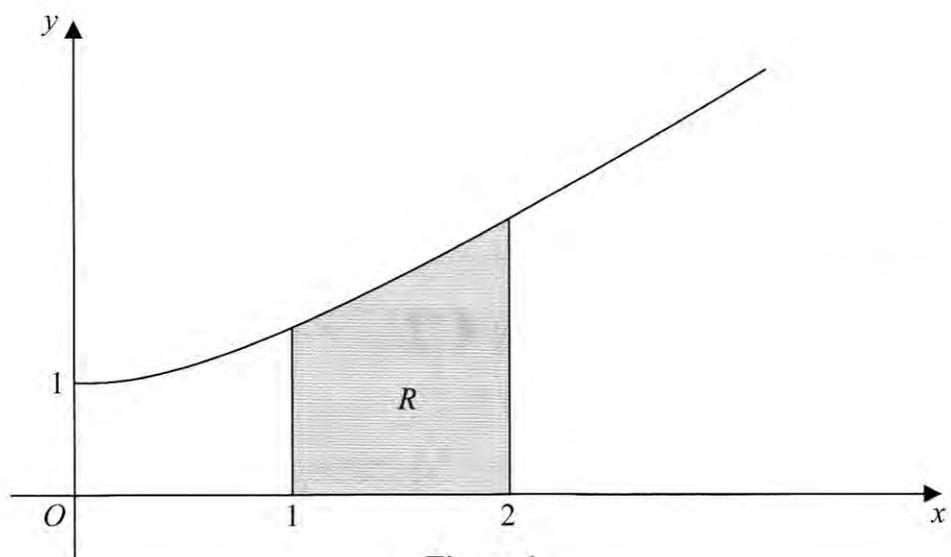C2 S14

1.

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \sqrt{(x^2 + 1)}$, $x \geqslant 0$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$

The table below shows corresponding values for x and y for $y = \sqrt{(x^2 + 1)}$.

$\longrightarrow h = 0.25$

x	1	1.25	1.5	1.75	2
y	1.414	1.601	1.803	2.016	2.236

(a) Complete the table above, giving the missing value of y to 3 decimal places.

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to find an approximate value for the area of R, giving your answer to 2 decimal places.

(4)

$$\text{Area} \approx \tfrac{1}{2}(0.25)\left[1.414 + 2(1.601 + 1.803 + 2.016) + 2.236\right]$$

$$\approx 1.81125 \qquad 1.81 \; (2dp)$$

2. $f(x) = 2x^3 - 7x^2 + 4x + 4$

(a) Use the factor theorem to show that $(x - 2)$ is a factor of f(x).

(2)

(b) Factorise f(x) completely.

(4)

a) $F(2) = 2(8) - 7(4) + 4(2) + 4 = 16 - 28 + 8 + 4 = 0$

∴ $(x-2)$ is a factor   [NOTE - YOU NEED THIS!]

b)

×	$2x^2$	$-3x$	-2
x	$2x^3$	$-3x^2$	$-2x$
-2	$-4x^2$	$+6x$	$+4$

$r = 0$

$(x-2)(x-2)(2x+1)$

3. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 - 3x)^6$$

giving each term in its simplest form.

(4)

(b) Hence, or otherwise, find the first 3 terms, in ascending powers of x, of the expansion of

$$\left(1 + \frac{x}{2}\right)(2 - 3x)^6$$

(3)

$(a-b)^6 = a^6 - 6a^5b + 15a^4b^2$

$(2-3x)^6 = 2^6 - 6(2)^5(3x) + 15(2)^4(3x)^2$

$(2-3x)^6 = 64 - 576x + 2160x^2$

```
        1 1
       1 2 1
      1 3 3 1
     1 4 6 4 1
   1 5 10 10 5 1
 1 6 15 20 15 6 1
```

b)

x	64	-576x	+2160x²
1	64	-576x	2160x²
+ $\frac{1}{2}x$	32x	-288x²	1080x³

$\cancel{\times}$ +

$$64 - 544x + 1872x^2$$

4. Use integration to find

$$\int_1^{\sqrt{3}} \left(\frac{x^3}{6} + \frac{1}{3x^2} \right) dx$$

giving your answer in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

(5)

$$\int_1^{\sqrt{3}} \frac{1}{6}x^3 + \frac{1}{3}x^{-2}\, dx = \left[\frac{1}{24}x^4 - \frac{1}{9}x^{-1} \right]_1^{\sqrt{3}}$$

