}{6}}$

Q2a) $\int [3x^{\frac{1}{2}} - 6x + 4]\,dx = \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{6x^2}{2} + 4x + c$

$= \boxed{2x^{3/2} - 3x^2 + 4x + c}$

bi) $y = 3x^{\frac{1}{2}} - 6x + 4$

$\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} - 6$

ii) $\frac{3}{2}x^{-\frac{1}{2}} - 6 = 0$

$\frac{3}{2\sqrt{x}} = 6 \quad \therefore \quad \frac{2\sqrt{x}}{3} = \frac{1}{6}$

so $\sqrt{x} = \frac{3}{2} \times \frac{1}{6} = \frac{3}{12} = \frac{1}{4}$ //

$\sqrt{x} = \frac{1}{4}$

$\boxed{x = \frac{1}{16}}$

(Q3a) $f(x) = x^2 - 10x + 23$

$= (x-5)^2 - 25 + 23$

$= \boxed{(x-5)^2 - 2}$ $a = -5$, $b = -2$

b) $(x-5)^2 - 2 = 0$

$(x-5)^2 = 2$

$x - 5 = \pm\sqrt{2}$

$x = 5 \pm \sqrt{2}$ so $\boxed{\begin{matrix} x = 5+\sqrt{2} \\ x = 5-\sqrt{2} \end{matrix}}$

c) let $x = y^{\frac{1}{2}}$,

then the eqn in (c) is identical to (b).

so . . . $y^{\frac{1}{2}} = 5 + \sqrt{2}$

$y = (5+\sqrt{2})^2 = 25 + 10\sqrt{2} + 2$

$= \boxed{27 + 10\sqrt{2}}$ //

Q4a)

year 1	year 2	year 3
600	720	840
a	$a+d$	$a+2d$

$a = 600$
$d = 120$

10^{th} term $= a + (10-1)d = 600 + 120(9)$
$= \boxed{£1680}$

b)

year 1	year 2	year 3
130	210	290
a	$a+d$	$a+2d$

$a = 130$
$d = 80$

For Kim: $S_N = \frac{N}{2}\left[2a + (n-1)d\right]$

$= \frac{N}{2}\left[260 + (N-1)80\right]$

For Andy: $S_N = \frac{N}{2}\left[1200 + (N-1)120\right]$

we are told... $\cancel{\frac{N}{2}}\left[1200 + (N-1)120\right] = 2 \times \cancel{\frac{N}{2}}\left[260 + (N-1)80\right]$

$\Rightarrow 1200 + 120N - 120 = 520 + 160N - 160$

$\Rightarrow 40N = 720$

$\therefore \boxed{N = 18}$

Q5a) + 2 in the x-direction $\longrightarrow$ $\boxed{(4,7)}$

b) $\times \frac{1}{2}$ to all x-values $\longrightarrow$ $\boxed{\left(\frac{5}{2}, 6\right)}$

c) $\boxed{y=1}$ (only x-values change).

d) $\boxed{y=7}$ (at highest point).

and $\boxed{y \leq 1}$

Q6a) $a_2 = \dfrac{4}{4+1} = \boxed{\dfrac{4}{5}}$

$a_3 = \dfrac{\frac{4}{5}}{\frac{4}{5}+1} = \dfrac{\frac{4}{5}}{\frac{9}{5}} = \boxed{\dfrac{4}{9}}$

$a_4 = \dfrac{\frac{4}{9}}{\frac{4}{9}+1} = \dfrac{\frac{4}{9}}{\frac{13}{9}} = \boxed{\dfrac{4}{13}}$

b)

$$\begin{array}{rl} 2p + q = 5 & \longrightarrow (a_2) \\ -\left[3p + q = 9\right] & \longrightarrow (a_3) \\ \hline -p + 0 = -4 & \end{array}$$