$$= \left(\frac{3}{8} - \frac{1}{9}\sqrt{3} \right) - \left(\frac{1}{24} - \frac{1}{3} \right) = \frac{2}{3} - \frac{1}{9}\sqrt{3}$$
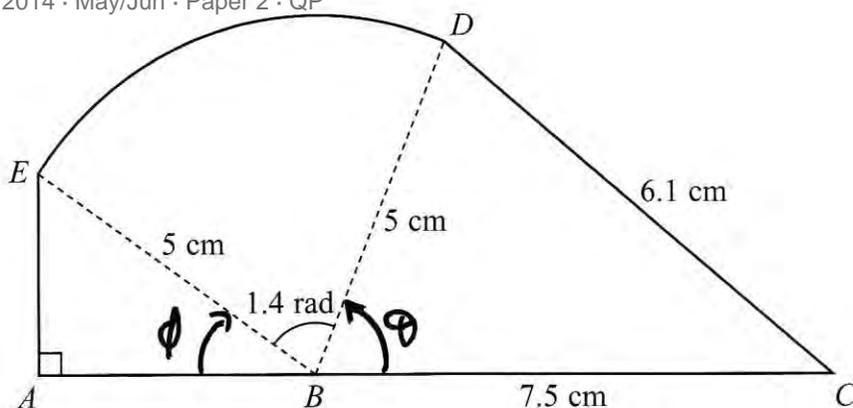
Figure 2

The shape $ABCDEA$, as shown in Figure 2, consists of a right-angled triangle EAB and a triangle DBC joined to a sector BDE of a circle with radius 5 cm and centre B.

The points A, B and C lie on a straight line with $BC = 7.5$ cm.

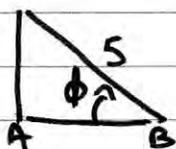Angle $EAB = \dfrac{\pi}{2}$ radians, angle $EBD = 1.4$ radians and $CD = 6.1$ cm.

(a) Find, in cm², the area of the sector BDE.

(2)

(b) Find the size of the angle DBC, giving your answer in radians to 3 decimal places.

(2)

(c) Find, in cm², the area of the shape $ABCDEA$, giving your answer to 3 significant figures.

(5)

a) $\text{area} = \frac{1}{2}(5)^2(1.4) = \underline{17.5 \text{ cm}^2}$

b) $\cos\theta = \dfrac{5^2 + 7.5^2 - 6.1^2}{2 \times 5 \times 7.5} \Rightarrow \theta = \underline{0.943^c}$

c)

$\text{area} = \frac{1}{2}(5)(7.5)\sin 0.943...$

$\text{area} = 15.177...$

$\phi = \pi - 1.4 - 0.943... = 0.79839...^c$

$AB = 5\cos 0.798...$

$AB = 3.489...$

$\therefore$ Total Area
$= \underline{38.9 \text{ cm}^2}$

$\text{area} = \frac{1}{2}(5)(3.489)\sin 0.798 = 6.2478...$

6. The first term of a geometric series is 20 and the common ratio is $\dfrac{7}{8}$

The sum to infinity of the series is S_∞

(a) Find the value of S_∞

(2)

The sum to N terms of the series is S_N

(b) Find, to 1 decimal place, the value of S_{12}

(2)

(c) Find the smallest value of N, for which

$$S_\infty - S_N < 0.5$$

(4)

a) $\quad S_\infty = \dfrac{a}{1-r} = \dfrac{20}{\left(\frac{1}{8}\right)} = \underline{160}$

b) $\quad S_n = \dfrac{a(1-r^n)}{1-r} = \dfrac{20\left(1-\left(\frac{7}{8}\right)^{12}\right)}{\frac{1}{8}} = \underline{127.7}$

c) $\quad 160 - S_N < 0.5 \quad \Rightarrow \quad S_N > 159.5$

$= \dfrac{20\left(1-\left(\frac{7}{8}\right)^N\right)}{\frac{1}{8}} > 159.5 \quad \Rightarrow \quad 1-\left(\frac{7}{8}\right)^N > 0.9968750$

$\Rightarrow \left(\frac{7}{8}\right)^N < \dfrac{1}{320} \quad \Rightarrow \quad \log\left(\frac{7}{8}\right)^N < \log\left(\dfrac{1}{320}\right)$

$\Rightarrow N \times \log\left(\frac{7}{8}\right) < \log\left(\dfrac{1}{320}\right) \quad \Rightarrow \quad N > \dfrac{\log\left(\frac{1}{320}\right)}{\log\left(\frac{7}{8}\right)}$

$\therefore N > 43.198... \quad \therefore \underline{N = 44}$

7. (i) Solve, for $0 \leqslant \theta < 360°$, the equation

$$9\sin(\theta + 60°) = 4$$

giving your answers to 1 decimal place.
You must show each step of your working.

(4)

(ii) Solve, for $-\pi \leqslant x < \pi$, the equation

$$2\tan x - 3\sin x = 0$$

giving your answers to 2 decimal places where appropriate.
[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(5)

a) $\sin(\theta + 60) = \frac{4}{9} \Rightarrow \theta + 60 = \sin^{-1}\left(\frac{4}{9}\right) = 23.39$

i)

$\theta + 60 = 23.39 , \quad 153.61 , \quad 386.39$

$\qquad\qquad\qquad 180 - \qquad\quad +360$

$\boxed{-60} \quad \therefore \theta = \quad \underset{z}{93.6} , \underset{z}{326.4}$

ii) $\quad 2\dfrac{\sin x}{\cos x} - 3\sin x = 0 \quad \boxed{\times \cos x} \quad 2\sin x - 3\sin x \cos x = 0$

$\therefore \sin x (2 - 3\cos x) = 0$

$\Rightarrow \sin x = 0 \qquad \cos x = \frac{2}{3} \quad \therefore x = \underset{z}{0.84^c} , \underset{z}{-0.84^c}$

$x = \underset{z}{-\pi}, \underset{z}{0}$

8. (a) Sketch the graph of

$$y = 3^x, \quad x \in \mathbb{R}$$

showing the coordinates of any points at which the graph crosses the axes.
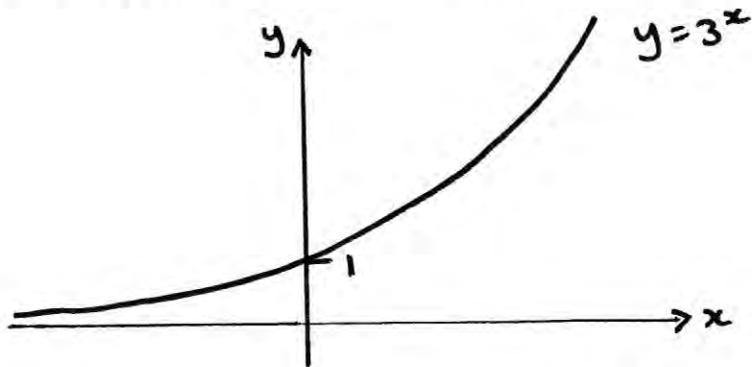
(2)

(b) Use algebra to solve the equation

$$3^{2x} - 9(3^x) + 18 = 0$$

giving your answers to 2 decimal places where appropriate.

(5)

b) $(3^x)^2 - 9(3^x) + 18 = 0 \Rightarrow (3^x - 6)(3^x - 3) = 0$

$\therefore 3^x = 3 \Rightarrow \underline{x = 1}$ $3^x = 6 \Rightarrow x = \log_3 6 = \underline{1.63}$
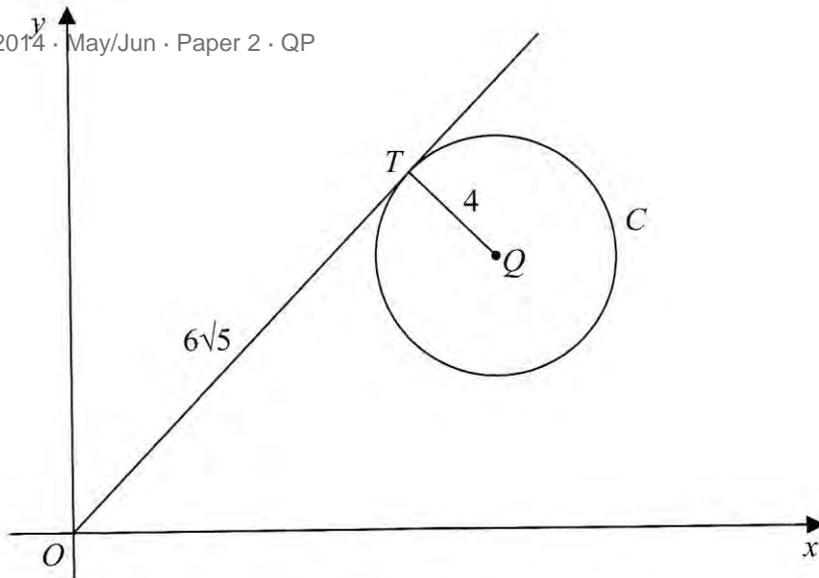
Figure 3

Figure 3 shows a circle C with centre Q and radius 4 and the point T which lies on C.

The tangent to C at the point T passes through the origin O and $OT = 6\sqrt{5}$

Given that the coordinates of Q are $(11, k)$, where k is a positive constant,

(a) find the exact value of k,

(3)

(b) find an equation for C.