$\boxed{p=4}$ so $q = 5 - 2(4) = \boxed{-3}$

c) $a_N = \frac{4}{4N-3} = \frac{4}{321}$

$\therefore \quad 4N - 3 = 321$

$4N = 324$

$N = \frac{324}{4} = \frac{162}{2} = \boxed{81}$

Q7a) $20x^2 = 4kx - 13kx^2 + 2$

$20x^2 + 13kx^2 - 4kx - 2 = 0$

$(20 + 13k)x^2 + (-4k)x + (-2) = 0$

No real roots $\longrightarrow b^2 - 4ac < 0$

$a = 20 + 13k$
$b = -4k$
$c = -2$

$b^2 - 4ac < 0$

$(-4k)^2 - 4(20 + 13k)(-2) < 0$

$16k^2 + 8(20 + 13k) < 0$

$16k^2 + 160 + 104k < 0$

$\underline{\div 8}$: $2k^2 + 20 + 13k < 0$

$2k^2 + 13k + 20 < 0$

b) $2u^2 + 13u + 20 = 0$

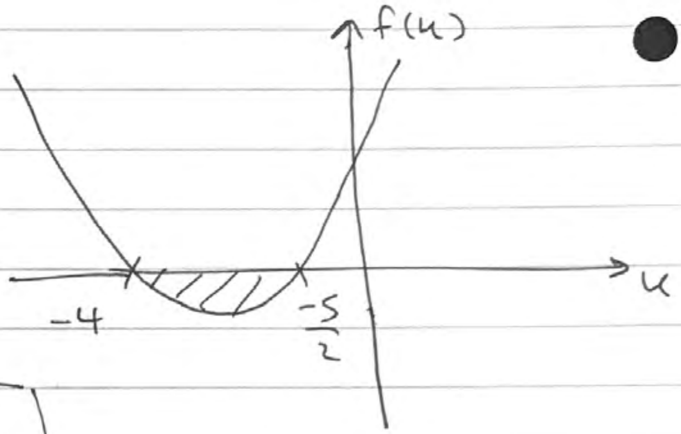
$$\left.\begin{array}{l} a = 2 \\ b = 13 \\ c = 20 \end{array}\right\} \quad u = \frac{-13 \pm \sqrt{169 - 4(2)(20)}}{4} = \frac{-13 \pm \sqrt{9}}{4}$$

$$u = \frac{-13 \pm 3}{4} \quad \text{so} \quad u = -\frac{5}{2}$$

$$u = -4$$

we want where $2u^2 + 13u + 20 < 0$

ie where y-values < 0



so $\boxed{-4 < u < -\frac{5}{2}}$

Q8a) $\div 4$: $y = \frac{5}{4}x + 3$

l_1 gradient $= \boxed{\frac{5}{4}}$

b) $y - 5 = \frac{5}{4}(x - 12)$

$y = \frac{5}{4}x - 15 + 5$

$\boxed{y = \frac{5}{4}x - 10}$

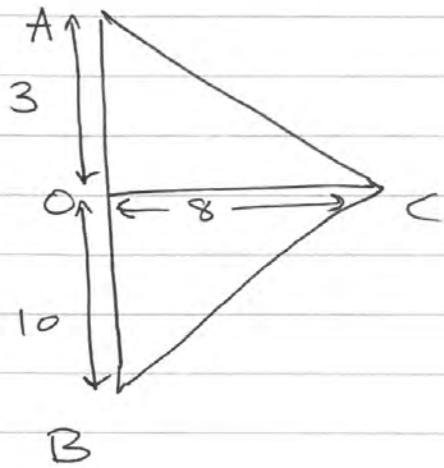
c) i) $\underline{x=0}$: $y = -10$

/ii)

$\underline{y=0}$: $\frac{5}{4}x = 10$ $\therefore x = \frac{10 \times 4}{5} = \frac{40}{5} = 8$

so $\boxed{B(0, -10)}$ $\boxed{C(8, 0)}$

d) consider ΔABC.



Area OAC $= \frac{1}{2}(8)(3) = 12$ //

Area OBC $= \frac{1}{2}(10)(8) = 40$ //

So Area $\Delta ABC = 52$ //

$\therefore$ Area parallelogram $= 2 \times 52$

$= \boxed{104 \text{ units}^2}$

Q9a) $f'(x) = (x-3)(3x+5) = 3x^2 + 5x - 9x - 15$

$= 3x^2 - 4x - 15$

$\therefore f(x) = \int f'(x)\,dx = \int (3x^2 - 4x - 15)\,dx$

$= \frac{3x^3}{3} - \frac{4x^2}{2} - 15x + c$

$= x^3 - 2x^2 - 15x + c = f(x)$ //

at $x=1$, $y=20$: $20 = 1^3 - 2(1)^2 - 15(1) + c$

$20 + 16 = c = 36$

so $f(x) = \boxed{x^3 - 2x^2 - 15x + 36}$

b) $(x-3)^2(x+A) \longrightarrow (x-3)^2(x+4)$.

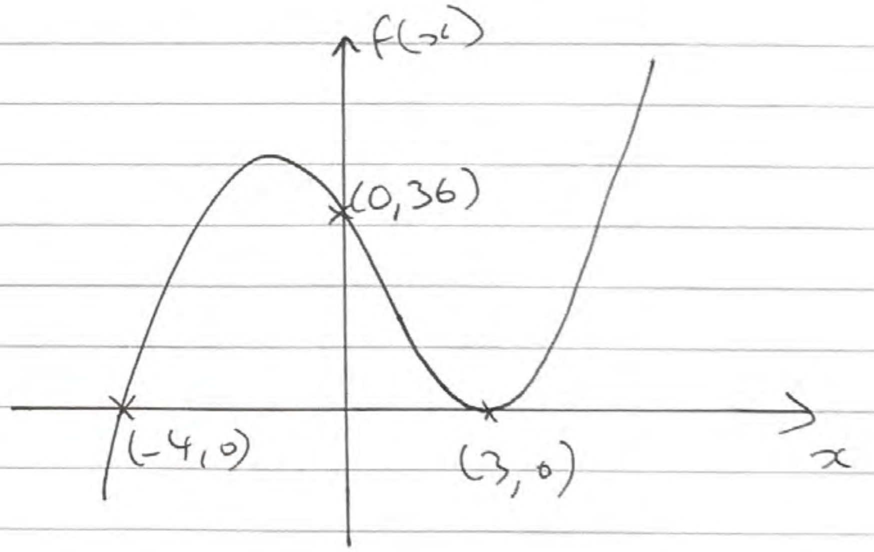
$A = 4$ $(36 = 3^2 \times A)$

$(x-3)^2(x+4) = (x^2 - 6x + 9)(x+4)$

$= (x^3 + 4x^2 - 6x^2 - 24x + 9x + 36)$

$= (x^3 - 2x^2 - 15x + 36) = f(x)$

c)



Q10a) $y = \frac{1}{2}x + 27x^{-1} - 12$

$\frac{dy}{dx} = \frac{1}{2} - 27x^{-2}$

at A, $\frac{dy}{dx} = \frac{1}{2} - 27(3^{-2}) = \frac{1}{2} - \frac{27}{9}$

$= \frac{1}{2} - 3 = -\frac{5}{2}$ //

so at normal, $m = \frac{2}{5}$ // $\left(\frac{2}{5} \times -\frac{5}{2} = -1\right)$

$\therefore \; y - -\frac{3}{2} = \frac{2}{5}(x - 3)$

$y + \frac{3}{2} = \frac{2}{5}x - \frac{6}{5}$

$y = \frac{2}{5}x - \frac{6}{5} - \frac{3}{2}$

$y = \frac{2}{5}x - \frac{27}{10}$

$\times 10$: $\boxed{10y = 4x - 27}$

b) $y = \frac{4x}{10} - \frac{27}{10} = \frac{1}{2}x + \frac{27}{x} - 12$

$\times 10x$: $4x^2 - 27x = 5x^2 + 270 - 120x$

$x^2 - 93x + 270 = 0$ //

$(x-90)(x-3) = 0$

$x = 90$ //

$x = 3 \longrightarrow$ this is A

$y = \frac{4(90)}{10} - \frac{27}{10} = 33.3$

so $\boxed{B\ (90,\ 33.3)}$