(2)

a)

$OQ^2 = 4^2 + (6\sqrt{5})^2 = 196$ $\therefore OQ = 14$

$\Rightarrow k^2 = 14^2 - 11^2 \Rightarrow k^2 = 75$

$\therefore k = 5\sqrt{3}$

b) $(x - 11)^2 + (y - 5\sqrt{3})^2 = 16$

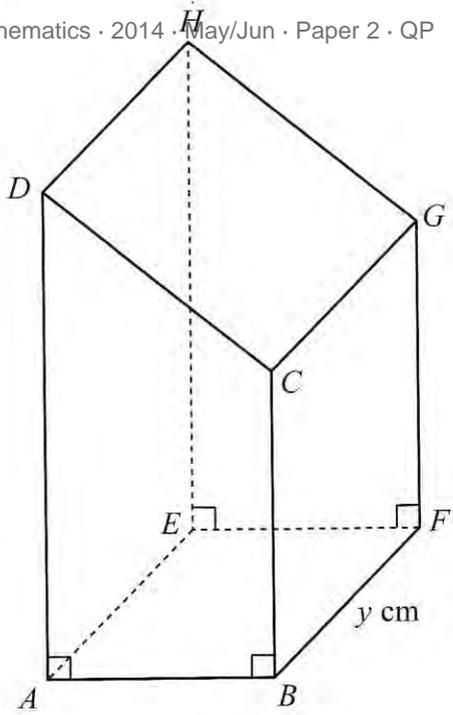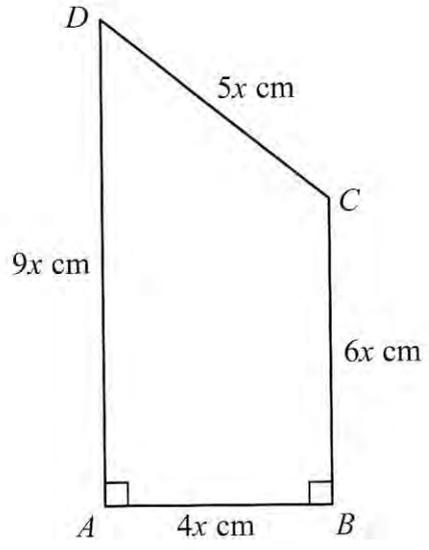**Figure 4**

Figure 5

Figure 4 shows a closed letter box *ABFEHGCD*, which is made to be attached to a wall of a house.

The letter box is a right prism of length y cm as shown in Figure 4. The base *ABFE* of the prism is a rectangle. The total surface area of the six faces of the prism is S cm^2.

The cross section *ABCD* of the letter box is a trapezium with edges of lengths $DA = 9x$ cm, $AB = 4x$ cm, $BC = 6x$ cm and $CD = 5x$ cm as shown in Figure 5.
The angle $DAB = 90°$ and the angle $ABC = 90°$.

The volume of the letter box is 9600 cm^3.

(a) Show that

$$y = \frac{320}{x^2}$$

(2)

(b) Hence show that the surface area of the letter box, S cm^2, is given by

$$S = 60x^2 + \frac{7680}{x}$$

(4)

(c) Use calculus to find the minimum value of S.

(6)

(d) Justify, by further differentiation, that the value of S you have found is a minimum.

(2)

a) Area of trapezium $= \dfrac{(9x+6x) \times 4x}{2} = 15x \times 2x = 30x^2$

Volume $= 30x^2 \times y = 9600$ $\therefore y = \dfrac{9600}{30x^2} = \dfrac{320}{x^2}$ #

b) SA $= 2 \times (30x^2) + 6xy + 5xy + 9xy + 4xy$

SA $= 60x^2 + 24xy = 60x^2 + 24x\left(\dfrac{320}{x^2}\right) = 60x^2 + \dfrac{7680}{x}$ #

c) $S = 60x^2 + 7680x^{-1}$ Min S when $S' = 0$

$S' = 120x - 7680x^{-2}$ $\Rightarrow 120x = \dfrac{7680}{x^2}$ $\Rightarrow x^3 = 64$

$S'' = 120 + 15360x^{-3}$ $\therefore x = 4$ cm

$\therefore$ Min S $= 60(4)^2 + \dfrac{7680}{4} = \dfrac{2880 \text{cm}^2}{2}$

d) when $x = 4$ $S'' = 120 + \dfrac{15360}{4^3} = 360$ $\therefore S'' > 0$ $\cup$

$\therefore$ S is at a maximum when $x = \dfrac{4}{2}$